Differentiating Functions & Expressions 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question No	Scheme	Marks
	(a)	
	$\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B	M1
	Applying vu'+uv' , $\ln(3x) \times 2x + x$	M1, A1 A1
		(4)
	(b)	
	Applying $\frac{vu' - uv'}{v^2}$	
	$\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 $\underline{\text{A1+A1}}$ A1
	$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	A1
		(5)
		(9 MARKS)

Question 2

Question No	Scheme	Marks
	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y + \frac{\pi}{12}\right)$	M1,A1
	substitute $y = \frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = 8$	M1, A1
	When $y = \frac{\pi}{4}$, $x = 2\sqrt{3}$ awrt 3.46	B1
	$\left(y - \frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$ oe	A1 (7 marks)

Question 3

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\tan 3x = -\sqrt{3}$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
		(6)
	(b) At x=0 $\quad \frac{dy}{dx} = 3$	B1
	Equation of normal is $-\frac{1}{3} = \frac{y-0}{x-0}$ or any equivalent $y = -\frac{1}{3}x$	M1A1
		(3)
		(9 marks)

Alternative in part (a) using the form $R\sin(3x+\alpha)$ JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$(\sqrt{12})\sin(3x + \frac{\pi}{3}) = 0$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
		(6)

Alternative to part (a) squaring both sides JUST LAST 3 MARKS

Question Number	Scheme	Marks
3.	(a) $\frac{dy}{dx} = \sqrt{3}e^{x\sqrt{3}}\sin 3x + 3e^{x\sqrt{3}}\cos 3x$	M1A1
	$\frac{dy}{dx} = 0$ $\quad e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\sqrt{3}\sin 3x = -3\cos 3x \Rightarrow \cos^2(3x) = \frac{1}{4} \text{ or } \sin^2(3x) = \frac{3}{4}$	A1
	$x = \frac{1}{3}\arccos(\pm\sqrt{\frac{1}{4}})$ oe	M1
	$x = \frac{2\pi}{9}$	A1

Question 4

Question Number	Scheme	Marks
	(a)(i) $\frac{d}{dx}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{d}{dx}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x) \times \frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}} \times \frac{3}{3x}$	M1A1
		(3)
	(ii)	
	$\frac{dy}{dx} = \frac{(2x-1)^5 \times -10 - (1-10x) \times 5(2x-1)^4 \times 2}{(2x-1)^{10}}$	M1A1
	$\frac{dy}{dx} = \frac{80x}{(2x-1)^6}$	A1
		(3)
	(b) $x = 3\tan 2y \quad \Rightarrow \frac{dx}{dy} = 6\sec^2 2y$	M1A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1 + \tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$	
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1A1
		(5) **(11 marks)**

Question 5

Question Number	Scheme	Marks
	(a) $-32 = (2w-3)^5 \Rightarrow w = \frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx} = 5\times(2x-3)^4 \times 2$ or $10(2x-3)^4$	M1A1
	When $x = \frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160' = \dfrac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y = 160x - 112$ cso	A1
		(5)
		(7 marks)

Question 6

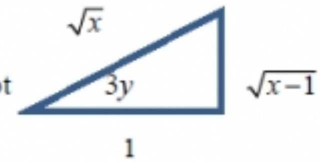
Question Number	Scheme	Marks
	(i)(a) $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$	M1A1A1
	$= 3x^2 \ln 2x + x^2$	
		(3)
	(i)(b) $\frac{dy}{dx} = 3(\boldsymbol{x} + \mathbf{sin2}\boldsymbol{x})^2 \times (1 + 2\cos 2x)$	**B1**M1A1
		(3)
	(ii) $\frac{dx}{dy} = -\text{cosec}^2 y$	M1A1
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y} = -\frac{1}{1+\cot^2 y} = -\frac{1}{1+x^2}$ cso	M1, A1*
		(5)
		(11 marks)

Question 7

Question Number	Scheme	Marks
	(a) $\dfrac{2}{x+2}+\dfrac{4}{x^2+5}-\dfrac{18}{(x+2)(x^2+5)}=\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\dfrac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\dfrac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $\text{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1A1
	$\text{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \text{h}(x)=\dfrac{\sqrt{5}}{5}$	M1,A1
	Range of $\text{h}(x)$ is $0\le h(x)\le\dfrac{\sqrt{5}}{5}$	A1ft
		(5)
		(12 marks)

Question 8

Question Number	Scheme	Marks
(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1 (2)
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1* (4)
(c)	$\frac{d^2y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
Alt 1 to (a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To (c)	$\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \frac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\frac{1}{2}) + (-1)(x-1)]$	dM1
	$= \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

Notes for Question	
(a)	
M1	Uses the chain rule to get $A\sec 3y \sec 3y \tan 3y = \left(A\sec^2 3y \tan 3y\right)$. There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$ or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y \sin 3y}{(\cos 3y)^4}$
A1	$\dfrac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.
(b)	
M1	Uses $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ to get an expression for $\dfrac{dy}{dx}$. Follow through on their $\dfrac{dx}{dy}$ Allow slips on the coefficient but not trig expression.
B1	Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x-1)^{\frac{1}{2}}$ All elements **must be present.** Accept  $\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$ If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$
M1	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\dfrac{dy}{dx}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$. It may be implied- see below
A1*	CSO. This is a given solution and you must be convinced that all steps are shown.
Note that the two method marks may occur the other way around Eg. $\dfrac{dx}{dy} = 6\sec^2 3y \tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6x(x-1)^{\frac{1}{2}}}$ Scores the 2nd method Scores the 1st method The above solution will score M1, B0, M1, A0	

Notes for Question Continued

Example 1- Scores 0 marks in part (b)

$$\frac{dx}{dy}=6\sec^2 3y\tan 3y \Rightarrow \frac{dy}{dx}=\frac{1}{6\sec^2 3x\tan 3x}=\frac{1}{6\sec^2 3x\sqrt{\sec^2 3x-1}}=\frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{dx}{dy}=2\sec^2 3y\tan 3y \Rightarrow \frac{dy}{dx}=\frac{1}{2\sec^2 3y\tan 3y}=\frac{1}{2\sec^2 3y\sqrt{\sec^2 3y-1}}=\frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u=1$ and $v=6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u'=0$ and $v'=A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\cancel{\left(\frac{d^2y}{dx^2}\right)}=\frac{0-[A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \cancel{\left(\frac{d^2y}{dx^2}\right)}=\frac{0-A(x-1)^{\frac{1}{2}}\pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{d^2y}{dx^2}=\frac{0-[6(x-1)^{\frac{1}{2}}+3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2}=\frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question Continued

(c) Using Product and Chain Rules

M1 Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and

v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct.

If the rules are not quoted nor implied then award if you see an expression of the form

$(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$

A1 $\cancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$

dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} == \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe*

(c) Using Quotient and Chain rules Rules

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ **and** achieving

$u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$.

If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form

$$\cancel{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)} = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$$

A1 Correct un simplified expression $\cancel{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$

dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = \frac{1}{(36x^3-36x^2)^{\frac{1}{2}}} = (36x^3-36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3-36x^2)^{-\frac{3}{2}}(Bx^2-Cx)$.
M1	Would automatically follow under this method if the first M has been scored

Question 9

Question Number	Scheme	Marks
(a)	$\frac{d}{dx}(\cos 2x) = -2\sin 2x$	B1
	Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$ $\quad = \frac{\sqrt{x}\times -2\sin 2x - \cos 2x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2}$	M1A1
	$= \frac{-2\sqrt{x}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$	
		(3)
(b)	$\frac{d}{dx}(\sec^2 3x) = 2\sec 3x \times 3\sec 3x \tan 3x \, (= 6\sec^2 3x \tan 3x)$	M1
	$= 6(1+\tan^2 3x)\tan 3x$	dM1
	$= 6(\tan 3x + \tan^3 3x)$	A1
		(3)
(c)	$x = 2\sin\left(\frac{y}{3}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3}\cos\left(\frac{y}{3}\right)$	M1A1
	$\frac{dy}{dx} = \frac{1}{\frac{2}{3}\cos\left(\frac{y}{3}\right)} = \frac{1}{\frac{2}{3}\sqrt{\left(1-\sin^2\left(\frac{y}{3}\right)\right)}} = \frac{1}{\frac{2}{3}\sqrt{1-\left(\frac{x}{2}\right)^2}}$	dM1
	$\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$ cao	A1
		(4)
		(10 marks)
Alt (c)	$y = 3\arcsin\left(\frac{x}{2}\right) \Rightarrow \frac{dy}{dx} = \frac{3}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \times \frac{1}{2}$	M1dM1A1
	$\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$	A1
	M1 Rearranging to $y = A\arcsin Bx$ and differentiating to $\frac{dy}{dx} = \frac{A}{\sqrt{1-Bx^2}}$	
	dM1 As above, but form of the rhs must be correct $\frac{dy}{dx} = \frac{C}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$	**(4)**
	A1 Correct but un simplified answer	

Notes for Question

(a)

B1 Award for the sight of $\frac{d}{dx}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{\sqrt{x} \times \pm A\sin 2x - \cos 2x \times Bx^{-\frac{1}{2}}}{(\sqrt{x})^2 \text{ or } x^{\frac{1}{4}}}$$

A1 Award for a correct answer. This does not need to be simplified.

Alt (a) using the product rule

B1 Award for the sight of $\frac{d}{dx}(\cos 2x) = -2\sin 2x$. This could be seen in their differential.

M1 Applies $vu'+uv'$ to $x^{-\frac{1}{2}}\cos 2x$. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their $vu'+uv'$) then only accept answers of the form

$$\pm Ax^{-\frac{1}{2}}\sin 2x - Bx^{-\frac{3}{2}}\cos 2x$$

A1 Award for a correct answer. This does not need to be simplified.

$$-2x^{-\frac{1}{2}}\sin 2x - \frac{1}{2}x^{-\frac{3}{2}}\cos 2x$$

(b)

M1 Award for a correct application of the chain rule on $\sec^2 3x$

Sight of $C\sec 3x \sec 3x \tan 3x$ is sufficient

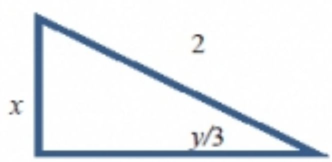
dM1 Replacing $\sec^2 3x = 1 + \tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored.

A1 The correct answer $6(\tan 3x + \tan^3 3x)$. There is no need to write $\mu = 6$

Alt (b) using the product rule

M1 Writes $\sec^2 3x$ as $\sec 3x \times \sec 3x$ and uses the product rule with $u' = A\sec 3x \tan 3x$ and $v' = B\sec 3x \tan 3x$ to produce a derivative of the form $A\sec 3x \tan 3x \sec 3x + B\sec 3x \tan 3x \sec 3x$

dM1 Replaces $\sec^2 3x$ with $1 + \tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored.

Notes for Question Continued
A1 The correct answer $6(\tan 3x+\tan^3 3x)$. There is no need to write $\mu=6$ **Alt (b) using** $\sec 3x=\dfrac{1}{\cos 3x}$ **and proceeding by the chain or quotient rule** M1 Writes $\sec^2 3x$ as $(\cos 3x)^{-2}$ and differentiates to $A(\cos 3x)^{-3}\sin 3x$ Alternatively writes $\sec^2 3x$ as $\dfrac{1}{(\cos 3x)^2}$ and achieves $\dfrac{(\cos 3x)^2\times 0-1\times A\cos 3x\sin 3x}{\left(\cos^2 3x\right)^2}$ dM1 Uses $\dfrac{\sin 3x}{\cos 3x}=\tan 3x$ and $\dfrac{1}{\cos^2 3x}=\sec^2 3x$ and $\sec^2 3x=1+\tan^2 3x$ in their derivative to create an expression in just $\tan 3x$. It is dependent upon the first M being scored. A1 The correct answer $6(\tan 3x+\tan^3 3x)$. There is no need to write $\mu=6$ **Alt (b) using** $\sec^2 3x=1+\tan^2 3x$ M1 Writes $\sec^2 3x$ as $1+\tan^2 3x$ and uses chain rule to produce a derivative of the form $A\tan 3x\sec^2 3x$ or the product rule to produce a derivative of the form $C\tan 3x\sec^2 3x+D\tan 3x\sec^2 3x$ dM1 Replaces $\sec^2 3x=1+\tan^2 3x$ to produce an expression in just $\tan 3x$. It is dependent upon the first M being scored. A1 The correct answer $6(\tan 3x+\tan^3 3x)$. There is no need to write $\mu=6$ (c) M1 Award for knowing the method that $\sin\left(\dfrac{y}{3}\right)$ differentiates to $\cos\left(\dfrac{y}{3}\right)$ The lhs does not need to be correct/present. Award for $2\sin\left(\dfrac{y}{3}\right)\rightarrow A\cos\left(\dfrac{y}{3}\right)$ A1 $x=2\sin\left(\dfrac{y}{3}\right)\Rightarrow\dfrac{dx}{dy}=\dfrac{2}{3}\cos\left(\dfrac{y}{3}\right)$. Both sides must be correct dM1 Award for inverting their $\dfrac{dx}{dy}$ and using $\sin^2\dfrac{y}{3}+\cos^2\dfrac{y}{3}=1$ to produce an expression for $\dfrac{dy}{dx}$ in terms of x only. It is dependent upon the first M 1 being scored. An alternative to Pythagoras is a triangle.  $\sin\left(\dfrac{y}{3}\right)=\dfrac{x}{2}\Rightarrow\cos\left(\dfrac{y}{3}\right)=\dfrac{\sqrt{4-x^2}}{2}$

Notes for Question Continued
Candidates who write $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2\cos\left(\arcsin\left(\frac{x}{2}\right)\right)}$ do not score the mark. BUT $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2\sqrt{1-\sin^2\left(\arcsin\left(\frac{x}{2}\right)\right)}}$ does score M1 as they clearly use a correct Pythagorean identity as required by the notes. A1 $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{\sqrt{4-x^2}}$. Expression must be in its simplest form. Do not accept $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{2\sqrt{1-\frac{1}{4}x^2}}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\frac{1}{3}\sqrt{4-x^2}}$ for the final A1

Question 10

Question Number	Scheme	Marks
(a)	$\text{f}(x)=\dfrac{4x+1}{x-2},\quad x>2$	
	Applies $\dfrac{vu'-uv'}{v^2}$ to get $\dfrac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$	M1A1
	$=\dfrac{-9}{(x-2)^2}$	A1*
		(3)
(b)	$\dfrac{-9}{(x-2)^2}=-1\Rightarrow x=..$	M1
	$(5,7)$	A1,A1
		(3)
		6 marks
Alt 1.(a)	$\text{f}(x)=\dfrac{4x+1}{x-2}=4+\dfrac{9}{x-2}$	
	Applies chain rule to get $\text{f}'(x)=A(x-2)^{-2}$	M1
	$=-9(x-2)^{-2}=\dfrac{-9}{(x-2)^2}$	A1, A1*
		(3)

(a)

M1 Applies the quotient rule to $f(x)=\dfrac{4x+1}{x-2}$ with $u=4x+1$ and $v=x-2$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=x-2, u'=..., v'=..$ followed by $\dfrac{vu'-uv'}{v^2}$.

If neither quoted nor implied only accept expressions of the form $\dfrac{(x-2)\times A-(4x+1)\times B}{(x-2)^2}$ $A, B>0$ allowing for a sign slip inside the brackets.

Condone missing brackets for the method mark but not the final answer mark.

Alternatively they could apply the product rule with $u=4x+1$ and $v=(x-2)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=(x-2)^{-1}, u'=..., v'=..$ followed by $vu'+uv'$.

If it is neither quoted nor implied only accept expressions of the form/ or equivalent to the form

$$(x-2)^{-1}\times C+(4x+1)\times D(x-2)^{-2}$$

A third alternative is to use the Chain rule. For this to score there must have been some attempt to divide first to achieve $f(x)=\dfrac{4x+1}{x-2}=..+\dfrac{..}{x-2}$ before applying the chain rule to get $f'(x)=A(x-2)^{-2}$

A1 A correct and unsimplified form of the answer.

Accept $\dfrac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$ from the quotient rule

Accept $\dfrac{4x-8-4x-1}{(x-2)^2}$ from the quotient rule even if the brackets were missing in line 1

Accept $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}$ or equivalent from the product rule

Accept $9\times -1(x-2)^{-2}$ from the chain rule

A1* Proceeds to achieve the given answer $=\dfrac{-9}{(x-2)^2}$. Accept $-9(x-2)^{-2}$

All aspects must be correct including the bracketing.

If they differentiated using the product rule the intermediate lines must be seen.

Eg. $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}=\dfrac{4}{(x-2)}-\dfrac{4x+1}{(x-2)^2}=\dfrac{4(x-2)-(4x+1)}{(x-2)^2}=\dfrac{-9}{(x-2)^2}$

(b)

M1 Sets $\dfrac{-9}{(x-2)^2}=-1$ and proceeds to $x=....$

The minimum expectation is that they multiply by $(x-2)^2$ and then either, divide by -1 before square rooting or multiply out before solving a 3TQ equation.

A correct answer of $x=5$ would also score this mark following $\dfrac{-9}{(x-2)^2}=-1$ as long as no incorrect work is seen.

A1 $x=5$

A1 (5, 7) or $x=5, y=7$. Ignore any reference to $x=-1$ (and $y=1$). Do not accept 21/3 for 7

If there is an extra solution, $x>2$, then withhold this final mark.

Question 11

Question Number	Scheme	Marks
(a)	$x = 8\dfrac{\pi}{8}\tan\left(2\times\dfrac{\pi}{8}\right) = \pi$	B1*
		(1)
(b)	$\dfrac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2y + 16y\sec^2(2y)$	M1A1A1
	At P $\dfrac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2\dfrac{\pi}{8} + 16\dfrac{\pi}{8}\sec^2\left(2\times\dfrac{\pi}{8}\right) = \{8+4\pi\}$	M1
	$\dfrac{y-\dfrac{\pi}{8}}{x-\pi} = \dfrac{1}{8+4\pi}$, accept $y - \dfrac{\pi}{8} = 0.049(x-\pi)$	M1A1
	$\Rightarrow (8+4\pi)y = x + \dfrac{\pi^2}{2}$	A1
		(7)
		(8 marks)

(a)

B1* Either sub $y = \dfrac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow x = 8\times\dfrac{\pi}{8}\tan\left(2\times\dfrac{\pi}{8}\right) = \pi$

Or sub $x = \pi$, $y = \dfrac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow \pi = 8\times\dfrac{\pi}{8}\tan\left(2\times\dfrac{\pi}{8}\right) = \pi\times 1 = \pi$

This is a proof and therefore an expectation that at least one intermediate line must be seen, including a term in tangent.

Accept as a minimum $y = \dfrac{\pi}{8} \Rightarrow x = \pi\tan\left(\dfrac{\pi}{4}\right) = \pi$

Or $\pi = \pi\times\tan\left(\dfrac{\pi}{4}\right) = \pi$

This is a given answer however, and as such there can be no errors.

(b)

M1 Applies the product rule to $8y\tan 2y$ achieving $A\tan 2y + By\sec^2(2y)$

A1 One term correct. Either $8\tan 2y$ or $+16y\sec^2(2y)$. There is no requirement for $\frac{dx}{dy} =$

A1 Both lhs and rhs correct. $\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$

It is an intermediate line and the expression does not need to be simplified.

Accept $\frac{dx}{dy} = \tan 2y \times 8 + 8y \times 2\sec^2(2y)$ or $\frac{dy}{dx} = \frac{1}{\tan 2y \times 8 + 8y \times 2\sec^2(2y)}$ or using implicit differentiation $1 = \tan 2y \times 8\frac{dy}{dx} + 8y \times 2\sec^2(2y)\frac{dy}{dx}$

M1 For fully substituting $y = \frac{\pi}{8}$ into their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ to find a 'numerical' value

Accept $\frac{dx}{dy} =$ awrt 20.6 or $\frac{dy}{dx} =$ awrt 0.05 as evidence

M1 For a correct attempt at an equation of the tangent at the point $\left(\pi, \frac{\pi}{8}\right)$.

The gradient must be an inverted numerical value of their $\frac{dx}{dy}$

Look for $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{\text{numerical } \frac{dx}{dy}}$.

Watch for negative reciprocals which is M0

If the form $y = mx + c$ is used it must be a full method to find a 'numerical' value to c.

A1 A correct equation of the tangent.

Accept $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{8 + 4\pi}$ or if $y = mx + c$ is used accept $m = \frac{1}{8 + 4\pi}$ and $c = \frac{\pi}{8} - \frac{\pi}{8 + 4\pi}$

Watch for answers like this which are correct $x - \pi = (8 + 4\pi)\left(y - \frac{\pi}{8}\right)$

Accept the decimal answers awrt 2sf $y = 0.049x + 0.24$, awrt 2sf $21y = x + 4.9$, $\frac{y - 0.39}{x - 3.1} = 0.049$

Accept a mixture of decimals and π's for example $20.6\left(y - \frac{\pi}{8}\right) = x - \pi$

A1 Correct answer and solution only. $(8 + 4\pi)y = x + \frac{\pi^2}{2}$

Accept exact alternatives such as $4(2 + \pi)y = x + 0.5\pi^2$ and because the question does not ask for a and b to be simplified in the form $ay = x + b$, accept versions like

$(8 + 4\pi)y = x + \frac{\pi}{8}(8 + 4\pi) - \pi$ and $(8 + 4\pi)y = x + (8 + 4\pi)\left(\frac{\pi}{8} - \frac{\pi}{8 + 4\pi}\right)$

Question 12

Question Number	Scheme	Marks
(i)	$\frac{dx}{dy} = 4\sec^2 2y \tan 2y$	B1
	Use $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$	M1
	Uses $\tan^2 2y = \sec^2 2y - 1$ and $\sec 2y = \sqrt{x}$ to get $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x	M1
	$\frac{dy}{dx} = \frac{1}{4x(x-1)^{\frac{1}{2}}}$ (conclusion stated with no errors previously)	A1*
		(4)
(ii)	$\frac{dy}{dx} = (x^2 + x^3) \times \frac{2}{2x} + (2x + 3x^2)\ln 2x$	M1 A1 A1
	When $x = \frac{e}{2}$, $\frac{dy}{dx} = 3(\frac{e}{2}) + 4(\frac{e}{2})^2 = 3(\frac{e}{2}) + e^2$	dM1 A1
		(5)
(iii)	$f'(x) = \frac{(x+1)^{\frac{1}{3}}(-3\sin x) - 3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{(x+1)^{\frac{2}{3}}}$	M1 A1
	$f'(x) = \frac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{4}{3}}}$	A1
		(3)
		12 marks

(i)

B1 $\frac{dx}{dy} = 4\sec^2 2y \tan 2y$ or equivalent such as $\frac{dx}{dy} = 4\frac{\sin 2y \cos 2y}{\cos^4 2y}$

Accept $\frac{dx}{dy} = 2\sec 2y \tan 2y \times \sec 2y + 2\sec 2y \tan 2y \times \sec 2y$, $1 = 4\sec^2 2y \tan 2y \frac{dy}{dx}$

M1 Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to get an expression for $\frac{dy}{dx}$ in terms of y.

It may be scored following the award of the next M1 if $\frac{dx}{dy}$ has been written in terms of x.

Follow through on their expression but condone errors on the coefficient.

For example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2y \tan 2y}$ is OK as is $\frac{dy}{dx} = \frac{2}{\sec^2 2y \tan 2y}$

Do not accept y' s going to x' s. So for example $\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sec^2 2x \tan 2x}$ is M0

M1 Uses $\tan^2 2y = \sec^2 2y - 1$ **and** $x = \sec^2 2y$ to get their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x

$\frac{dx}{dy} = 2\sec^2 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2x\sqrt{(\sec^2 2y - 1)} = 2x\sqrt{x-1}$ is incorrect but scores M1

$\frac{dx}{dy} = 2\sec 2y \tan 2y \Rightarrow \frac{dx}{dy} = 2\sec 2y\sqrt{(\sec^2 2y - 1)} = 2\sqrt{x}\sqrt{x-1}$ is incorrect but scores M1

The stating and use $1 + \tan^2 x = \sec^2 x$ is unlikely to score this mark.

Accept $1 + \tan^2 2y = \sec^2 2y \Rightarrow 1 + \tan^2 2y = x \Rightarrow \tan 2y = \sqrt{x-1}$. So $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan 2y} = \frac{1}{4x\sqrt{x-1}}$

Condone examples where the candidate adapts something to get the given answer

Eg. $\frac{dy}{dx} = \frac{1}{4\sec^2 2y \tan^2 2y} = \frac{1}{4\sec^2 2y\left(\sec^2 2y - 1\right)} = \frac{1}{4x\sqrt{(x-1)}}$

A1* Completely correct solution. This is a 'show that' question and it is a requirement that all elements are seen.

(ii)

M1 Uses the product rule to differentiate $(x^2 + x^3)\ln 2x$. If the rule is stated it must be correct. It may be implied by their $u = ..., u' = ..., v = ..., v' = ..$ followed by $vu' + uv'$. If the rule is neither stated nor implied only accept expressions of the form $\ln 2x \times (ax + bx^2) + (x^2 + x^3) \times \frac{C}{x}$

It is acceptable to multiply out the expression to get $x^2 \ln 2x + x^3 \ln 2x$ but the product rule must be applied to both terms

A1 One term correct (unsimplified). Either $(x^2 + x^3) \times \frac{2}{2x}$ or $(2x + 3x^2)\ln 2x$

If they have multiplied out before differentiating the equivalent would be two of the four terms correct.

A1 A completely correct (unsimplified) expression $\frac{dy}{dx} = (x^2 + x^3) \times \frac{2}{2x} + (2x + 3x^2)\ln 2x$

dM1 Fully substitutes $x = \frac{e}{2}$ (dependent on previous M mark) into their expression for $\frac{dy}{dx} = ...$ Implied by awrt 11.5

A1 $\frac{dy}{dx} = 3(\frac{e}{2}) + e^2$ Accept equivalent simplified forms such as $\frac{dy}{dx} = 1.5e + e^2$, $\frac{dy}{dx} = e(1.5 + e)$, $\frac{dy}{dx} = \frac{e(2e+3)}{2}$

(iii)

M1 Uses quotient rule with $u = 3\cos x$, $v = (x+1)^{\frac{1}{3}}$, $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{2}{3}}$.

If the rule is quoted it must be correct. It may be implied by their $u = 3\cos x$, $v = (x+1)^{\frac{1}{3}}$, $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{2}{3}}$ followed by $\frac{vu' - uv'}{v^2}$

Additionally this could be scored by using the product rule with $u = 3\cos x$, $v = (x+1)^{-\frac{1}{3}}$ $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{4}{3}}$. If the rule is quoted it must be correct. It may be implied by their $u = 3\cos x$, $v = (x+1)^{-\frac{1}{3}}$ $u' = \pm A\sin x$ and $v' = B(x+1)^{-\frac{4}{3}}$ followed by $vu' + uv'$

If it is not quoted nor implied only accept either of the two expressions

1) Using quotient form $\frac{(x+1)^{\frac{1}{3}} \times \pm A\sin x - 3\cos x \times B(x+1)^{-\frac{2}{3}}}{\left((x+1)^{\frac{1}{3}}\right)^2}$ or $\frac{(x+1)^{\frac{1}{3}} \times \pm A\sin x - 3\cos x \times B(x+1)^{-\frac{2}{3}}}{(x+1)^{\frac{1}{9}}}$

2) Using product form $(x+1)^{-\frac{1}{3}} \times \pm A\sin x + 3\cos x \times B(x+1)^{-\frac{4}{3}}$

A1 A correct gradient. Accept $f'(x) = \frac{(x+1)^{\frac{1}{3}}(-3\sin x) - 3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{\left((x+1)^{\frac{1}{3}}\right)^2}$

or $f'(x) = (x+1)^{-\frac{1}{3}} \times -3\sin x + 3\cos x \times -\frac{1}{3}(x+1)^{-\frac{4}{3}}$

A1 $f'(x) = \frac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{4}{3}}}$ oe. or a statement that $g(x) = -3(x+1)(\sin x) - \cos x$ oe.

Question 13

Question Number	Scheme	Marks
(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1 (1)
(b)	$x=(4y-\sin 2y)^2 \Rightarrow \frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	M1A1
	Sub $y=\frac{\pi}{2}$ into $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$	
	$\Rightarrow \frac{dx}{dy}=24\pi \quad (=75.4)$ / $\frac{dy}{dx}=\frac{1}{24\pi}(=0.013)$	M1
	Equation of tangent $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$	M1
	Using $y-\frac{\pi}{2}=\frac{1}{24\pi}(x-4\pi^2)$ with $x=0 \Rightarrow y=\frac{\pi}{3}$ cso	M1, A1 (6) (7 marks)
Alt (b) I	$x=(4y-\sin 2y)^2 \Rightarrow x^{0.5}=4y-\sin 2y$ $\Rightarrow 0.5x^{-0.5}\frac{dx}{dy}=4-2\cos 2y$	M1A1
Alt (b) II	$x=(16y^2-8y\sin 2y+\sin^2 2y)$ $\Rightarrow 1=32y\frac{dy}{dx}-8\sin 2y\frac{dy}{dx}-16y\cos 2y\frac{dy}{dx}+4\sin 2y\cos 2y\frac{dy}{dx}$ Or $1dx=32y\,dy-8\sin 2y\,dy-16y\cos 2y\,dy+4\sin 2y\cos 2y\,dy$	M1A1

(a)

B1 $p = 4\pi^2$ or exact equivalent $(2\pi)^2$

Also allow $x = 4\pi^2$

(b)

M1 Uses the chain rule of differentiation to get a form
$A(4y-\sin 2y)(B \pm C\cos 2y)$, $A, B, C \neq 0$ on the right hand side
Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x=\left(16y^2-8y\sin 2y+\sin^2 2y\right)\Rightarrow\frac{dx}{dy}=Py\pm Q\sin 2y\pm Ry\cos 2y\pm S\sin 2y\cos 2y \quad P,Q,R,S\neq 0$
A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4-Q\cos 2y$
A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

A1 $\frac{dx}{dy}=2(4y-\sin 2y)(4-2\cos 2y)$ or $\frac{dy}{dx}=\frac{1}{2(4y-\sin 2y)(4-2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs.
In the alternative $\frac{dx}{dy}=32y-8\sin 2y-16y\cos 2y+4\sin 2y\cos 2y$

M1 Sub $y=\frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y=\frac{\pi}{2}\Rightarrow\frac{dx}{dy}=...$
It is not dependent upon the previous M1 but it must be a changed $x=(4y-\sin 2y)^2$

M1 Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \frac{\pi}{2}\right)$.

Allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical}\left(\frac{dx}{dy}\right)}\left(x-\text{their } 4\pi^2\right)$

Allow for $\left(y-\frac{\pi}{2}\right)\times\text{their numerical}\left(\frac{dx}{dy}\right)=\left(x-\text{their } 4\pi^2\right)$

Even allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical}\left(\frac{dx}{dy}\right)}(x-p)$

It is possible to score this by stating the equation $y=\frac{1}{24\pi}x+c$ as long as $\left('4\pi^2', \frac{\pi}{2}\right)$ is used in a subsequent line.

M1 Score for writing their equation in the form $y=mx+c$ and stating the value of 'c'
Or setting $x=0$ in their $y-\frac{\pi}{2}=\frac{1}{24\pi}\left(x-4\pi^2\right)$ and solving for y.
Alternatively using the gradient of the line segment AP = gradient of tangent.
Look for $\frac{\frac{\pi}{2}-y}{4\pi^2}=\frac{1}{24\pi}\Rightarrow y=..$ Such a method scores the previous M mark as well.
At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1 cso $y=\frac{\pi}{3}$. You do not have to see $\left(0, \frac{\pi}{3}\right)$

Question 14

Question Number	Scheme	Marks
(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)e^{-2x}$	
	$g'(x)=\left(x^2-x^3\right)\times -2e^{-2x}+\left(2x-3x^2\right)\times e^{-2x}$	M1 A1
	$g'(x)=\left(2x^3-5x^2+2x\right)e^{-2x}$	A1
		(3)
(b)	Sets $\left(2x^3-5x^2+2x\right)e^{-2x}=0 \Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0 \Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2}, 2$ into $g(x)=\left(x^2-x^3\right)e^{-2x} \Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8e}, \quad g(2)=-\frac{4}{e^4}$	dM1,A1
	Range $-\frac{4}{e^4}\leqslant g(x)\leqslant\frac{1}{8e}$	A1
		(6)
(c)	Accept $g(x)$ is NOT a ONE to ONE function	
	Accept $g(x)$ is a MANY to ONE function	B1
	Accept $g^{-1}(x)$ would be ONE to MANY	(1)
		(10 marks)

Note that parts (a) and (b) can be scored together. Eg accept work in part (b) for part (a)

(a)

M1 Uses **the product rule** $vu'+uv'$ with $u=x^2-x^3$ and $v=e^{-2x}$ or vice versa. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $vu'+uv'$. If the rule is not quoted nor implied only accept expressions of the form $\left(x^2-x^3\right)\times \pm Ae^{-2x}+\left(Bx\pm Cx^2\right)\times e^{-2x}$ condoning bracketing issues

Method 2: multiplies out and **uses the product rule** on each term of $x^2e^{-2x}-x^3e^{-2x}$
Condone issues in the signs of the last two terms for the method mark
Uses the product rule for $uvw=u'vw+uv'w+uvw'$ applied as in method 1

Method 3:Uses **the quotient rule** with $u=x^2-x^3$ and $v=e^{2x}$. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $\frac{vu'-uv'}{v^2}$ If the rule is not quoted nor implied accept expressions of the form $\frac{e^{2x}\left(Ax-Bx^2\right)-\left(x^2-x^3\right)\times Ce^{2x}}{\left(e^{2x}\right)^2}$

condoning missing brackets on the numerator and e^{2x^2} on the denominator.

Method 4: Apply implicit differentiation to $ye^{2x}=x^2-x^3 \Rightarrow e^{2x}\times\frac{dy}{dx}+y\times 2e^{2x}=2x-3x^2$

Condone errors on coefficients and signs

A1 A correct (unsimplified form) of the answer
$g'(x) = (x^2 - x^3) \times -2e^{-2x} + (2x - 3x^2) \times e^{-2x}$ by one use of the product rule
or $g'(x) = x^2 \times -2e^{-2x} + 2xe^{-2x} - x^3 \times -2e^{-2x} - 3x^2 \times e^{-2x}$ using the first alternative
or $g'(x) = 2x(1-x)e^{-2x} + x^2 \times -1 \times e^{-2x} + x^2(1-x) \times -2e^{-2x}$ using the product rule on 3 terms
or $g'(x) = \dfrac{e^{2x}(2x-3x^2) - (x^2-x^3) \times 2e^{2x}}{(e^{2x})^2}$ using the quotient rule.

A1 Writes $g'(x) = (2x^3 - 5x^2 + 2x)e^{-2x}$. You do not need to see $f(x)$ stated and award even if a correct $g'(x)$ is followed by an incorrect $f(x)$. If the f(x) is not simplified at this stage you need to see it simplified later for this to be awarded.

(b) Note: The last mark in e-pen has been changed from a 'B' to an A mark

M1 For setting their f(x) =0. The = 0 may be implied by subsequent working.
Allow even if the candidate has failed to reach a 3TC for f(x).
Allow for $f(x) \geqslant 0$ or $f(x) \leqslant 0$ as they can use this to pick out the relevant sections of the curve

M1 For solving their 3TC = 0 by ANY correct method.
Allow for division of x or factorising out the x followed by factorisation of 3TQ. Check first and last terms of the 3TQ. Allow for solutions from either $f(x) \geqslant 0$ or $f(x) \leqslant 0$
Allow solutions **from the cubic equation** just appearing from a Graphical Calculator

A1 $x = \frac{1}{2}, 2$. Correct answers from a correct $g'(x)$ would imply all 3 marks so far in (b)

dM1 Dependent upon both previous M's being scored. For substituting their **two** (non zero) values of x into g(x) to find both y values. Minimal evidence is required $x = .. \Rightarrow y = ..$ is OK.

A1 Accept decimal answers for this mark. $g\left(\frac{1}{2}\right) = \frac{1}{8e}$ =awrt 0.046 AND $g(2) = -\frac{4}{e^4}$ =awrt −0.073

A1 CSO Allow $-\frac{4}{e^4} \leqslant \text{Range} \leqslant \frac{1}{8e}$, $-\frac{4}{e^4} \leqslant y \leqslant \frac{1}{8e}$, $\left[-\frac{4}{e^4}, \frac{1}{8e}\right]$. Condone $y \geqslant -\frac{4}{e^4}$ $y \leqslant \frac{1}{8e}$

Note that the question states hence and part (a) must have been used for all marks. Some students will just write down the answers for the range from a graphical calculator.
Seeing just $-\frac{4}{e^4} \leqslant g(x) \leqslant \frac{1}{8e}$ or $-0.073 \leqslant g(x) \leqslant 0.046$ special case 100000.
They know what a range is!

(c)

B1 If the candidate states 'NOT ONE TO ONE' then accept unless the explicitly link it to $g^{-1}(x)$.
So accept 'It is not a one to one function'. 'The function is not one to one' '$g(x)$ is not one to one'
If the candidate states 'IT IS MANY TO ONE' then accept unless the candidate explicitly links it to $g^{-1}(x)$. So accept 'It is a many to one function.' 'The function is many to one' '$g(x)$ is many to one'
If the candidate states 'IT IS ONE TO MANY' then accept unless the candidate explicitly links it to $g(x)$
Accept an explanation like " one value of x would map/ go to more than one value of y"
Incorrect statements scoring B0 would be $g^{-1}(x)$ is not one to one, $g^{-1}(x)$ is many to one and $g(x)$ is one to many.

Question 15

Question Number	Scheme	Marks
(a)	$x^2-3kx+2k^2=(x-2k)(x-k)$	B1
	$2-\dfrac{(x-5k)(x-k)}{(x-2k)(x-k)}=2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$=\dfrac{x+k}{(x-2k)}$	A1*
		(3)
(b)	Applies $\dfrac{vu'-uv'}{v^2}$ to $y=\dfrac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$	
	$\Rightarrow f'(x)=\dfrac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$	M1, A1
	$\Rightarrow f'(x)=\dfrac{-3k}{(x-2k)^2}$	A1
		(3)
(c)	If $f'(x)=\dfrac{-Ck}{(x-2k)^2}\Rightarrow f(x)$ is an increasing function as $f'(x)>0$,	M1
	$f'(x)=\dfrac{-3k}{(x-2k)^2}>0$ for all values of x as $\dfrac{\text{negative}\times\text{negative}}{\text{positive}}=\text{positive}$	A1
		(2)
		(8 marks)

(a)

B1 For seeing $x^2-3kx+2k^2=(x-2k)(x-k)$ anywhere in the solution

M1 For writing as a single term or two terms with the same denominator

Score for $2-\dfrac{(x-5k)}{(x-2k)}=\dfrac{2(x-2k)-(x-5k)}{(x-2k)}$ or

$$2-\frac{(x-5k)(x-k)}{(x-2k)(x-k)}=\frac{2(x-2k)(x-k)-(x-5k)(x-k)}{(x-2k)(x-k)} \qquad \left(=\frac{x^2-k^2}{x^2-3kx+2k^2}\right)$$

A1* Proceeds without any errors (including bracketing) to $=\dfrac{x+k}{(x-2k)}$

(b)

M1 Applies $\frac{vu'-uv'}{v^2}$ to $y=\frac{x+k}{x-2k}$ with $u=x+k$ and $v=x-2k$.

If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=x-2k$ with their u',v' and $\frac{vu'-uv'}{v^2}$

If it is neither stated nor implied only accept expressions of the form $f'(x)=\frac{x-2k-x\pm k}{(x-2k)^2}$

The mark can be scored for applying the product rule to $y=(x+k)(x-2k)^{-1}$ If the rule it is stated it must be correct. It can be implied by $u=x+k$ and $v=(x-2k)^{-1}$ with their u',v' and $vu'+uv'$

If it is neither stated nor implied only accept expressions of the form

$f'(x)=(x-2k)^{-1}\pm(x+k)(x-2k)^{-2}$

Alternatively writes $y=\frac{x+k}{x-2k}$ as $y=1+\frac{3k}{x-2k}$ and differentiates to $\frac{dy}{dx}=\frac{A}{(x-2k)^2}$

A1 Any correct form (unsimplified) form of $f'(x)$.

$f'(x)=\frac{(x-2k)\times1-(x+k)\times1}{(x-2k)^2}$ by quotient rule

$f'(x)=(x-2k)^{-1}-(x+k)(x-2k)^{-2}$ by product rule

and $f'(x)=\frac{-3k}{(x-2k)^2}$ by the third method

A1 cao $f'(x)=\frac{-3k}{(x-2k)^2}$. Allow $f'(x)=\frac{-3k}{x^2-4kx+4k^2}$

As this answer is not given candidates you may allow recovery from missing brackets

(c) Note that this is B1 B1 on e pen. We are scoring it M1 A1

M1 If in part (b) $f'(x)=\frac{-Ck}{(x-2k)^2}$, look for f(x) is an increasing function as $f'(x)$ / gradient >0

Accept a version that states as $k<0\Rightarrow -Ck>0$ hence increasing

If in part (b) $f'(x)=\frac{(+)Ck}{(x-2k)^2}$, look for f(x) is an decreasing function as $f'(x)$ / gradient<0

Similarly accept a version that states as $k<0\Rightarrow (+)Ck<0$ hence decreasing

A1 Must have $f'(x)=\frac{-3k}{(x-2k)^2}$ and give a reason that links the gradient with its sign.

There must have been reference to the sign of both numerator and denominator to justify the overall positive sign.

Question 16.

Question Number	Scheme		Marks
(a)	$\{y = 3^x \Rightarrow\}\ \frac{dy}{dx} = 3^x \ln 3$	$\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$	B1
	Either T: $y - 9 = 3^2 \ln 3(x-2)$ **or T:** $y = (3^2 \ln 3)x + 9 - 18\ln 3$, where $9 = (3^2 \ln 3)(2) + c$	See notes	M1
	{Cuts x-axis $\Rightarrow y = 0 \Rightarrow$}		
	$-9 = 9\ln 3(x-2)$ or $0 = (3^2 \ln 3)x + 9 - 18\ln 3,$	Sets $y = 0$ in their tangent equation and progresses to $x = ...$	M1
	So, $x = 2 - \frac{1}{\ln 3}$	$2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ o.e.	A1 cso
			[4]

		Question Notes
(a)	B1	$\frac{dy}{dx} = 3^x \ln 3$ or $\ln 3\left(e^{x\ln 3}\right)$ or $y\ln 3$. Can be implied by later working.
	M1	Substitutes either $x = 2$ **or** $y = 9$ into their $\frac{dy}{dx}$ which is a function of x or y to find m_T **and** • **either** applies $y - 9 = \left(\text{their } m_T\right)(x-2)$, where m_T is a numerical value. • **or** applies $y = \left(\text{their } m_T\right)x$ + their c, where m_T is a numerical value and c is found by solving $9 = \left(\text{their } m_T\right)(2) + c$
	Note	The first M1 mark can be implied from later working.
	M1	Sets $y = 0$ in their *tangent* equation, where m_T is a numerical value, (seen or implied) and progresses to $x = ...$
	A1	An exact value of $2 - \frac{1}{\ln 3}$ or $\frac{2\ln 3 - 1}{\ln 3}$ or $\frac{\ln 9 - 1}{\ln 3}$ by a correct solution only.
	Note	Allow A1 for $2 - \frac{\lambda}{\lambda \ln 3}$ or $\frac{\lambda(2\ln 3 - 1)}{\lambda \ln 3}$ or $\frac{\lambda(\ln 9 - 1)}{\lambda \ln 3}$ or $2 - \frac{\lambda}{\lambda \ln 3}$, where λ is an integer, and ignore subsequent working.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$) is M0 M0 in part (a).
	Note	Candidates who invent a value for m_T (which bears no resemblance to their gradient function) cannot gain the 1st M1 and 2nd M1 mark in part (a).
	Note	A decimal answer of 1.089760773... (without a correct exact answer) is A0.

Question 17

Question	Scheme	Marks
(a)	$y=\frac{4x}{(x^2+5)}\Rightarrow\left(\frac{dy}{dx}\right)=\frac{4(x^2+5)-4x\times 2x}{(x^2+5)^2}$	M1A1
	$\Rightarrow\left(\frac{dy}{dx}\right)=\frac{20-4x^2}{(x^2+5)^2}$	M1A1
		(4)
(b)	$\frac{20-4x^2}{(x^2+5)^2}<0\Rightarrow x^2>\frac{20}{4}$ Critical values of $\pm\sqrt{5}$	M1
	$x<-\sqrt{5},\ x>\sqrt{5}$ or equivalent	dM1A1
		(3)
		7 marks

(a)M1 Attempt to use the **quotient rule** $\frac{vu'-uv'}{v^2}$ with $u=4x$ and $v=x^2+5$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,\ v=x^2+5, v'=Bx$ followed by their $\frac{vu'-uv'}{v^2}$

If the rule is neither quoted nor implied only accept expressions of the form

$\frac{A(x^2+5)-4x\times Bx}{(x^2+5)^2}, A,B>0$ You may condone missing (invisible) brackets

Alternatively uses the **product rule** with $u(/v)=4x$ and $v(/u)=(x^2+5)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x, u'=A,\ v=x^2+5, v'=Bx(x^2+5)^{-2}$ followed by their $vu'+uv'$. If the rule is neither quoted nor implied only accept expressions of the form $A(x^2+5)^{-1}\pm 4x\times Bx(x^2+5)^{-2}$

A1 $\mathrm{f}'(x)$ correct (unsimplified). For the product rule look for versions of $4(x^2+5)^{-1}-4x\times 2x(x^2+5)^{-2}$

M1 Simplifies to the form $\mathrm{f}'(x)=\frac{A+Bx^2}{(x^2+5)^2}$ oe. This is not dependent so could be scored from $\frac{v'u-u'v}{v^2}$

When the product rule has been used the A of $A(x^2+5)^{-1}$ must be adapted.

A1 CAO. Accept exact equivalents such as $(\mathrm{f}'(x))=\frac{4(5-x^2)}{(x^2+5)^2}$, $-\frac{4x^2-20}{(x^2+5)^2}$ or $\frac{-4(x^2-5)}{x^4+10x^2+25}$

Remember to isw after a correct answer

(b)

M1 Sets their numerator either $=0$, <0, $\leqslant 0$, >0, $\geqslant 0$ and proceeds to at least **one** value for x

For example $20-4x^2\geqslant 0 \Rightarrow x\geqslant\sqrt{5}$ will be M1 dM0 A0.

It cannot be scored from a numerator such as 4 or indeed $20+4x^2$

dM1 Achieves **two** critical values for their numerator $=0$ and chooses the outside region

Look for $x<$ smaller root, $x>$ bigger root. Allow decimals for the roots.

Condone $x\leqslant -\sqrt{5}, x\geqslant\sqrt{5}$ and expressions like $-\sqrt{5}>x>\sqrt{5}$

If they have $4x^2-20<0$ following an incorrect derivative they should be choosing the inside region

A1 Allow $x<-\sqrt{5}, x>\sqrt{5}$ $\quad x<-\sqrt{5}$ or $x>\sqrt{5}$ $\quad \{x:-\infty<x<-\sqrt{5}\cup\sqrt{5}<x<\infty\}$ $\quad |x|>\sqrt{5}$

Do not allow for the A1 $x<-\sqrt{5}$ and $x>\sqrt{5}$. $\sqrt{5}<x<-\sqrt{5}$ or $\{x:-\infty<x<-\sqrt{5}\cap\sqrt{5}<x<\infty\}$ but you may isw following a correct answer.

Question 18

Question	Scheme	Marks
(i)	$y = e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \frac{1}{4}\arctan\frac{3}{4}$	M1
	$\Rightarrow x = \text{awrt } 0.9463$ 4dp	A1
		(5)
(ii)	$x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5) (10 marks)
(ii) Alt I	$x = \sin^2 2y \Rightarrow x = \frac{1}{2} - \frac{1}{2}\cos 4y$	2nd M1
	$\frac{dx}{dy} = 2\sin 4y$	1st M1 A1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt II	$x^{\frac{1}{2}} = \sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\frac{dy}{dx}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)
(ii) Alt III	$x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \text{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2}x^{-\frac{1}{2}}$	M1A1
	Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$	M1A1
		(5)

(i)

M1 Uses the product rule $uv' + vu'$ to achieve $\left(\frac{dy}{dx}\right) = Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A, B \neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx} = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx} = 0$ factorises/cancels)by e^{3x} to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x} \equiv \tan 4x$, moves from $\tan 4x = C, C \neq 0$ using correct order of operations to $x = ...$ Accept $x =$ awrt 0.16 (radians) $x =$ awrt 9.22 (degrees) for this mark.

If a candidate elects to pursue a more difficult method using $R\cos(\theta + \alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of R and α correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x - \text{awrt } 0.64)$ before using the correct order of operations to $x = ...$

Similarly candidates who square $3\cos 4x - 4\sin 4x = 0$ then use a Pythagorean identity should proceed from either $\sin 4x = \frac{3}{5}$ or $\cos 4x = \frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x =$ awrt 0.9463.

Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.

There is no need for lhs to be seen/ correct

If the product rule is used look for $\cancel{\frac{dy}{dx}} = \pm A\sin 2y\cos 2y \pm B\sin 2y\cos 2y$,

A1 Both lhs and rhs correct (unsimplified). $\frac{dx}{dy} = 2\sin 2y \times 2\cos 2y = (4\sin 2y\cos 2y)$ or

$1 = 2\sin 2y \times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression.
You may just see a statement such as $4\sin 2y\cos 2y = 2\sin 4y$ which is fine.
Candidates who write $\cancel{\frac{dy}{dx}} = A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dy}{dx}} = \frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx} = 1 \Big/ \frac{dx}{dy}$ **for their expression in** y. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = 2\text{cosec}4y$ is condoned for the M1

If $\frac{dx}{dy} = a + b$ do not allow $\frac{dy}{dx} = \frac{1}{a} + \frac{1}{b}$

A1 $\frac{dy}{dx} = \frac{1}{2}\text{cosec}4y$ If a candidate then proceeds to write down incorrect values of p and q then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x = \sin^2 2y \Rightarrow x = \pm\frac{1}{2} \pm \frac{1}{2}\cos 4y$ from $\cos 4y = \pm 1 \pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}} = \sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}} = Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y = \text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\cancel{\frac{dy}{dx}} = N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}} \times x^{-0.5}$

Question 19

Question	Scheme	Marks
(a)	$x^2+x-6\overline{)x^4+x^3-3x^2+7x-6}$ quotient x^2+3 $\underline{x^4+x^3-6x^2}$ $3x^2+7x-6$ $\underline{3x^2+3x-18}$ $4x+12$	M1 A1
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6} \equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(4)
(b)	$f'(x)=2x-\dfrac{4}{(x-2)^2}$	M1A1ft
	Subs $x=3$ into $f'(x=3)=2\times3-\dfrac{4}{(3-2)^2}=(2)$	M1
	Uses $m=-\dfrac{1}{f'(3)}=\left(-\dfrac{1}{2}\right)$ with $(3, f(3))=(3,16)$ to form eqn of normal	
	$y-16=-\dfrac{1}{2}(x-3)$ or equivalent cso	M1A1
		(5)
		(9 marks)

(a)

M1 Divides $x^4+x^3-3x^2+7x-6$ by x^2+x-6 to get a quadratic quotient and a linear or constant remainder. To award this look for a minimum of the following

$$\begin{array}{r} x^2(+..x)+A \\ x^2+x-6\overline{)x^4+x^3-3x^2+7x-6} \\ \underline{x^4+x^3-6x^2} \\ \\ \overline{(Cx)+D} \end{array}$$

If they divide by $(x+3)$ first they must then divide their by result by $(x-2)$ before they score this method mark. Look for a cubic quotient with a constant remainder followed by a quadratic quotient and a constant remainder

Note: FYI Dividing by $(x+3)$ gives x^3-2x^2+3x-2 and $(x^3-2x^2+3x-2)\div(x-2)=x^2+3$ with a remainder of 4.

Division by $(x-2)$ first is possible but difficult.....please send to review any you feel deserves credit.

A1 Quotient = x^2+3 and Remainder = $4x+12$

M1 Factorises x^2+x-6 and writes their expression in the appropriate form.

$$\left(\frac{x^4+x^3-3x^2+7x-6}{x^2+x-6}\right)\equiv \text{Their Quadratic Quotient}+\frac{\text{Their Linear Remainder}}{(x+3)(x-2)}$$

It is possible to do this part by partial fractions. To score M1 under this method the terms must be correct and it must be a full method to find both "numerators"

A1 $x^2+3+\dfrac{4}{(x-2)}$ or $A=3, B=4$ but don't penalise after a correct statement.

(b)

M1 $x^2+A+\dfrac{B}{x-2}\to 2x\pm\dfrac{B}{(x-2)^2}$

If they fail in part (a) to get a function in the form $x^2+A+\dfrac{B}{x-2}$ allow candidates to pick up this method mark for differentiating a function of the form $x^2+Px+Q+\dfrac{Rx+S}{x\pm T}$ using the quotient rule oe.

A1ft $x^2+A+\dfrac{B}{x-2}\to 2x-\dfrac{B}{(x-2)^2}$ oe. FT on their numerical A, B for for $x^2+A+\dfrac{B}{x-2}$ only

M1 Subs $x=3$ into their $f'(x)$ in an attempt to find a numerical gradient

M1 For the correct method of finding an equation of a normal. The gradient must be $-\dfrac{1}{\text{their f}'(3)}$ and the point must be $(3, f(3))$. Don't be overly concerned about how they found their f(3), ie accept x=3 y =.

Look for $y-f(3)=-\dfrac{1}{f'(3)}(x-3)$ or $(y-f(3))\times -f'(3)=(x-3)$

If the form $y=mx+c$ is used they must proceed as far as $c=$

A1 cso $y-16=-\dfrac{1}{2}(x-3)$ oe such as $2y+x-35=0$ but remember to isw after a correct answer.

Alt (a) attempted by equating terms.

Alt (a)	$x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x+3)$ Compare 2 terms (or substitute 2 values) AND solve simultaneously ie $x^2 \Rightarrow A-6=-3, \quad x \Rightarrow A+B=7, \quad \text{const} \Rightarrow -6A+3B=-6$ $A=3, B=4$	M1 M1 A1,A1

1st Mark M1 Scored for multiplying by (x^2+x-6) and cancelling/dividing to achieve $x^4+x^3-3x^2+7x-6 \equiv (x^2+A)(x^2+x-6)+B(x\pm 3)$

3rd Mark M1 Scored for comparing two terms (or for substituting two values) **AND** solving simultaneously to get values of A and B.

2nd Mark A1 Either $A=3$ or $B=4$. One value may be correct by substitution of say $x=-3$

4th Mark A1 Both $A=3$ and $B=4$

Alt (b) is attempted by the quotient (or product rule)

ALT (b) 1st 3 marks	$f'(x)=\dfrac{(x^2+x-6)(4x^3+3x^2-6x+7)-(x^4+x^3-3x^2+7x-6)(2x+1)}{(x^2+x-6)^2}$ Subs $x=3$ into	M1A1 M1

M1 Attempt to use the **quotient rule** $\dfrac{vu'-uv'}{v^2}$ with $u=x^4+x^3-3x^2+7x-6$ and $v=x^2+x-6$ and achieves an expression of the form $f'(x)=\dfrac{(x^2+x-6)(..x^3........)-(x^4+x^3-3x^2+7x-6)(..x..)}{(x^2+x-6)^2}$.

Use a similar approach to the product rule with $u=x^4+x^3-3x^2+7x-6$ and $v=(x^2+x-6)^{-1}$

Note that this can score full marks from a partially solved part (a) where $f(x)\equiv x^2+3+\dfrac{4x+12}{x^2+x-6}$

Question 20

Question Number	Scheme	Marks
(i) (a)	$y=2x\left(x^2-1\right)^5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^5\times 2+2x\times 10x\left(x^2-1\right)^4$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\left(x^2-1\right)^4\left(2x^2-2+20x^2\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$	M1 A1
		(4)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}\ldots 0\Rightarrow\left(22x^2-2\right)\ldots 0\Rightarrow$ critical values of $\pm\frac{1}{\sqrt{11}}$	M1
	$x\ldots\frac{1}{\sqrt{11}}\quad x,,\ -\frac{1}{\sqrt{11}}$	A1
		(2)
(ii)	$x=\ln(\sec 2y)\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{1}{\sec 2y}\times 2\sec 2y\tan 2y$	B1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\tan 2y}=\frac{1}{2\sqrt{\sec^2 2y-1}}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	M1 M1 A1
		(4)
		10 marks

Alt 1 (ii)	$x=\ln(\sec 2y)\Rightarrow\sec 2y=\mathrm{e}^x$	
	$\Rightarrow 2\sec 2y\tan 2y\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^x$	B1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^x}{2\sec 2y\tan 2y}=\frac{\mathrm{e}^x}{2\mathrm{e}^x\sqrt{\sec^2 2y-1}}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	M1M1A1
		(4)
Alt 2 (ii)	$y=\frac{1}{2}\arccos\left(\mathrm{e}^{-x}\right)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}\times\frac{1}{\sqrt{1-\left(\mathrm{e}^{-x}\right)^2}}\times -\mathrm{e}^{-x}$	B1M1M1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	A1
		(4)

(i)(a)

M1 Attempts the product rule to differentiate $2x(x^2-1)^5$ to a form $A\left(x^2-1\right)^5+Bx^n\left(x^2-1\right)^4$ where $n=1$ or 2. and $A, B>0$ If the rule is stated it must be correct, and not with a "$-$" sign.

A1 Any unsimplified but correct form $\left(\dfrac{dy}{dx}\right)=2\left(x^2-1\right)^5+20x^2\left(x^2-1\right)^4$

M1 For taking a common factor of $\left(x^2-1\right)^4$ out of a suitable expression

Look for $A\left(x^2-1\right)^5 \pm Bx^n\left(x^2-1\right)^4=\left(x^2-1\right)^4\left\{A\left(x^2-1\right)\pm Bx^n\right\}$ but you may condone missing brackets

It can be scored from a *vu'-uv'* or similar.

A1 $\left(\dfrac{dy}{dx}\right)=\left(x^2-1\right)^4\left(22x^2-2\right)$ Expect $g(x)$ to be simplified but accept $\dfrac{dy}{dx}=\left(x^2-1\right)^4 2\left(11x^2-1\right)$

There is no need to state $g(x)$ and remember to isw after a correct answer. This must be in part (a).

(i)(b)

M1 Sets their $\dfrac{dy}{dx}\geqslant 0, >0$ or $\dfrac{dy}{dx}=0$ and proceeds to find one of the critical values for **their** $g(x)$ or their $\dfrac{dy}{dx}=0$ rearranged and $\div\left(x^2-1\right)^4$ if $g(x)$ not found. $g(x)$ should be at least a 2TQ with real roots. If $g(x)$ is factorised, the usual rules apply. The M cannot be awarded from work **just** on $\left(x^2-1\right)^4\geqslant 0$ ie $x=\pm 1$

You may see and accept decimals for the M.

A1 cao $x\geqslant\dfrac{1}{\sqrt{11}}$ $x\leqslant-\dfrac{1}{\sqrt{11}}$ or exact equivalent only. Condone $x\geqslant\dfrac{1}{\sqrt{11}}$ $x\leqslant-\dfrac{1}{\sqrt{11}}$, with $x\geqslant 1, x\leqslant -1$

Accept exact equivalents such as $x\geqslant\dfrac{\sqrt{11}}{11}$ $x\leqslant-\dfrac{\sqrt{11}}{11}$; $|x|\geqslant\dfrac{1}{\sqrt{11}}$; $\left\{\left(-\infty,-\dfrac{\sqrt{11}}{11}\right]\cup\left[\dfrac{\sqrt{11}}{11},\infty\right)\right\}$

Condone the word "and" appearing between the two sets of values.

Withhold the final mark if $x\geqslant\dfrac{1}{\sqrt{11}}$ $x\leqslant-\dfrac{1}{\sqrt{11}}$, appears with values not in this region eg $x\leqslant 1, x\geqslant -1$

(ii)

B1 Differentiates and achieves a correct line involving $\frac{dy}{dx}$ or $\frac{dx}{dy}$

Accept $\frac{dx}{dy} = \frac{1}{\sec 2y} \times 2\sec 2y \tan 2y$, $\frac{dx}{dy} = -\frac{1}{\cos 2y} \times -2\sin 2y$ $\quad 2\sec 2y \tan 2y \frac{dy}{dx} = e^x$

M1 For inverting their expression for $\frac{dx}{dy}$ to achieve an expression for $\frac{dy}{dx}$.

The variables (on the rhs) must be consistent, you may condone slips on the coefficients but not the terms. In the alternative method it is for correctly changing the subject

M1 Scored for using $\tan^2 2y = \pm 1 \pm \sec^2 2y$ **and** $\sec 2y = e^x$ to achieve $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of x

Alternatively they could use $\sin^2 2y + \cos^2 2y = 1$ with $\cos 2y = e^{-x}$ to achieve $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of x

For the M mark you may condone $\sec^2 2y = \left(e^x\right)^2$ appearing as e^{x^2}

A1 cso $\frac{dy}{dx} = \frac{1}{2\sqrt{e^{2x} - 1}}$ Final answer, do not allow if students then simplify this to eg. $\frac{dy}{dx} = \frac{1}{2e^x - 1}$

Condone $\frac{dy}{dx} = \pm \frac{1}{2\sqrt{e^{2x} - 1}}$ but do not allow $\frac{dy}{dx} = -\frac{1}{2\sqrt{e^{2x} - 1}}$

Allow a misread on $x = \ln(\sec y)$ for the two method marks only

Question 21

Question Number	Scheme	Marks
(a)	$P_0 = \dfrac{100}{1+3} + 40 = 65$	B1
		(1)
	$\dfrac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$	M1
(b)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{\left(1+3\mathrm{e}^{-0.9t}\right)\times -10\mathrm{e}^{-0.1t} - 100\mathrm{e}^{-0.1t}\times -2.7\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$	M1 A1
		(3)
(c)(i)	At maximum $\quad -10\mathrm{e}^{-0.1t} - 30\mathrm{e}^{-0.1t}\times \mathrm{e}^{-0.9t} + 270\mathrm{e}^{-0.1t}\times \mathrm{e}^{-0.9t} = 0$	
	$\mathrm{e}^{-0.1t}\left(-10+240\mathrm{e}^{-0.9t}\right) = 0$	
	$\mathrm{e}^{-0.9t} = \dfrac{10}{240}$ oe $\mathrm{e}^{0.9t} = 24$	M1
	$-0.9t = \ln\left(\dfrac{1}{24}\right) \Rightarrow t = \dfrac{10}{9}\ln(24) = 3.53$	M1, A1
(c) (ii)	Sub $t = 3.53 \Rightarrow P_T = 102$	A1
		(4)
(d)	40	B1
		(1)
		9 marks

(a)

B1 $\left(P_0 =\right)65$

(b)

M1 For sight of $\frac{d}{dt}e^{kt} = Ce^{kt}$ (Allow C =1)This may be within an incorrect product or quotient rule

M1 Scored for a full application of the quotient rule. If the formula is quoted it should be correct.

The denominator should be present even when the correct formula has been quoted.

In cases where a formula has not been quoted it is very difficult to judge that a correct formula has been used (due to the signs between the terms). So…………….

if the formula has not been quoted look for the **order** of the terms $\frac{(1+3e^{-0.9t})\times pe^{-0.1t} - qe^{-0.1t}\times e^{-0.9t}}{(1+3e^{-0.9t})^2}$ $\frac{(1+3e^{-0.9t})\times pe^{-0.1t} + qe^{-0.1t}\times e^{-0.9t}}{(1+3e^{-0.9t})^2}$

For the product rule. Look for $ae^{-0.1t}\left(1+3e^{-0.9t}\right)^{-1} \pm be^{-0.1t}e^{-0.9t}\left(1+3e^{-0.9t}\right)^{-2}$ either way around

Penalise if an incorrect formula is quoted . Condone missing brackets in both cases.

A1 A correct **unsimplified** answer.

Eg using quotient rule $\left(\frac{dP}{dt}\right) = \frac{-10e^{-0.1t}\left(1+3e^{-0.9t}\right)+270e^{-0.1t}e^{-0.9t}}{\left(1+3e^{-0.9t}\right)^2}$ oe $\frac{-10e^{-0.1t}+240e^{-1t}}{\left(1+3e^{-0.9t}\right)^2}$ simplified

Eg using product rule $\left(\frac{dP}{dt}\right) = -10e^{-0.1t}\left(1+3e^{-0.9t}\right)^{-1} + 270e^{-0.1t}e^{-0.9t}\left(1+3e^{-0.9t}\right)^{-2}$ oe

Remember to isw after a correct (unsimplified) answer.

There is no need to have the $\frac{dP}{dt}$ and it could be called $\frac{dy}{dx}$

(c)(i) Do NOT allow any marks in here without sight/implication of $\frac{dP}{dt}=0$, $\frac{dP}{dt}<0$ OR $\frac{dP}{dt}>0$

The question requires the candidate to find t using part (b) so it is possible to do this part using inequalities using the same criteria as we apply for the equality. All marks in (c) can be scored from an incorrect denominator (most likely v), no denominator, or using a numerator the wrong way around ie $uv'-u'v$

M1 Sets their $\frac{dP}{dt}=0$ or the numerator of their $\frac{dP}{dt}=0$, factorises out or cancels a term in $e^{-0.1t}$ to reach a form $Ae^{\pm 0.9t}=B$ oe. Alternatively they could combine terms to reach $Ae^{-t}=Be^{-0.1t}$ or equivalent

Condone a double error on $e^{-0.1t}\times e^{-0.9t}=e^{-0.1t\times -0.9t}$ or similar before factorising. **Look for correct indices.**

If they use the product rule then expect to see their $\frac{dP}{dt}=0$ followed by multiplication of $\left(1+3e^{-0.9t}\right)^2$ before similar work to the quotient rule leads to a form $Ae^{\pm 0.9t}=B$

M1 Having set the numerator of their $\frac{dP}{dt}=0$ and obtained either $e^{\pm kt}=C$ (k may be incorrect) or $Ae^{-t}=Be^{-0.1t}$ it is awarded for the correct order of operations, taking ln's leading to $t=..$

It cannot be awarded from impossible equations Eg $e^{\pm 0.9t}=-0.3$

A1 cso $t=$ awrt 3.53 Accept $t=\frac{10}{9}\ln(24)$ or exact equivalent.

(c)(ii)

A1 awrt 102 following 3.53 The M's must have been awarded. This is not a B mark.

(d)

B1 Sight of 40

Condone statements such as $P\rightarrow 40$ $k\ldots 40$ or likewise