

Implicit Differentiation 2 - Edexcel Past Exam Questions **MARK SCHEME**
Question 1

Question Number	Scheme	Marks
1. (a)	$\left\{ \frac{\cancel{dy}}{\cancel{dx}} \right\} \times \left\{ 2 + 6y \frac{dy}{dx} + \left(\underline{\underline{6xy + 3x^2 \frac{dy}{dx}}} \right) = \underline{\underline{8x}} \right.$ $\left\{ \frac{dy}{dx} = \frac{8x - 2 - 6xy}{6y + 3x^2} \right\} \quad \text{not necessarily required.}$ $\text{At } P(-1, 1), \quad m(T) = \frac{dy}{dx} = \frac{8(-1) - 2 - 6(-1)(1)}{6(1) + 3(-1)^2} = -\frac{4}{9}$	M1 A1 B1 dM1 A1 cso [5]
(b)	<p>So, $m(N) = \frac{-1}{-\frac{4}{9}} \left\{ = \frac{9}{4} \right\}$</p> <p>N: $y - 1 = \frac{9}{4}(x + 1)$</p> <p>N: $9x - 4y + 13 = 0$</p>	M1 M1 A1 [3] 8

(a)	<p>M1: Differentiates implicitly to include either $\pm ky \frac{dy}{dx}$ or $3x^2 \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).</p> <p>A1: $(2x + 3y^2) \rightarrow \left(2 + 6y \frac{dy}{dx} \right)$ and $(4x^2 \rightarrow \underline{\underline{8x}})$. Note: If an extra "sixth" term appears then award A0.</p> <p>B1: $6xy + 3x^2 \frac{dy}{dx}$.</p> <p>dM1: Substituting $x = -1$ and $y = 1$ into an equation involving $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dy}{dx} = \frac{8x - 2 - 6xy}{6y + 3x^2}$ is substituted into or evaluated correctly.</p> <p>If it is clear, however, that the candidate is intending to substitute $x = 1$ and $y = -1$, then award M0.</p> <p>Candidates who substitute $x = 1$ and $y = -1$, will usually achieve $m(T) = -4$</p> <p>Note that this mark is dependent on the previous method mark being awarded.</p> <p>A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or -0.4 or awrt -0.44</p> <p>If the candidate's solution is not completely correct, then do not give this mark.</p>	
(b)	<p>M1: Applies $m(N) = -\frac{1}{\text{their } m(T)}$.</p> <p>M1: Uses $y - 1 = (m_N)(x - -1)$ or finds c using $x = -1$ and $y = 1$ and uses $y = (m_N)x + "c"$,</p> <p>Where $m_N = -\frac{1}{\text{their } m(T)}$ or $m_N = \frac{1}{\text{their } m(T)}$ or $m_N = -\text{their } m(T)$.</p> <p>A1: $9x - 4y + 13 = 0$ or $-9x + 4y - 13 = 0$ or $4y - 9x - 13 = 0$ or $18x - 8y + 26 = 0$ etc.</p> <p>Must be "$= 0$". So do not allow $9x + 13 = 4y$ etc.</p> <p>Note: $m_N = -\left(\frac{6y + 3x^2}{8x - 2 - 6xy} \right)$ is M0M0 unless a numerical value is then found for m_N.</p>	

Question 2

Question Number	Scheme	Marks
	<p>(a) Differentiating implicitly to obtain $\pm ay^2 \frac{dy}{dx}$ and/or $\pm bx^2 \frac{dy}{dx}$</p> <p>$48y^2 \frac{dy}{dx} + \dots - 54 \dots$</p> <p>$9x^2 y \rightarrow 9x^2 \frac{dy}{dx} + 18xy$ or equivalent</p> <p>$(48y^2 + 9x^2) \frac{dy}{dx} + 18xy - 54 = 0$</p> <p>$\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2} \left(= \frac{18 - 6xy}{16y^2 + 3x^2} \right)$</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p>
	<p>(b) $18 - 6xy = 0$</p> <p>Using $x = \frac{3}{y}$ or $y = \frac{3}{x}$</p> <p>$16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0$ or $16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0$</p> <p>Leading to</p> <p>$16y^4 + 81 - 162 = 0$ or $16 + x^4 - 2x^4 = 0$</p> <p>$y^4 = \frac{81}{16}$ or $x^4 = 16$</p> <p>$y = \frac{3}{2}, -\frac{3}{2}$ or $x = 2, -2$</p> <p>Substituting either of their values into $xy = 3$ to obtain a value of the other variable.</p> <p>$\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right)$ both</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>[12]</p>

Question 3

Question Number	Scheme	Marks
(a)	$x^2 + 4xy + y^2 + 27 = 0$ $\left\{ \begin{array}{l} \cancel{x^2} \times \\ \cancel{4x} \end{array} \right\} \underline{2x} + \left(4y + 4x \frac{dy}{dx} \right) + 2y \frac{dy}{dx} = 0$ $2x + 4y + (4x + 2y) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y} \left\{ = \frac{-x - 2y}{2x + y} \right\}$	<p>M1 A1 B1</p> <p>dM1</p> <p>A1 cso oe</p> <p>[5]</p>
(b)	$4x + 2y = 0$ $y = -2x$ $x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$ $-3x^2 + 27 = 0$ $x^2 = 9$ $x = -3$ <p>When $x = -3$, $y = -2(-3)$</p> $y = 6$	<p>M1</p> <p>A1</p> <p>M1*</p> <p>dM1*</p> <p>A1</p> <p>ddM1*</p> <p>A1 cso</p> <p>[7]</p> <p>12</p>
Notes for Question		
(a)	<p>M1: Differentiates implicitly to include either $4x \frac{dy}{dx}$ or $\pm ky \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).</p> <p>A1: $(x^2) \rightarrow (2x)$ and $\left(\dots + y^2 + 27 = 0 \rightarrow + 2y \frac{dy}{dx} = 0 \right)$.</p> <p>Note: If an extra term appears then award A0.</p> <p>Note: The "$= 0$" can be implied by rearrangement of their equation.</p> <p>i.e.: $2x + 4y + 4x \frac{dy}{dx} + 2y \frac{dy}{dx}$ leading to $4x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 4y$ will get A1 (implied).</p> <p>B1: $4y + 4x \frac{dy}{dx}$ or $4 \left(y + x \frac{dy}{dx} \right)$ or equivalent</p> <p>dM1: An attempt to factorise out $\frac{dy}{dx}$ as long as there are at least two terms in $\frac{dy}{dx}$.</p> <p>ie. $\dots + (4x + 2y) \frac{dy}{dx} = \dots$ or $\dots + 2(2x + y) \frac{dy}{dx} = \dots$</p> <p>Note: This mark is dependent on the previous method mark being awarded.</p> <p>A1: For $\frac{-2x - 4y}{4x + 2y}$ or equivalent. Eg: $\frac{+2x + 4y}{-4x - 2y}$ or $\frac{-2(x + 2y)}{4x + 2y}$ or $\frac{-x - 2y}{2x + y}$</p> <p>cso: If the candidate's solution is not completely correct, then do not give this mark.</p>	

Notes for Question Continued	
(b)	<p>M1: Sets the denominator of their $\frac{dy}{dx}$ equal to zero (or the numerator of their $\frac{dx}{dy}$ equal to zero) oe.</p> <p>A1: Rearranges to give either $y = -2x$ or $x = -\frac{1}{2}y$. (correct solution only).</p> <p>The first two marks can be implied from later working, i.e. for a correct substitution of either $y = -2x$ into y^2 or for $x = -\frac{1}{2}y$ into $4xy$.</p> <p>M1*: Substitutes $y = \pm \lambda x$ or $x = \pm \mu y$ or $y = \pm \lambda x \pm a$ or $x = \pm \mu y \pm b$ ($\lambda \neq 0, \mu \neq 0$) into $x^2 + 4xy + y^2 + 27 = 0$ to form an equation in one variable.</p> <p>dM1*: leading to at least either $x^2 = A, A > 0$ or $y^2 = B, B > 0$</p> <p>Note: This mark is dependent on the previous method mark (M1*) being awarded.</p> <p>A1: For $x = -3$ (ignore $x = 3$) or if y was found first, $y = 6$ (ignore $y = -6$) (correct solution only).</p> <p>ddM1*: Substitutes their value of x into $y = \pm \lambda x$ to give $y = \text{value}$ or substitutes their value of x into $x^2 + 4xy + y^2 + 27 = 0$ to give $y = \text{value}$.</p> <p>Alternatively, substitutes their value of y into $x = \pm \mu y$ to give $x = \text{value}$ or substitutes their value of y into $x^2 + 4xy + y^2 + 27 = 0$ to give $x = \text{value}$</p> <p>Note: This mark is dependent on the two previous method marks (M1* and dM1*) being awarded.</p> <p>A1: $(-3, 6)$ cso.</p> <p>Note: If a candidate offers two sets of coordinates without either rejecting the incorrect set or accepting the correct set then award A0. DO NOT APPLY ISW ON THIS OCCASION.</p> <p>Note: $x = -3$ followed later in working by $y = 6$ is fine for A1.</p> <p>Note: $y = 6$ followed later in working by $x = -3$ is fine for A1.</p> <p>Note: $x = -3, 3$ followed later in working by $y = 6$ is A0, unless candidate indicates that they are rejecting $x = 3$</p> <p>Note: Candidates who set the numerator of $\frac{dy}{dx}$ equal to 0 (or the denominator of their $\frac{dx}{dy}$ equal to zero) can <i>only achieve a maximum of 3 marks</i> in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a maximum marking profile of M0A0M1M1A0M1A0. They will usually find $(-6, 3)$ { or even $(6, -3)$ }.</p> <p>Note: Candidates who set <i>the numerator or the denominator</i> of $\frac{dy}{dx}$ equal to $\pm k$ (usually $k = 1$) can <i>only achieve a maximum of 3 marks</i> in this part. They can only achieve the 2nd, 3rd and 4th Method marks to give a marking profile of M0A0M1M1A0M1A0.</p> <p>Special Case: It is possible for a candidate who does not achieve full marks in part (a), (but has a correct denominator for $\frac{dy}{dx}$) to gain all 7 marks in part (b).</p> <p>Eg: An incorrect part (a) answer of $\frac{dy}{dx} = \frac{2x - 4y}{4x + 2y}$ can lead to a correct $(-3, 6)$ in part (b) and 7 marks.</p>

Question 4

Question Number	Scheme	Marks
	$3^{x-1} + xy - y^2 + 5 = 0$ $\left\{ \begin{array}{l} \frac{dy}{dx} \times \\ \frac{dy}{dx} \end{array} \right\} 3^{x-1} \ln 3 + \left(y + x \frac{dy}{dx} \right) - 2y \frac{dy}{dx} = 0$ <p>(ignore)</p> $\{(1, 3) \Rightarrow\} 3^{(1-1)} \ln 3 + 3 + (1) \frac{dy}{dx} - 2(3) \frac{dy}{dx} = 0$ $\ln 3 + 3 + \frac{dy}{dx} - 6 \frac{dy}{dx} = 0 \Rightarrow 3 + \ln 3 = 5 \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{3 + \ln 3}{5}$ $\frac{dy}{dx} = \frac{1}{5} (\ln e^3 + \ln 3) = \frac{1}{5} \ln(3e^3)$	<p>$3^{x-1} \rightarrow 3^{x-1} \ln 3$</p> <p>Differentiates implicitly to include either $\pm \lambda x \frac{dy}{dx}$ or $\pm ky \frac{dy}{dx}$.</p> <p>$xy \rightarrow +y + x \frac{dy}{dx}$</p> <p>$\dots + y + x \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$</p> <p>Substitutes $x = 1, y = 3$ into their differentiated equation or expression.</p> <p>Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5} \ln(3e^3)$</p> <p>B1 oe</p> <p>M1*</p> <p>B1</p> <p>A1</p> <p>dM1*</p> <p>dM1*</p> <p>A1 cso</p> <p>[7]</p> <p>7</p>
Notes for Question		
<p>B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1} \rightarrow 3^{x-1} \ln 3$ or $3^{x-1} = \frac{1}{3}(3^x) \rightarrow \frac{1}{3}(3^x) \ln 3$</p> <p>or $3^{x-1} = e^{(x-1)\ln 3} \rightarrow \ln 3 e^{(x-1)\ln 3}$ or $3^{x-1} = \frac{1}{3}(3^x) = \frac{1}{3}e^{x \ln 3} \rightarrow \frac{1}{3}(\ln 3)e^{x \ln 3}$</p> <p>M1: Differentiates implicitly to include either $\pm \lambda x \frac{dy}{dx}$ or $\pm ky \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).</p> <p>B1: $xy \rightarrow +y + x \frac{dy}{dx}$</p> <p>1st A1: $\dots + y + x \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$ Note: The 1st A0 follows from an award of the 2nd B0.</p> <p>Note: The "$= 0$" can be implied by rearrangement of their equation.</p> <p>ie: $3^{x-1} \ln 3 + y + x \frac{dy}{dx} - 2y \frac{dy}{dx}$ leading to $3^{x-1} \ln 3 + y = 2y \frac{dy}{dx} - x \frac{dy}{dx}$ will get A1 (implied).</p> <p>2nd M1: Note: This method mark is dependent upon the 1st M1* mark being awarded.</p> <p>Substitutes $x = 1, y = 3$ into their differentiated equation or expression. Allow one slip.</p> <p>3rd M1: Note: This method mark is dependent upon the 1st M1* mark being awarded.</p> <p>Candidate has two differentiated terms in $\frac{dy}{dx}$ and rearranges to make $\frac{dy}{dx}$ the subject.</p> <p>Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.</p> <p>Eg: Candidate may write $\frac{dy}{dx} = \frac{y + 3^{x-1} \ln 3}{2y - x}$ before substituting in $x = 1$ and $y = 3$</p> <p>2nd A1: cso. Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5} \ln(3e^3)$, $\left(= \frac{1}{\lambda} \ln(\mu e^3) \right)$, $\lambda = 5$ and $\mu = 3$</p> <p>Note: $3 = \ln e^3$ needs to be seen in their proof.</p>		

Notes for Question Continued		
<p><i>Aliter</i> Way 2</p>	<p><u>Alternative Method: Multiplying both sides by 3</u></p> $3^{x-1} + xy - y^2 + 5 = 0$ $3^x + 3xy - 3y^2 + 15 = 0$	
	<p>$\left\{ \frac{dy}{dx} \right\} \times \left\{ 3^x \ln 3 + \left(3y + 3x \frac{dy}{dx} \right) - 6y \frac{dy}{dx} = 0 \right.$ (ignore)</p> <p>$\{(1, 3) \Rightarrow\} 3^1 \ln 3 + 3(3) + (3)(1) \frac{dy}{dx} - 6(3) \frac{dy}{dx} = 0$</p> $3 \ln 3 + 9 + 3 \frac{dy}{dx} - 18 \frac{dy}{dx} = 0 \Rightarrow 9 + 3 \ln 3 = 15 \frac{dy}{dx}$ $\frac{dy}{dx} = \frac{9 + 3 \ln 3}{15} \left\{ = \frac{3 + \ln 3}{5} \right\}$ $\frac{dy}{dx} = \frac{1}{5} (\ln e^3 + \ln 3)$ $\frac{dy}{dx} = \frac{1}{5} (\ln e^3 + \ln 3) = \frac{1}{5} \ln(3e^3)$	<p>$3^x \rightarrow 3^x \ln 3$ B1</p> <p>Differentiates implicitly to include either $\pm \lambda x \frac{dy}{dx}$ or $\pm ky \frac{dy}{dx}$ M1*</p> <p>$3xy \rightarrow + 3y + 3x \frac{dy}{dx}$ B1</p> <p>$\dots + 3y + 3x \frac{dy}{dx} - 6y \frac{dy}{dx} = 0$ A1</p> <p>Substitutes $x = 1, y = 3$ into their differentiated equation or expression. dM1*</p> <p>dM1*</p> <p>Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5} \ln(3e^3)$ A1 cso</p>
<p>NOTE: Only apply this scheme if the candidate has multiplied both sides of their equation by 3.</p> <p>NOTE: For reference, $\frac{dy}{dx} = \frac{3y + 3^x \ln 3}{6y - 3x}$</p> <p>NOTE: If the candidate applies this method then $3xy \rightarrow + 3y + 3x \frac{dy}{dx}$ must be seen for the 2nd B1 mark.</p>		

[7]
7

Question 5

Question Number	Scheme		Marks
	$x^3 + 2xy - x - y^3 - 20 = 0$		
(a)	$\left\{ \frac{dy}{dx} \right\} \times \left\{ 3x^2 + \left(2y + 2x \frac{dy}{dx} \right) - 1 - 3y^2 \frac{dy}{dx} = 0 \right.$ $3x^2 + 2y - 1 + (2x - 3y^2) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x} \quad \text{or} \quad \frac{1 - 3x^2 - 2y}{2x - 3y^2}$		M1 <u>A1</u> <u>B1</u> dM1 A1 cso [5]
(b)	At $P(3, -2)$, $m(T) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)} = \frac{22}{6}$ or $\frac{11}{3}$ and either T: $y - -2 = \frac{11}{3}(x - 3)$ or $(-2) = \left(\frac{11}{3} \right)(3) + c \Rightarrow c = \dots$, T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$		see notes M1 A1 cso
			[2] 7
(a)	<u>Alternative method for part (a)</u> $\left\{ \frac{dx}{dy} \right\} \times \left\{ 3x^2 \frac{dx}{dy} + \left(2y \frac{dx}{dy} + 2x \right) - \frac{dx}{dy} - 3y^2 = 0 \right.$ $2x - 3y^2 + (3x^2 + 2y - 1) \frac{dx}{dy} = 0$ $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x} \quad \text{or} \quad \frac{1 - 3x^2 - 2y}{2x - 3y^2}$		M1 <u>A1</u> <u>B1</u> dM1 A1 cso [5]
Question Notes			
(a) General	Note	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ from no working is full marks.	
	Note	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\frac{1 - 3x^2 - 2y}{3y^2 - 2x}$ from no working is M1A0B0M1A0	
	Note	Few candidates will write $3x^2 + 2y + 2x \frac{dy}{dx} - 1 - 3y^2 \frac{dy}{dx} = 0$ leading to $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$, o.e. This should get full marks.	
(a)	M1	Differentiates implicitly to include either $2x \frac{dy}{dx}$ or $-y^3 \rightarrow \pm k y^2 \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).	
	A1	$x^3 \rightarrow 3x^2$ and $-x - y^3 - 20 = 0 \rightarrow -1 - 3y^2 \frac{dy}{dx} = 0$	
	B1	$2xy \rightarrow 2y + 2x \frac{dy}{dx}$	
	Note	If an extra term appears then award 1 st A0.	

(a) ctd	<p>Note</p> <p>dM1</p> <p>Note</p> <p>A1</p>	$3x^2 + 2y + 2x \frac{dy}{dx} - 1 - 3y^2 \frac{dy}{dx} \rightarrow 3x^2 + 2y - 1 = 3y^2 \frac{dy}{dx} - 2x \frac{dy}{dx}$ <p>will get 1st A1 (implied) as the "= 0" can be implied by rearrangement of their equation.</p> <p>dependent on the first method mark being awarded.</p> <p>An attempt to factorise out all the terms in $\frac{dy}{dx}$ as long as there are <i>at least two terms</i> in $\frac{dy}{dx}$.</p> <p>ie. $\dots + (2x - 3y^2) \frac{dy}{dx} = \dots$</p> <p>Placing an extra $\frac{dy}{dx}$ at the beginning and then including it in their factorisation is fine for dM1.</p> <p>For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$</p> <p>cso: If the candidate's solution is not completely correct, then do not give this mark.</p> <p>isw: You can, however, ignore subsequent working following on from correct solution.</p>
(b)	<p>M1</p> <p>Note</p> <p>A1</p> <p>cso</p> <p>isw</p>	<p>Some attempt to substitute both $x = 3$ and $y = -2$ into their $\frac{dy}{dx}$ which contains both x and y to find m_T and</p> <ul style="list-style-type: none"> either applies $y - -2 = (\text{their } m_T)(x - 3)$, where m_T is a numerical value. or finds c by solving $(-2) = (\text{their } m_T)(3) + c$, where m_T is a numerical value. <p>Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ is M0).</p> <p>Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.</p> <p>A correct solution is required from a correct $\frac{dy}{dx}$.</p> <p>You can ignore subsequent working following a correct solution.</p>
(a)	<p>M1</p> <p>A1</p> <p>B1</p> <p>dM1</p> <p>A1</p>	<p>Alternative method for part (a): Differentiating with respect to y</p> <p>Differentiates implicitly to include either $2y \frac{dx}{dy}$ or $x^3 \rightarrow \pm kx^2 \frac{dx}{dy}$ or $-x \rightarrow -\frac{dx}{dy}$</p> <p>(Ignore $\left(\frac{dx}{dy} = \right)$).</p> <p>$x^3 \rightarrow 3x^2 \frac{dx}{dy}$ and $-x - y^3 - 20 = 0 \rightarrow -\frac{dx}{dy} - 3y^2 = 0$</p> <p>$2xy \rightarrow 2y \frac{dx}{dy} + 2x$</p> <p>dependent on the first method mark being awarded.</p> <p>An attempt to factorise out all the terms in $\frac{dx}{dy}$ as long as there are <i>at least two terms</i> in $\frac{dx}{dy}$.</p> <p>For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$</p> <p>cso: If the candidate's solution is not completely correct, then do not give this mark.</p>

Question 6

Question Number	Scheme	Marks
(a)	$x^2 + y^2 + 10x + 2y - 4xy = 10$ $\left\{ \frac{dy}{dx} \right\} \times \left\{ 2x + 2y \frac{dy}{dx} + 10 + 2 \frac{dy}{dx} - \left(4y + 4x \frac{dy}{dx} \right) \right\} = 0$ $2x + 10 - 4y + (2y + 2 - 4x) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{2x + 10 - 4y}{4x - 2y - 2}$ <p>Simplifying gives $\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1} \left\{ = \frac{-x - 5 + 2y}{-2x + y + 1} \right\}$</p>	<p>See notes</p> <p>M1 A1 M1</p> <p>Dependent on the first M1 mark.</p> <p>dM1</p> <p>A1 cso oe</p> <p>[5]</p>
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} x + 5 - 2y = 0$ <p>So $x = 2y - 5$,</p> $(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$ $4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$ <p>gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$</p> $(3y - 7)(y - 5) = 0 \text{ and } y = \dots$ $y = \frac{7}{3}, 5$	<p>M1</p> <p>M1</p> <p>A1 oe</p> <p>see notes</p> <p>Method mark for solving a quadratic equation.</p> <p>ddM1</p> <p>A1 cao</p> <p>[5]</p>
(b)	<p>Alternative method for part (b)</p> $\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} x + 5 - 2y = 0$ <p>So $y = \frac{x + 5}{2}$,</p> $x^2 + \left(\frac{x + 5}{2} \right)^2 + 10x + 2 \left(\frac{x + 5}{2} \right) - 4x \left(\frac{x + 5}{2} \right) = 10$ $x^2 + \frac{x^2 + 10x + 25}{4} + 10x + x + 5 - 2x^2 - 10x = 10$ $4x^2 + x^2 + 10x + 25 + 40x + 4x + 20 - 8x^2 - 40x = 40$ <p>gives $-3x^2 + 14x + 5 = 0$ or $3x^2 - 14x - 5 = 0$</p> $(3x + 1)(x - 5) = 0, x = \dots$ $y = \frac{-\frac{1}{3} + 5}{2}, \frac{5 + 5}{2}$ $y = \frac{7}{3}, 5$	<p>M1</p> <p>M1</p> <p>A1 oe</p> <p>see notes</p> <p>Solves a quadratic and finds at least one value for y.</p> <p>ddM1</p> <p>A1 cao</p> <p>[5]</p>
		10

	Question	Notes
(a)	M1	Differentiates implicitly to include either $\pm 4x \frac{dy}{dx}$ or $y^2 \rightarrow 2y \frac{dy}{dx}$ or $2y \rightarrow 2 \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1	$x^2 + y^2 + 10x + 2y \rightarrow 2x + 2y \frac{dy}{dx} + 10 + 2 \frac{dy}{dx}$ and $10 \rightarrow 0$
	M1	$-4xy \rightarrow \pm 4y \pm 4x \frac{dy}{dx}$
	Note	If an extra term appears then award 1 st A0.
	Note	$2x + 2y \frac{dy}{dx} + 10 + 2 \frac{dy}{dx} - 4y - 4x \frac{dy}{dx} \rightarrow 2x + 10 - 4y = -2y \frac{dy}{dx} - 2 \frac{dy}{dx} + 4x \frac{dy}{dx}$ will get 1 st A1 (implied) as the "= 0" can be implied by rearrangement of their equation.
	dm1	dependent on the first method mark being awarded. An attempt to factorise out all the terms in $\frac{dy}{dx}$ as long as there are <i>at least two terms in</i> $\frac{dy}{dx}$.
	A1	$\frac{x+5-2y}{2x-y-1}$ or $\frac{-x-5+2y}{-2x+y+1}$ (must be simplified).
	cso:	If the candidate's solution is not completely correct, then do not give this mark.
(b)	M1	Sets the numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) oe.
	NOTE	If the numerator involves one variable only then <i>only</i> the 1 st M1 mark is possible in part (b).
	M1	Substitutes their x or their y into the printed equation to give an equation in one variable only.
	A1	For obtaining either $-3y^2 + 22y - 35 \{= 0\}$ or $3y^2 - 22y + 35 \{= 0\}$
	Note	This mark can also awarded for a correct three term equation, eg. either $-3y^2 + 22y = 35$ $3y^2 - 22y = -35$ or $3y^2 + 35 = 22y$ are all fine for A1.
	ddM1	Dependent on the previous 2 M marks. See notes at the beginning of the mark scheme: Method mark for solving a 3 term quadratic
		<ul style="list-style-type: none"> $(3y - 7)(y - 5) = 0 \Rightarrow y = \dots$ $y = \frac{22 \pm \sqrt{(-22)^2 - 4(3)(35)}}{2(3)}$ $y^2 - \frac{22}{3}y - \frac{35}{3} = 0 \Rightarrow \left(y - \frac{11}{3}\right)^2 - \frac{121}{9} + \frac{35}{3} = 0 \Rightarrow y = \dots$ Or writes down at least one correct y-root from their quadratic equation. This is usually found from their calculator.
	Note	If a candidate applies <i>the alternative method</i> then they also need to use their $y = \frac{x+5}{2}$ in order to find at least one value for y in order to gain the final M1.
	A1	$y = \frac{7}{3}, 5$. cao. (2.33 or 2.3 without reference to $\frac{7}{3}$ or $2\frac{1}{3}$ is not allowed for this mark.)
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 5 marks in part (b).

Question 7

Question Number	Scheme	Marks
(a)	$x^2 - 3xy - 4y^2 + 64 = 0$	
	$\left\{ \frac{dx}{dx} \right\} 2x - \left(3y + 3x \frac{dy}{dx} \right) - 8y \frac{dy}{dx} = 0$	M1A1 M1
	$2x - 3y + (-3x - 8y) \frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} \text{ or } \frac{3y - 2x}{-3x - 8y}$	o.e. A1 cso
		[5]
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} 2x - 3y = 0$	M1
	$y = \frac{2}{3}x$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$	
	$\left\{ \Rightarrow x^2 = \frac{576}{25} \Rightarrow \right\} x = \frac{24}{5} \text{ or } -\frac{24}{5}$	A1 cso
	When $x = \pm \frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ and $-\frac{2}{3}\left(\frac{24}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ or $x = \frac{24}{5}$, $y = \frac{16}{5}$ and $x = -\frac{24}{5}$, $y = -\frac{16}{5}$	ddM1 cso A1
		[6] 11
(a)	Alternative method for part (a)	
	$\left\{ \frac{dx}{dx} \right\} 2x \frac{dx}{dy} - \left(3y \frac{dx}{dy} + 3x \right) - 8y = 0$	M1A1 M1
	$(2x - 3y) \frac{dx}{dy} - 3x - 8y = 0$	dM1
	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} \text{ or } \frac{3y - 2x}{-3x - 8y}$	o.e. A1 cso
		[5]

Question Notes		
(a) General	Note	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} \text{ or } \frac{3y - 2x}{-3x - 8y}$ from no working is full marks
	Note	Writing down $\frac{dy}{dx} = \frac{2x - 3y}{-3x - 8y} \text{ or } \frac{3y - 2x}{3x + 8y}$ from no working is M1A0B1M1A0
	Note	Few candidates will write $2x dx - 3y dx - 3x dy - 8y dy = 0$ leading to $\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$, o.e. This should get full marks.

(a)	M1	Differentiates implicitly to include either $\pm 3x \frac{dy}{dx}$ or $-4y^2 \rightarrow \pm ky \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).
	A1	Both $x^2 \rightarrow 2x$ and $\dots - 4y^2 + 64 = 0 \rightarrow -8y \frac{dy}{dx} = 0$
	Note	If an extra term appears then award A0.
	M1	$-3xy \rightarrow -3x \frac{dy}{dx} - 3y$ or $-3x \frac{dy}{dx} + 3y$ or $3x \frac{dy}{dx} - 3y$ or $3x \frac{dy}{dx} + 3y$
	Note	$2x - 3y - 3x \frac{dy}{dx} - 8y \frac{dy}{dx} \rightarrow 2x - 3y = 3x \frac{dy}{dx} + 8y \frac{dy}{dx}$ will get 1 st A1 (implied) as the "= 0" can be implied by the rearrangement of their equation.
	dM1	dependent on the FIRST method mark being awarded. An attempt to factorise out all the terms in $\frac{dy}{dx}$ as long as there are <i>at least two terms</i> in $\frac{dy}{dx}$. i.e. $\dots + (-3x - 8y) \frac{dy}{dx} = \dots$ or $\dots = (3x + 8y) \frac{dy}{dx}$. (Allow combining in 1 variable).
	A1	$\frac{2x - 3y}{3x + 8y}$ or $\frac{3y - 2x}{-3x - 8y}$ or equivalent.
	Note	cso If the candidate's solution is not completely correct, then do not give this mark.
	Note	You cannot recover work for part (a) in part (b).
(b)	M1	Sets their numerator of their $\frac{dy}{dx}$ equal to zero (or the denominator of their $\frac{dx}{dy}$ equal to zero) o.e.
	Note	1 st M1 can also be gained by setting $\frac{dy}{dx}$ equal to zero in their " $2x - 3y - 3x \frac{dy}{dx} - 8y \frac{dy}{dx} = 0$ "
	Note	If their numerator involves one variable only then only the 1 st M1 mark is possible in part (b).
	Note	If their numerator is a constant then no marks are available in part (b)
	Note	If their numerator is in the form $\pm ax^2 \pm by^2 = 0$ or $\pm ax \pm by^2 = 0$ then the first 3 marks are possible in part (b).
	Note	$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y} = 0$ is not sufficient for M1.
	A1ft	Either <ul style="list-style-type: none"> Sets $2x - 3y$ to zero and obtains either $y = \frac{2}{3}x$ or $x = \frac{3}{2}y$ the follow through result of making either y or x the subject from setting their numerator of their $\frac{dy}{dx}$ equal to zero
	dM1	dependent on the first method mark being awarded. Substitutes <i>either</i> their $y = \frac{2}{3}x$ or their $x = \frac{3}{2}y$ into the original equation to give an equation in one variable only.
	A1	Obtains either $x = \frac{24}{5}$ or $-\frac{24}{5}$ or $y = \frac{16}{5}$ or $-\frac{16}{5}$, (or equivalent) <i>by correct solution only</i> . i.e. You can allow for example $x = \frac{48}{10}$ or 4.8, etc.
	Note	$x = \sqrt{\frac{576}{25}}$ (not simplified) or $y = \sqrt{\frac{256}{25}}$ (not simplified) is not sufficient for A1.

(b) ctd	ddM1	<p>dependent on both previous method marks being awarded in this part.</p> <p>Method 1</p> <p>Either:</p> <ul style="list-style-type: none"> substitutes their x into their $y = \frac{2}{3}x$ or substitutes their y into their $x = \frac{3}{2}y$, or substitutes <i>the other of</i> their $y = \frac{2}{3}x$ or their $x = \frac{3}{2}y$ into the original equation, <p>and achieves either:</p> <ul style="list-style-type: none"> exactly two sets of two coordinates or exactly two distinct values for x and exactly two distinct values for y. <p>Method 2</p> <p>Either:</p> <ul style="list-style-type: none"> substitutes their first x-value, x_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one y-value, y_1 and substitutes their second x-value, x_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain 1 y-value y_2 or substitutes their first y-value, y_1 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_1 and substitutes their second y-value, y_2 into $x^2 - 3xy - 4y^2 + 64 = 0$ to obtain one x-value x_2.
	Note	Three or more sets of coordinates given (without identification of two sets of coordinates) is ddM0.
	A1	Both $\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{16}{5}\right)$, only by cso. Note that decimal equivalents are fine.
	Note	Also allow $x = \frac{24}{5}, y = \frac{16}{5}$ and $x = -\frac{24}{5}, y = -\frac{16}{5}$ all seen in their working to part (b).
	Note	Allow $x = \pm \frac{24}{5}, y = \pm \frac{16}{5}$ for 3 rd A1.
	Note	$x = \pm \frac{24}{5}, y = \pm \frac{16}{5}$ followed by eg. $\left(\frac{16}{5}, \frac{24}{5}\right)$ and $\left(-\frac{16}{5}, -\frac{24}{5}\right)$ (eg. coordinates stated the wrong way round) is 3 rd A0.
	Note	It is possible for a candidate who does not achieve full marks in part (a), (but has a correct numerator for $\frac{dy}{dx}$) to gain all 6 marks in part (b).
	Note	Decimal equivalents to fractions are fine in part (b). i.e. $(4.8, 3.2)$ and $(-4.8, -3.2)$.
	Note	$\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ from no working is M0A0M0A0M0A0.
	Note	Candidates could potentially lose the final 2 marks for setting both their numerator and denominator to zero.
	Note	No credit in this part can be gained by only setting the denominator to zero.

Question 8

Question Number	Scheme	Notes	Marks
	$2x^2y + 2x + 4y - \cos(\pi y) = 17$		
(a) Way 1	$\left\{ \begin{array}{l} \cancel{\frac{d}{dx}} \times \end{array} \right\} \left(\underline{4xy + 2x^2 \frac{dy}{dx}} \right) + 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = 0$		M1 <u>A1</u> <u>B1</u>
	$\frac{dy}{dx}(2x^2 + 4 + \pi \sin(\pi y)) + 4xy + 2 = 0$		dM1
	$\left\{ \frac{dy}{dx} = \right\} \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$	Correct answer or equivalent	A1 cso
			[5]
(b)	At $\left(3, \frac{1}{2}\right)$, $m_T = \frac{dy}{dx} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi \sin(\frac{1}{2}\pi)} \left\{ = \frac{-8}{22 + \pi} \right\}$	Substituting $x = 3$ & $y = \frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N = \frac{22 + \pi}{8}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N Can be implied by later working	M1
	<ul style="list-style-type: none"> $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$ $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$ 	$y - \frac{1}{2} = m_N(x - 3)$ or $y = m_N x + c$ where $\frac{1}{2} = (\text{their } m_N)3 + c$ with a numerical $m_N (\neq m_T)$ where m_N is in terms of π and sets $y = 0$ in their normal equation.	dM1
	Cuts x-axis $\Rightarrow y = 0$ $\Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$		
	So, $\left\{ x = \frac{-4}{22 + \pi} + 3 \Rightarrow \right\} x = \frac{3\pi + 62}{\pi + 22}$	$\frac{3\pi + 62}{\pi + 22}$ or $\frac{6\pi + 124}{2\pi + 44}$ or $\frac{62 + 3\pi}{22 + \pi}$	A1 o.e.
			[4]
			9
(a) Way 2	$\left\{ \begin{array}{l} \cancel{\frac{d}{dx}} \times \end{array} \right\} \left(\underline{4xy \frac{dx}{dy} + 2x^2} \right) + 2 \frac{dx}{dy} + 4 + \pi \sin(\pi y) = 0$		M1 <u>A1</u> <u>B1</u>
	$\frac{dx}{dy}(4xy + 2) + 2x^2 + 4 + \pi \sin(\pi y) = 0$		dM1
	$\frac{dx}{dy} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)} \text{ or } \frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$	Correct answer or equivalent	A1 cso
			[5]

	Question	Notes
(a)	Note	Writing down <i>from no working</i> <ul style="list-style-type: none"> $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi \sin(\pi y)}$ scores M1A1B1M1A1 $\frac{dy}{dx} = \frac{4xy + 2}{2x^2 + 4 + \pi \sin(\pi y)}$ scores M1A0B1M1A0
	Note	Few candidates will write $4xy dx + 2x^2 dy + 2dx + 4dy + \pi \sin(\pi y) dy = 0$ leading to $\frac{dy}{dx} = \frac{-4xy - 2}{2x^2 + 4 + \pi \sin(\pi y)}$ or equivalent. This should get full marks.

	Question	Notes Continued
(a) Way 1	M1	Differentiates implicitly to include either $2x^2 \frac{dy}{dx}$ or $4y \rightarrow 4 \frac{dy}{dx}$ or $-\cos(\pi y) \rightarrow \pm \lambda \sin(\pi y) \frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ is a constant which can be 1.
	1st A1	$2x + 4y - \cos(\pi y) = 17 \rightarrow 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = 0$
	Note	$4xy + 2x^2 \frac{dy}{dx} + 2 + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} \rightarrow 2x^2 \frac{dy}{dx} + 4 \frac{dy}{dx} + \pi \sin(\pi y) \frac{dy}{dx} = -4xy - 2$ will get 1 st A1 (implied) as the " $= 0$ " can be implied by the rearrangement of their equation.
	B1	$2x^2 y \rightarrow 4xy + 2x^2 \frac{dy}{dx}$
	Note	If an extra term appears then award 1 st A0.
	dM1	Dependent on the first method mark being awarded. An attempt to factorise out all the terms in $\frac{dy}{dx}$ as long as there are <i>at least two terms</i> in $\frac{dy}{dx}$. ie. $\frac{dy}{dx}(2x^2 + 4 + \pi \sin(\pi y)) + \dots = \dots$
	Note	Writing down an extra $\frac{dy}{dx} = \dots$ and then including it in their factorisation is fine for dM1.
	Note	Final A1 cso: If the candidate's solution is not completely correct, then do not give this mark.
	Note	Final A1 isw: You can, however, ignore subsequent working following on from correct solution.
(a)	Way 2	Apply the mark scheme for Way 2 in the same way as Way 1.
(b)	1st M1	M1 can be gained by seeing at least one example of substituting $x = 3$ and at least one example of substituting $y = \frac{1}{2}$. E.g. " $-4xy$ " \rightarrow " -6 " in their $\frac{dy}{dx}$ would be sufficient for M1, unless it is clear that they are instead applying $x = \frac{1}{2}, y = 3$.
	3rd M1	<i>is dependent on the first M1.</i>
	Note	The 2 nd M1 mark can be implied by later working. Eg. Award 2 nd M1 3 rd M1 for $\frac{\frac{1}{2}}{3-x} = \frac{-1}{\text{their } m_T}$
	Note	We can accept $\sin \pi$ or $\sin\left(\frac{\pi}{2}\right)$ as a numerical value for the 2 nd M1 mark. But, $\sin \pi$ by itself or $\sin\left(\frac{\pi}{2}\right)$ by itself are not allowed as being in terms of π for the 3 rd M1 mark. The 3 rd M1 can be accessed for terms containing $\pi \sin\left(\frac{\pi}{2}\right)$.

Question 9

Question Number	Scheme	Notes	Marks
	$4x^2 - y^3 - 4xy + 2^y = 0$		
(a) Way 1	$\left\{ \frac{dy}{dx} \times \right\} \underline{8x - 3y^2 \frac{dy}{dx} - 4y - 4x \frac{dy}{dx} + 2^y \ln 2 \frac{dy}{dx} = 0}$		M1 A1 M1 B1
	$8(-2) - 3(4)^2 \frac{dy}{dx} - 4(4) - 4(-2) \frac{dy}{dx} + 2^4 \ln 2 \frac{dy}{dx} = 0$	dependent on the first M mark	dM1
	$-16 - 48 \frac{dy}{dx} - 16 + 8 \frac{dy}{dx} + 16 \ln 2 \frac{dy}{dx} = 0$		
	$\frac{dy}{dx} = \frac{32}{-40 + 16 \ln 2}$ or $\frac{-32}{40 - 16 \ln 2}$ or $\frac{4}{-5 + 2 \ln 2}$ or $\frac{4}{-5 + \ln 4}$ or exact equivalent		A1 cso
	NOTE: You can recover work for part (a) in part (b)		[6]
(b)	e.g. $m_N = \frac{-40 + 16 \ln 2}{-32}$ or $\frac{40 - 16 \ln 2}{32}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N Can be implied by later working	M1
	<ul style="list-style-type: none">$y - 4 = \left(\frac{40 - 16 \ln 2}{32} \right)(x - -2)$ <p>Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40 - 16 \ln 2}{32} \right)(2)$</p>	Using a numerical $m_N (\neq m_T)$, either $y - 4 = m_N(x - -2)$ and sets $x = 0$ in their normal equation or $4 = (\text{their } m_N)(-2) + c$	M1
	<ul style="list-style-type: none">$4 = \left(\frac{40 - 16 \ln 2}{32} \right)(-2) + c$		
	$\left\{ \Rightarrow c = 4 + \frac{40 - 16 \ln 2}{16}, \text{ so } y = \frac{104 - 16 \ln 2}{16} \Rightarrow \right\}$		
	$y \text{ (or } c) = \frac{13}{2} - \ln 2$	$\frac{104}{16} - \ln 2$ or $\frac{13}{2} - \ln 2$ or $-\ln 2 + \frac{13}{2}$	A1 cso isw
	Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		[3]

(a) Way 2	$\left\{ \frac{dx}{dy} \times \right\} 8x \frac{dx}{dy} - 3y^2 - 4y \frac{dx}{dy} - 4x + 2^y \ln 2 = 0$		M1 <u>A1</u> <u>M1</u> <u>B1</u>
	$8(-2) \frac{dx}{dy} - 3(4)^2 - 4(4) \frac{dx}{dy} - 4(-2) + 2^4 \ln 2 = 0$		dependent on the first M mark dM1
	$\frac{dy}{dx} = \frac{32}{-40+16 \ln 2}$ or $\frac{-32}{40-16 \ln 2}$ or $\frac{4}{-5+2 \ln 2}$ or $\frac{4}{-5+ \ln 4}$ or exact equivalent		A1 cso
	Note: You must be clear that Way 2 is being applied before you use this scheme		[6]
Question Notes			
(a)	Note	For the first four marks	
		Writing down <i>from no working</i>	
		<ul style="list-style-type: none"> $\frac{dy}{dx} = \frac{4y - 8x}{-3y^2 - 4x + 2^y \ln 2}$ or $\frac{8x - 4y}{3y^2 + 4x - 2^y \ln 2}$ scores M1A1M1B1 $\frac{dy}{dx} = \frac{8x - 4y}{-3y^2 - 4x + 2^y \ln 2}$ or $\frac{4y - 8x}{3y^2 + 4x - 2^y \ln 2}$ scores M1A0M1B1 	
		Writing $8x dx - 3y^2 dy - 4y dx - 4x dy + 2^y \ln 2 dy = 0$ scores M1A1M1B1	

Question Notes Continued		
(a)	1st M1	Differentiates implicitly to include <i>either</i> $\pm 4x \frac{dy}{dx}$ <i>or</i> $-y^3 \rightarrow \pm \lambda y^2 \frac{dy}{dx}$ <i>or</i> $2^y \rightarrow \pm \mu 2^y \frac{dy}{dx}$ (Ignore $\left(\frac{dy}{dx} = \right)$). λ, μ are constants which can be 1
	1st A1	Both $4x^2 - y^3 \rightarrow 8x - 3y^2 \frac{dy}{dx}$ and $= 0 \rightarrow = 0$
	Note	e.g. $8x - 3y^2 \frac{dy}{dx} - 4y - 4x \frac{dy}{dx} + 2^y \ln 2 \frac{dy}{dx} \rightarrow -3y^2 \frac{dy}{dx} - 4x \frac{dy}{dx} + 2^y \ln 2 \frac{dy}{dx} = 4y - 8x$ or e.g. $-16 - 48 \frac{dy}{dx} - 16 + 8 \frac{dy}{dx} + 16 \ln 2 \frac{dy}{dx} \rightarrow -48 \frac{dy}{dx} + 8 \frac{dy}{dx} + 16 \ln 2 \frac{dy}{dx} = 32$ will get 1 st A1 (implied) as the " $= 0$ " can be implied by the rearrangement of their equation.
	2nd M1	$-4xy \rightarrow -4y - 4x \frac{dy}{dx}$ or $4y - 4x \frac{dy}{dx}$ or $-4y + 4x \frac{dy}{dx}$ or $4y + 4x \frac{dy}{dx}$
	B1	$2^y \rightarrow 2^y \ln 2 \frac{dy}{dx}$ or $2^y \rightarrow e^{y \ln 2} \ln 2 \frac{dy}{dx}$
	Note	If an extra term appears then award 1 st A0
	3rd dM1	dependent on the first M mark For substituting $x = -2$ and $y = 4$ into an equation involving $\frac{dy}{dx}$
	Note	M1 can be gained by seeing at least one example of substituting $x = -2$ and at least one example of substituting $y = 4$ unless it is clear that they are instead applying $x = 4$ and $y = -2$. Otherwise, you will NEED to check (with your calculator) that $x = -2, y = 4$ that has been substituted into their equation involving $\frac{dy}{dx}$
(b)	Note	A1 cso: If the candidate's solution is not completely correct, then do not give this mark.
	Note	isw: You can, however, ignore subsequent working following on from correct solution.
	Note	The 2 nd M1 mark can be implied by later working. Eg. Award 1st M1 and 2nd M1 for $\frac{y-4}{2} = \frac{-1}{\text{their } m_T \text{ evaluated at } x = -2 \text{ and } y = 4}$
	Note	A1: Allow the alternative answer $\left\{ y = \right\} \ln\left(\frac{1}{2}\right) + \frac{13}{2 \ln 2} (\ln 2)$ which is in the form $p + q \ln 2$

(a) Way 2	1st M1	Differentiates implicitly to include <i>either</i> $\pm 4y \frac{dx}{dy}$ <i>or</i> $4x^2 \rightarrow \pm \lambda x \frac{dx}{dy}$ (Ignore $\left(\frac{dx}{dy} = \right)$). λ is a constant which can be 1
	1st A1	Both $4x^2 - y^3 \rightarrow 8x \frac{dx}{dy} - 3y^2$ and $= 0 \rightarrow = 0$
	2nd M1	$-4xy \rightarrow -4y \frac{dx}{dy} - 4x$ or $4y \frac{dx}{dy} - 4x$ or $-4y \frac{dx}{dy} + 4x$ or $4y \frac{dx}{dy} + 4x$
	B1	$2^y \rightarrow 2^y \ln 2$
	3rd dM1	dependent on the first M mark For substituting $x = -2$ and $y = 4$ into an equation involving $\frac{dx}{dy}$