## Connected Rates of Change 2 - Edexcel Past Exam Questions MARK SCHEME

### Question 1

| Question Number | Scheme | Marks |
|---|---|---|
| | (a) $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ * cso | B1 (1) |
| | (b) $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{0.048}{3x^2}$ | M1 |
| | At $x = 8$ $\frac{dx}{dt} = \frac{0.048}{3(8^2)} = 0.00025 \ (\text{cm s}^{-1})$ $2.5 \times 10^{-4}$ | A1 (2) |
| | (c) $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$ | B1 |
| | $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 12x\left(\frac{0.048}{3x^2}\right)$ | M1 |
| | At $x = 8$ $\frac{dS}{dt} = 0.024 \ (\text{cm}^2 \text{s}^{-1})$ | A1 (3) |
| | | [6] |

## Question 2

| Question Number | Scheme | Marks |
|---|---|---|
| | $\frac{dV}{dt} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$, | |
| | $\frac{dV}{dh} = 8\pi h + 16\pi$ $\quad \pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $\quad 8\pi h + 16\pi$ | M1 A1 |
| | $\left\{\frac{dV}{dh}\times\frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ (8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\quad \left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$ | M1 oe |
| | $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}\ \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$ $\quad$ or $\quad 80\pi \div \text{Candidate's } \frac{dV}{dh}$ | |
| | When $h = 6$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi \left\{= \frac{80\pi}{64\pi}\right\}$ $\quad$ **dependent on the previous M1 see notes** | dM1 |
| | $\frac{dh}{dt} = \underline{1.25}\ (\text{cms}^{-1})$ $\quad 1.25$ or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ | A1 oe |
| | | [5] 5 |
| | **Alternative Method for the first M1A1** | |
| | Product rule: $\begin{cases} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{cases}$ | |
| | $\frac{dV}{dh} = 4\pi(h+4) + 4\pi h$ $\quad \pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$ $\quad 4\pi(h+4) + 4\pi h$ | M1 A1 |

**Question Notes**

| | |
|---|---|
| **M1** | An expression of the form $\pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$. **Can be simplified or un-simplified.** |
| **A1** | Correct simplified or un-simplified differentiation of $V$.<br>eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent. |
| **Note** | Some candidates will use the product rule to differentiate $V$ with respect to $h$. **(See Alt Method 1).** |
| **Note** | $\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their $V$. |
| **M1** | $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$ |
| **Note** | Also allow 2nd M1 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80$ or $80 \div \text{Candidate's } \frac{dV}{dh}$ |
| **Note** | Give 2nd M0 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi\text{t}$ or $80\text{k}$ or $80\pi\text{t}$ or $80\text{k} \div \text{Candidate's } \frac{dV}{dh}$ |
| **dM1** | **which is dependent on the previous M1 mark.**<br>Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and $80\pi$ (or 80) |
| **A1** | 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required). |
| **Note** | $\frac{80\pi}{64\pi}$ as a final answer is A0. |
| **Note** | Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives $64\pi$ but the final M1 mark can only be awarded if this is used as a quotient with $80\pi$ (or 80) |

## Question 3

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | From question, $V=\frac{4}{3}\pi r^3$, $S=4\pi r^2$, $\frac{dV}{dt}=3$ | |
| | $\left\{V=\frac{4}{3}\pi r^3 \Rightarrow\right\}\ \frac{dV}{dr}=4\pi r^2$ $\qquad$ $\frac{dV}{dr}=4\pi r^2$ (Can be implied) | B1 oe |
| | $\left\{\frac{dV}{dr}\times\frac{dr}{dt}=\frac{dV}{dt}\Rightarrow\right\}\ (4\pi r^2)\frac{dr}{dt}=3$ $\qquad$ $\left(\text{Candidate's }\frac{dV}{dr}\right)\times\frac{dr}{dt}=3$ | M1 oe |
| | $\left\{\frac{dr}{dt}=\frac{dV}{dt}\div\frac{dV}{dr}\Rightarrow\right\}\ \frac{dr}{dt}=(3)\frac{1}{4\pi r^2};\left\{=\frac{3}{4\pi r^2}\right\}$ $\qquad$ or $3\div\text{Candidate's }\frac{dV}{dr}$; | |
| | When $r=4\text{cm}$, $\frac{dr}{dt}=\frac{3}{4\pi(4)^2}\ \left\{=\frac{3}{64\pi}\right\}$ $\qquad$ **dependent on previous M1. see notes** | dM1 |
| | Hence, $\frac{dr}{dt}=0.01492077591...(\text{cm}^2\,\text{s}^{-1})$ $\qquad$ anything that rounds to 0.0149 | A1 |
| | | **[4]** |
| (b) | $\left\{\frac{dS}{dt}=\frac{dS}{dr}\times\frac{dr}{dt}=\right\}\Rightarrow\frac{dS}{dt}=8\pi r\times\frac{3}{4\pi r^2}\ \left\{\text{or }\frac{6}{r}\text{ or }8\pi r\times0.0149...\right\}$ $\qquad$ $8\pi r\times\text{Candidate's }\frac{dr}{dt}$ | M1; oe |
| | When $r=4\text{cm}$, $\frac{dr}{dt}=8\pi(4)\times\frac{3}{4\pi(4)^2}$ or $\frac{6}{4}$ or $8\pi(4)\times0.0149...$ | |
| | Hence, $\frac{dS}{dt}=1.5\ (\text{cm}^2\,\text{s}^{-1})$ $\qquad$ anything that rounds to 1.5 | A1 cso |
| | | **[2]** |
| | | **6** |

**Question Notes**

(a)

**B1** $\frac{dV}{dr}=4\pi r^2$ Can be implied by later working.

**M1** $\left(\text{Candidate's }\frac{dV}{dr}\right)\times\frac{dr}{dt}=3$ or $3\div\text{Candidate's }\frac{dV}{dr}$

**dM1** **(dependent on the previous method mark)**

Substitutes $r=4$ into an expression which is a result of a quotient of "3" and their $\frac{dV}{dr}$.

**A1** anything that rounds to 0.0149 (units are not required)

(b)

**M1** $8\pi r\times\text{Candidate's }\frac{dr}{dt}$

**A1** anything that rounds to 1.5 (units are not required). **Correct solution only.**

**Note** Using $\frac{dr}{dt}=0.0149$ gives $\frac{dS}{dt}=1.4979...$ which is fine for A1.