Finding Areas using Integration 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question Number	Scheme	Marks
	(a) $\int x^{\frac{1}{2}} \ln 2x \, dx = \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{3}{2}} \times \frac{1}{x} dx$	M1 A1
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \int \frac{2}{3}x^{\frac{1}{2}} \, dx$	
	$= \frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}} \quad (+C)$	M1 A1 (4)
	(b) $\left[\frac{2}{3}x^{\frac{3}{2}} \ln 2x - \frac{4}{9}x^{\frac{3}{2}}\right]_1^4 = \left(\frac{2}{3}4^{\frac{3}{2}} \ln 8 - \frac{4}{9}4^{\frac{3}{2}}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$	M1
	$= (16 \ln 2 - \ ...) - \ ...$ Using or implying $\ln 2^n = n \ln 2$	M1
	$= \frac{46}{3}\ln 2 - \frac{28}{9}$	A1 (3)

Question 2

Question Number	Scheme		Marks
	$\left\{u = 1 + \sqrt{x}\right\} \Rightarrow \underline{\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{dx}{du} = 2(u-1)}$		<u>B1</u>
	$\left\{\int \frac{x}{1+\sqrt{x}}\, dx = \right\} \int \frac{(u-1)^2}{u} . 2(u-1)\, du$	$\int \frac{(u-1)^2}{u}$......	M1
		$\int \frac{(u-1)^2}{u} . 2(u-1)$	A1
	$= 2\int \frac{(u-1)^3}{u}\, du \; = \{2\}\int \frac{(u^3 - 3u^2 + 3u - 1)}{u}\, du$	Expands to give a "four term" cubic in u. Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$	M1
	$= \{2\}\int \left(u^2 - 3u + 3 - \frac{1}{u}\right) du$	An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$= \{2\}\left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	$\int \frac{(u-1)^3}{u} \to \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	A1
	$\text{Area}(R) = \left[\frac{2u^3}{3} - 3u^2 + 6u - 2\ln u\right]_2^3$		
	$= \left(\frac{2(3)^3}{3} - 3(3)^2 + 6(3) - 2\ln 3\right) - \left(\frac{2(2)^3}{3} - 3(2)^2 + 6(2) - 2\ln 2\right)$	Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$= \frac{11}{3} + 2\ln 2 - 2\ln 3$ or $\frac{11}{3} + 2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3} - \ln\left(\frac{9}{4}\right)$, etc	Correct exact answer or equivalent.	A1
			[8]

Question 3

Question Number	Scheme		Marks
	$\left\{\int (4t\mathrm{e}^{-\frac{1}{3}t}+3)\,\mathrm{d}t\right\} = -12t\mathrm{e}^{-\frac{1}{3}t} - \int -12\mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$	$\pm At\mathrm{e}^{-\frac{1}{3}t} \pm B\int \mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A \neq 0$, $B \neq 0$	M1
		See notes.	A1
	$+\,3t$	$3 \to 3t$	B1
	$= -12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}\{+3t\}$	$-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}$	A1
	$\left[-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t} + 3t\right]_0^8 =$		
	$= \left(-12(8)\mathrm{e}^{-\frac{1}{3}(8)} - 36\mathrm{e}^{-\frac{1}{3}(8)} + 3(8)\right) - \left(-12(0)\mathrm{e}^{-\frac{1}{3}(0)} - 36\mathrm{e}^{-\frac{1}{3}(0)} + 3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t}$ or $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.	dM1
	$= \left(-96\mathrm{e}^{-\frac{8}{3}} - 36\mathrm{e}^{-\frac{8}{3}} + 24\right) - (0 - 36 + 0)$		
	$= 60 - 132\mathrm{e}^{-\frac{8}{3}}$	$60 - 132\mathrm{e}^{-\frac{8}{3}}$	A1
			[6]

Question 4

Question Number	Scheme		Marks
(a)	$y=4x-xe^{\frac{1}{2}x},\ x\geqslant 0$ $\left\{y=0\Rightarrow 4x-xe^{\frac{1}{2}x}=0\Rightarrow x(4-e^{\frac{1}{2}x})=0\ \Rightarrow\right\}$		
	$e^{\frac{1}{2}x}=4\Rightarrow x_A=4\ln 2$	Attempts to solve $e^{\frac{1}{2}x}=4$ giving $x=\dots$ in terms of $\pm\lambda\ln\mu$ where $\mu>0$	M1
		$4\ln 2$ cao (Ignore $x=0$)	A1
			[2]
(b)	$\left\{\int xe^{\frac{1}{2}x}\,dx\right\}=\ 2xe^{\frac{1}{2}x}-\int 2e^{\frac{1}{2}x}\{dx\}$	$\alpha xe^{\frac{1}{2}x}-\beta\int e^{\frac{1}{2}x}\{dx\},\ \alpha>0,\beta>0$	M1
		$2xe^{\frac{1}{2}x}-\int 2e^{\frac{1}{2}x}\{dx\}$, with or without dx	A1 (M1 on ePEN)
	$=\ 2xe^{\frac{1}{2}x}-4e^{\frac{1}{2}x}\ \{+c\}$	$2xe^{\frac{1}{2}x}-4e^{\frac{1}{2}x}$ o.e. with or without $+c$	A1
			[3]
(c)	$\left\{\int 4x\,dx\right\}=\ 2x^2$	$4x\to 2x^2$ or $\dfrac{4x^2}{2}$ o.e.	B1
	$\left\{\int_0^{4\ln 2}(4x-xe^{\frac{1}{2}x})\,dx\right\}=\left[2x^2-\left(2xe^{\frac{1}{2}x}-4e^{\frac{1}{2}x}\right)\right]_0^{4\ln 2\text{ or }\ln 16\text{ or their limits}}$		
	$=\left(2(4\ln 2)^2-2(4\ln 2)e^{\frac{1}{2}(4\ln 2)}+4e^{\frac{1}{2}(4\ln 2)}\right)-\left(2(0)^2-2(0)e^{\frac{1}{2}(0)}+4e^{\frac{1}{2}(0)}\right)$	See notes	M1
	$=\left(32(\ln 2)^2-32(\ln 2)+16\right)-(4)$ $=32(\ln 2)^2-32(\ln 2)+12$	$32(\ln 2)^2-32(\ln 2)+12$, see notes	A1
			[3] 8

	Question Notes	
(a)	M1	Attempts to solve $e^{\frac{1}{2}x}=4$ giving $x=\dots$ in terms of $\pm\lambda\ln\mu$ where $\mu>0$
	A1	$4\ln 2$ cao stated in part (a) only (Ignore $x=0$)
(b)	NOTE	**Part (b) appears as M1M1A1 on ePEN, but is now marked as M1A1A1.**
	M1	Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x}-\beta\int e^{\frac{1}{2}x}\{dx\}$, where $\alpha>0,\beta>0$. (must be in this form) with or without dx
	A1	$2xe^{\frac{1}{2}x}-\int 2e^{\frac{1}{2}x}\{dx\}$ or equivalent, with or without dx. **Can be un-simplified.**
	A1	$2xe^{\frac{1}{2}x}-4e^{\frac{1}{2}x}$ or equivalent with or without $+c$. **Can be un-simplified.**
	Note	You can also allow $2e^{\frac{1}{2}x}(x-2)$ or $e^{\frac{1}{2}x}(2x-4)$ for the final A1.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	<u>SPECIAL CASE:</u> A candidate who uses $u=x$, $\dfrac{dv}{dx}=e^{\frac{1}{2}x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. (Applying their v counts for one consistent error.)

(c)	B1	$4x \to 2x^2$ or $\frac{4x^2}{2}$ oe
	M1	**Complete** method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bxe^{\frac{1}{2}x} \pm Ce^{\frac{1}{2}x}$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So subtracting 0 is M0.
	Note	$\ln 16$ or $2\ln 4$ or equivalent is fine as an upper limit.
	A1	A correct three term exact quadratic expression in $\ln 2$. For example allow for A1 • $32(\ln 2)^2 - 32(\ln 2) + 12$ • $8(2\ln 2)^2 - 8(4\ln 2) + 12$ • $2(4\ln 2)^2 - 32(\ln 2) + 12$ • $2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 12$
	Note	Note that the constant term of 12 needs to be combined from $4e^{\frac{1}{2}(4\ln 2)} - 4e^{\frac{1}{2}(0)}$ o.e.
	Note	Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \frac{12}{32\ln 2}\right)$ for A1.
	Note	***Do not apply "ignore subsequent working" for incorrect simplification.*** Eg: $32(\ln 2)^2 - 32(\ln 2) + 12 \to 64(\ln 2) - 32(\ln 2) + 12$ or $32(\ln 4) - 32(\ln 2) + 12$
	Note	**Bracketing error:** $32\ln 2^2 - 32(\ln 2) + 12$, unless recovered is final A0.
	Note	**Notation:** Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.
	Note	$5.19378\ldots$ without seeing $32(\ln 2)^2 - 32(\ln 2) + 12$ is A0.
	Note	$5.19378\ldots$ following from a correct $2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)$ is M1A0.
	Note	$5.19378\ldots$ from no working is M0A0.

Question 5

Question Number	Scheme		Marks
(a)	$A = \int_0^3 \sqrt{(3-x)(x+1)}\,\mathrm{d}x$, $x = 1 + 2\sin\theta$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ or $2\cos\theta$ used correctly in their working. Can be implied.	B1
	$\left\{\int \sqrt{(3-x)(x+1)}\,\mathrm{d}x \text{ or } \int \sqrt{(3+2x-x^2)}\,\mathrm{d}x\right\}$		
	$= \int \sqrt{(3-(1+2\sin\theta))((1+2\sin\theta)+1)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$	Substitutes for both x and $\mathrm{d}x$, where $\mathrm{d}x \neq \lambda \mathrm{d}\theta$. Ignore $\mathrm{d}\theta$	M1
	$= \int \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ $= \int \sqrt{(4-4\sin^2\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$		
	$= \int \sqrt{(4-4(1-\cos^2\theta)}\ 2\cos\theta\ \{\mathrm{d}\theta\}$ or $\int \sqrt{4\cos^2\theta}\ 2\cos\theta\ \{\mathrm{d}\theta\}$	Applies $\cos^2\theta = 1 - \sin^2\theta$ see notes	M1
	$= 4\int \cos^2\theta\,\mathrm{d}\theta,\ \{k = 4\}$	$4\int \cos^2\theta\mathrm{d}\theta$ or $\int 4\cos^2\theta\mathrm{d}\theta$ Note: $\mathrm{d}\theta$ is required here.	A1
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2} \Rightarrow \underline{\theta = -\frac{\pi}{6}}$ **and** $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \underline{\theta = \frac{\pi}{2}}$	See notes	B1
			[5]
(b)	$\left\{k\int \cos^2\theta\,\{\mathrm{d}\theta\}\right\} = \{k\}\int\left(\frac{1+\cos 2\theta}{2}\right)\{\mathrm{d}\theta\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral	M1
	$= \{k\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	Integrates to give $\pm\alpha\theta \pm \beta\sin 2\theta,\ \alpha \neq 0, \beta \neq 0$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$	M1 (A1 on ePEN)
	$\left\{\text{So } 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\mathrm{d}\theta = \left[2\theta + \sin 2\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}\right\}$ $= \left(2\left(\frac{\pi}{2}\right) + \sin\left(\frac{2\pi}{2}\right)\right) - \left(2\left(-\frac{\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right)\right)$		
	$\left\{= (\pi) - \left(-\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)\right\} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$	$\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ or $\frac{1}{6}(8\pi + 3\sqrt{3})$	A1 cao cso
			[3] 8

	Question Notes	
(a)	B1	$\frac{dx}{d\theta} = 2\cos\theta$. Also allow $dx = 2\cos\theta d\theta$. This mark can be implied by later working.
	Note	You can give B1 for $2\cos\theta$ used correctly in their working.
	M1	Substitutes $x = 1 + 2\sin\theta$ and their dx $\left(\text{from their rearranged } \frac{dx}{d\theta}\right)$ into $\sqrt{(3-x)(x+1)}\ dx$.
	Note	Condone bracketing errors here.
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$.
	Note	Condone substituting $dx = \cos\theta$ for the 1st M1 after a correct $\frac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta d\theta$
	M1	Applies either • $1 - \sin^2\theta = \cos^2\theta$ • $\lambda - \lambda\sin^2\theta$ or $\lambda(1 - \sin^2\theta) = \lambda\cos^2\theta$ • $4 - 4\sin^2\theta = 4 + 2\cos 2\theta - 2 = 2 + 2\cos 2\theta = 4\cos^2\theta$ to their expression where λ is a numerical value.
	A1	Correctly proves that $\int \sqrt{(3-x)(x+1)}\ dx$ is equal to $4\int \cos^2\theta d\theta$ or $\int 4\cos^2\theta d\theta$
	Note	All three previous marks must have been awarded before A1 can be awarded.
	Note	Their final answer must include $d\theta$.
	Note	You can ignore limits for the final A1 mark.
	B1	Evidence of a correct equation in $\sin\theta$ or $\sin^{-1}\theta$ for both x-values leading to both θ values. Eg: • $0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2}$ which then leads to $\theta = -\frac{\pi}{6}$, **and** • $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1$ which then leads to $\theta = \frac{\pi}{2}$
	Note	Allow B1 for $x = 1 + 2\sin\left(-\frac{\pi}{6}\right) = 0$ **and** $x = 1 + 2\sin\left(\frac{\pi}{2}\right) = 3$
	Note	Allow B1 for $\sin\theta = \left(\frac{x-1}{2}\right)$ or $\theta = \sin^{-1}\left(\frac{x-1}{2}\right)$ followed by $x = 0,\ \theta = -\frac{\pi}{6};\ x = 3,\ \theta = \frac{\pi}{2}$
(b)	NOTE	**Part (b) appears as M1A1A1 on ePEN, but is now marked as M1M1A1.**
	M1	Writes down a correct equation involving $\cos 2\theta$ and $\cos^2\theta$ **Eg:** $\cos 2\theta = 2\cos^2\theta - 1$ or $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$ or $\lambda\cos^2\theta = \lambda\left(\frac{1 + \cos 2\theta}{2}\right)$ and *applies* it to their integral. **Note:** Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.
	M1	Integrates to give an expression of the form $\pm\alpha\theta \pm \beta\sin 2\theta$ or $k(\pm\alpha\theta \pm \beta\sin 2\theta)$, $\alpha \neq 0, \beta \neq 0$ (can be simplified or un-simplified).
	A1	A *correct solution in part (b)* leading to a "two term" exact answer. Eg: $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ or $\frac{8\pi}{6} + \frac{\sqrt{3}}{2}$ or $\frac{1}{6}(8\pi + 3\sqrt{3})$
	Note	5.054815… from no working is M0M0A0.
	Note	Candidates can work in terms of k (note that k is not given in (a)) for the M1M1 marks in part (b).
	Note	If they incorrectly obtain $4\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta d\theta$ in part (a) (or guess $k = 4$) then the final A1 is available for a correct solution in part (b) only.

Question 6

Question Number	Scheme		Marks
Way 1	$\left\{I=\int x^2\ln x\,\mathrm{d}x\right\}$, $\left\{\begin{matrix} u=\ln x \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{x} \\ \dfrac{\mathrm{d}v}{\mathrm{d}x}=x^2 \Rightarrow v=\dfrac{1}{3}x^3 \end{matrix}\right\}$		
	$=\dfrac{x^3}{3}\ln x-\int\dfrac{x^3}{3}\left(\dfrac{1}{x}\right)\{\mathrm{d}x\}$	Either $x^2\ln x \to \pm\lambda x^3\ln x-\int\mu x^3\left(\dfrac{1}{x}\right)\{\mathrm{d}x\}$ or $\pm\lambda x^3\ln x-\int\mu x^2\{\mathrm{d}x\}$, where $\lambda,\mu>0$	M1
		$x^2\ln x \to \dfrac{x^3}{3}\ln x-\int\dfrac{x^3}{3}\left(\dfrac{1}{x}\right)\{\mathrm{d}x\}$, simplified or un-simplified	A1
	$=\dfrac{x^3}{3}\ln x-\dfrac{x^3}{9}$	$\dfrac{x^3}{3}\ln x-\dfrac{x^3}{9}$, simplified or un-simplified	A1
	$\text{Area}(R)=\left\{\left[\dfrac{x^3}{3}\ln x-\dfrac{x^3}{9}\right]_1^2\right\}=\left(\dfrac{8}{3}\ln 2-\dfrac{8}{9}\right)-\left(0-\dfrac{1}{9}\right)$	**dependent on the previous M mark.** Applies limits of 2 and 1 and subtracts the correct way round	dM1
	$=\dfrac{8}{3}\ln 2-\dfrac{7}{9}$	$\dfrac{8}{3}\ln 2-\dfrac{7}{9}$ or $\dfrac{1}{9}(24\ln 2-7)$	A1 oe cso
			[5]

Way 2	$I=x^2(x\ln x-x)-\int 2x(x\ln x-x)\,\mathrm{d}x$	$\left\{\begin{matrix} u=x^2 \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}=2x \\ \dfrac{\mathrm{d}v}{\mathrm{d}x}=\ln x \Rightarrow v=x\ln x-x \end{matrix}\right\}$	
	So, $3I=x^2(x\ln x-x)+\int 2x^2\{\mathrm{d}x\}$		
	and $I=\dfrac{1}{3}x^2(x\ln x-x)+\dfrac{1}{3}\int 2x^2\{\mathrm{d}x\}$	A full method of applying $u=x^2,\ v'=\ln x$ to give $\pm\lambda x^2(x\ln x-x)\pm\mu\int x^2\{\mathrm{d}x\}$	M1
		$\dfrac{1}{3}x^2(x\ln x-x)+\dfrac{1}{3}\int 2x^2\{\mathrm{d}x\}$ simplified or un-simplified	A1
	$=\dfrac{1}{3}x^2(x\ln x-x)+\dfrac{2}{9}x^3$	$\dfrac{x^3}{3}\ln x-\dfrac{x^3}{9}$, simplified or un-simplified	A1
		Then award dM1A1 in the same way as above	M1 A1
			[5]
			9

Question 7

Question Number	Scheme	Notes	Marks
(a)	$\left\{u = e^x \text{ or } x = \ln u \Rightarrow\right\}$		
	$\frac{du}{dx} = e^x$ or $\frac{du}{dx} = u$ or $\frac{dx}{du} = \frac{1}{u}$ or $du = u\,dx$ etc., and $\int \frac{6}{(e^x+2)}\,dx = \int \frac{6}{(u+2)u}\,du$	**See notes**	B1 *
	$\{x=0\} \Rightarrow a = e^0 \Rightarrow \underline{a=1}$ $\{x=1\} \Rightarrow b = e^1 \Rightarrow \underline{b=e}$	$a=1$ and $b=e$ or $b=e^1$ or evidence of $0 \to 1$ and $1 \to e$	B1
	NOTE: 1st B1 mark CANNOT be recovered for work in part (d) **NOTE: 2nd B1 mark CAN be recovered for work in part (d)**		**[2]**
(b) **Way 1**	$\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$ $\Rightarrow 6 \equiv A(u+2) + Bu$	Writing $\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$, o.e. **or** $\frac{1}{u(u+2)} \equiv \frac{P}{u} + \frac{Q}{(u+2)}$, o.e., and a complete method for finding the value of at least one of **their** A or **their** B (or **their** P or **their** Q)	M1
	$u=0 \Rightarrow A=3$ $u=-2 \Rightarrow B=-3$	Both **their** $A=3$ **and their** $B=-3$. (Or **their** $P=\frac{1}{2}$ **and their** $Q=-\frac{1}{2}$ with the factor of 6 in front of the integral sign)	A1
	$\int \frac{6}{u(u+2)}\,du = \int \left(\frac{3}{u} - \frac{3}{(u+2)}\right) du$	Integrates $\frac{M}{u} \pm \frac{N}{u \pm k}$, $M, N, k \neq 0$; (i.e. ***a two term partial fraction***) to obtain either $\pm\lambda\ln(\alpha u)$ **or** $\pm\mu\ln(\beta(u \pm k))$; $\lambda, \mu, \alpha, \beta \neq 0$	M1
	$= 3\ln u - 3\ln(u+2)$ or $= 3\ln 2u - 3\ln(2u+4)$	Integration of both terms is **correctly followed through** from **their** M and from **their** N.	A1 ft
	$\left\{\text{So } \left[3\ln u - 3\ln(u+2)\right]_1^e\right\}$ $= \left(3\ln(e) - 3\ln(e+2)\right) - \left(3\ln 1 - 3\ln 3\right)$ [Note: A proper consideration of the limit of $u=1$ is required for this mark]	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b>0, b \neq 1, a>0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$= 3 - 3\ln(e+2) + 3\ln 3$ **or** $3(1 - \ln(e+2) + \ln 3)$ **or** $3 + 3\ln\left(\frac{3}{e+2}\right)$ **or** $3\ln\left(\frac{e}{e+2}\right) - 3\ln\left(\frac{1}{3}\right)$ **or** $3 - 3\ln\left(\frac{e+2}{3}\right)$ **or** $3\ln\left(\frac{3e}{e+2}\right)$ **or** $\ln\left(\frac{27e^3}{(e+2)^3}\right)$	**see notes**	A1 cso
	Note: Allow e^1 in place of e for the final A1 mark.		**[6]**
	Note: Give final A0 for $3 - 3\ln e + 2 + 3\ln 3$ (i.e. bracketing error) unless recovered.		**12**
	Note: Give final A0 for $3 - 3\ln(e+2) + 3\ln 3 - 3\ln 1$, where $3\ln 1$ has not been simplified to 0		
	Note: Give final A0 for $3\ln e - 3\ln(e+2) + 3\ln 3$, where $3\ln e$ has not been simplified to 3		

(a)	**1^{st} B1**	Must start from either • $\int y\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(e^x+2)}\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(e^x+2)}\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u$, with integral sign and $\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ **and** state either $\frac{\mathrm{d}u}{\mathrm{d}x}=e^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\mathrm{d}x$ **and** end at $\int \frac{6}{u(u+2)}\,\mathrm{d}u$, with integral sign and $\mathrm{d}u$, **with no incorrect working.**
	Note	So, just writing $\frac{\mathrm{d}u}{\mathrm{d}x}=e^x$ **and** $\int \frac{6}{(e^x+2)}\,\mathrm{d}x=\int \frac{6}{u(u+2)}\,\mathrm{d}u$ is sufficient for 1^{st} B1
	Note	Give 2^{nd} B0 for $b=2.718...$, without reference to $a=1$ and $b=e$ or $b=e^1$
	Note	You can also give the 1^{st} B1 mark for using a reverse process. i.e. Proceeding from $\int \frac{6}{u(u+2)}\,\mathrm{d}u$ to $\int \frac{6}{(e^x+2)}\,\mathrm{d}x$, **with no incorrect working,** and stating either $\frac{\mathrm{d}u}{\mathrm{d}x}=e^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\mathrm{d}x$
(b)	**Note**	Give final A0 for $3-3\ln(e+2)+3\ln 3$ simplifying to $1-\ln(e+2)+\ln 3$ (i.e. dividing their correct final answer by 3) Otherwise, you can ignore incorrect working (isw) following on from a correct exact value.
	Note	A decimal answer of 1.641502724... (without a correct **exact** answer) is final A0
	Note	$\left[-3\ln(u+2)+3\ln u\right]_1^e$ followed by awrt 1.64 (without a correct **exact** answer) is final M1A0

	Question	Notes Continued
(b)	**Note**	**BE CAREFUL! Candidates will assign their own "A" and "B" for this question.**
	Note	*Writing down* $\dfrac{6}{(u+2)u}$ in the form $\dfrac{A}{(u+2)}+\dfrac{B}{u}$ with at least one of A or B correct is 1st M1
	Note	*Writing down* $\dfrac{6}{(u+2)u}$ as $\dfrac{-3}{(u+2)}+\dfrac{3}{u}$ is 1st M1 1st A1.
	Note	**Condone** $\int\left(\dfrac{3}{u}-\dfrac{3}{(u+2)}\right)\mathrm{d}u$ to give $3\ln u-3\ln u+2$ (poor bracketing) for 2nd A1.
	Note	**Award M0A0M1A1ft** for a candidate who writes down e.g. $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u=\int\left(\dfrac{6}{u}+\dfrac{6}{(u+2)}\right)\mathrm{d}u=6\ln u+6\ln(u+2)$ **AS EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ **AS PARTIAL FRACTIONS.**
	Note	**Award M0A0M0A0** for a candidate who writes down $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u=6\ln u+6\ln(u+2)$ **or** $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u=\ln u+6\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	**Award M1A1M1A1** for a candidate who writes down $\int\dfrac{6}{u(u+2)}\,\mathrm{d}u=3\ln u-3\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\dfrac{6}{u(u+2)}$ as partial fractions.
	Note	If they lose the "6" and find $\int_1^e\dfrac{1}{u(u+2)}\mathrm{d}u$ we can allow a maximum of M1A0M1A1ftM1A0

	Question Notes Continued		
(b) **Way 2**	$\left\{\int \frac{6}{u^2+2u}\,\mathrm{d}u = \int \frac{3(2u+2)}{u^2+2u}\,\mathrm{d}u - \int \frac{6u}{u^2+2u}\,\mathrm{d}u\right\}$		
	$= \int \frac{3(2u+2)}{u^2+2u}\,\mathrm{d}u - \int \frac{6}{u+2}\,\mathrm{d}u$	$\int \frac{\pm\alpha(2u+2)}{u^2+2u}\{\mathrm{d}u\} \pm \int \frac{\delta}{u+2}\{\mathrm{d}u\},\ \alpha, \beta, \delta \neq 0$	M1
		Correct expression	A1
	$= 3\ln(u^2+2u) - 6\ln(u+2)$	Integrates $\frac{\pm M(2u+2)}{u^2+2u} \pm \frac{N}{u\pm k}$, $M, N, k \neq 0$, to obtain any one of $\pm\lambda\ln(u^2+2u)$ **or** $\pm\mu\ln(\beta(u\pm k))$; $\lambda, \mu, \beta \neq 0$	M1
		Integration of both terms is **correctly followed through** from **their** M and from **their** N	A1 ft
	$\left\{\text{So, } \left[3\ln(u^2+2u)-6\ln(u+2)\right]_1^{\mathrm{e}}\right\}$ $= \left(3\ln(\mathrm{e}^2+2\mathrm{e}) - 6\ln(\mathrm{e}+2)\right) - \left(3\ln 3 - 6\ln 3\right)$	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b>0, b\neq 1, a>0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$= 3\ln(\mathrm{e}^2+2\mathrm{e}) - 6\ln(\mathrm{e}+2) + 3\ln 3$	$3\ln(\mathrm{e}^2+2\mathrm{e}) - 6\ln(\mathrm{e}+2) + 3\ln 3$	A1 o.e.
			[6]
(b) **Way 3**	Applying $u = \theta - 1$		
	$\left\{\int_1^{\mathrm{e}} \frac{6}{u(u+2)}\,\mathrm{d}u = \right\} \int_2^{1+\mathrm{e}} \frac{6}{(\theta-1)(\theta+1)}\,\mathrm{d}\theta = \int_2^{1+\mathrm{e}} \frac{6}{\theta^2-1}\,\mathrm{d}u = \left[3\ln\left(\frac{\theta-1}{\theta+1}\right)\right]_2^{1+\mathrm{e}}$		M1A1M1A1
	$= 3\ln\left(\frac{1+\mathrm{e}-1}{\mathrm{e}+1+1}\right) - 3\ln\left(\frac{2-1}{2+1}\right) = 3\ln\left(\frac{\mathrm{e}}{\mathrm{e}+2}\right) - 3\ln\left(\frac{1}{3}\right)$	3rd M mark is dependent on 2nd M mark	dM1A1
			[6]