Integration by Parts 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question Number	Scheme	Marks
(a)	$\int x\sin 3x \, dx = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \{dx\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x \, dx = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \{dx\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \{+c\}$	A1 isw
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \{+c\}\right\}$ *Ignore subsequent working*	[3]
		6
(a)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$). This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant. If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \{dx\}$. Can be un-simplified. Ignore the $\{dx\}$. **A1:** $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without $+c$. Can be un-simplified.	
(b)	**M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$). This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$ or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$. If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0. **A1:** $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \{dx\}$. Can be un-simplified. Ignore the $\{dx\}$. **A1:** $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without $+c$, can be un-simplified. You can ignore subsequent working here. **Special Case:** If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}$(their follow through part(a) answer).	

Question 2

Question Number	Scheme	
2. (a)	$\int \frac{1}{x^3}\ln x \, dx$, $\left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array} \right\}$	
	In the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$	M1
	$= \underline{\frac{-1}{2x^2}\ln x} - \underline{\underline{\int \frac{-1}{2x^2}.\frac{1}{x} dx}}$ $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified.	A1
	$\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ simplified or un-simplified.	A1
	$\left\{ = \frac{-1}{2x^2}\ln x + \frac{1}{2}\int \frac{1}{x^3} dx \right\}$	
	$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ $\pm\int \mu \frac{1}{x^2}.\frac{1}{x} \to \pm\beta x^{-2}$.	dM1
	Correct answer, with/without $+c$	A1
		[5]
(b)	$\left\{\left[-\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\right]_1^2\right\} = \left(-\frac{1}{2(2)^2}\ln 2 - \frac{1}{4(2)^2}\right) - \left(-\frac{1}{2(1)^2}\ln 1 - \frac{1}{4(1)^2}\right)$ Applies limits of 2 and 1 to their part (a) answer and subtracts the correct way round.	M1
	$= \frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3 - 2\ln 2)$, etc, or awrt 0.1 or equivalent.	A1
		[2] **7**
(a)	**M1**: Integration by parts is applied in the form $\frac{\pm\lambda}{x^2}\ln x \pm \int \mu \frac{1}{x^2}.\frac{1}{x}$ or equivalent. **A1**: $\underline{\frac{-1}{2x^2}\ln x}$ simplified or un-simplified. **A1**: $\underline{\underline{-\int \frac{-1}{2x^2}.\frac{1}{x}}}$ or equivalent. You can ignore the dx. **dM1**: Depends on the previous M1. $\pm\int \mu \frac{1}{x^2}.\frac{1}{x} \to \pm\beta x^{-2}$. **A1**: $-\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)\{+c\}$ or $= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\{+c\}$ or $\frac{x^{-2}}{-2}\ln x - \frac{x^{-2}}{4}\{+c\}$ or $\frac{-1 - 2\ln x}{4x^2}\{+c\}$ or equivalent. You can ignore subsequent working after a correct stated answer.	
(b)	**M1**: Some evidence of applying limits of 2 and 1 to their part (a) answer and subtracts the correct way round. **A1**: ***Two term exact answer*** of either $\frac{3}{16} - \frac{1}{8}\ln 2$ or $\frac{3}{16} - \ln 2^{\frac{1}{8}}$ or $\frac{1}{16}(3 - 2\ln 2)$ or $\frac{\ln\left(\frac{1}{4}\right) + 3}{16}$ or $0.1875 - 0.125\ln 2$. Also allow awrt 0.1. Also note the fraction terms must be combined. **Note:** Award the final A0 in part (b) for a candidate who achieves awrt 0.1 in part (b), when their answer to part (a) is incorrect.	

2. (b) ctd	**Note:** Decimal answer is 0.100856... in part (b).	
	Alternative Solution	
	$\int \frac{1}{x^3}\ln x\,dx$, $\left\{ \begin{array}{lll} u = x^{-3} & \Rightarrow & \frac{du}{dx} = -3x^{-4} \\ \frac{dv}{dx} = \ln x & \Rightarrow & v = x\ln x - x \end{array} \right\}$	
	$\int \frac{1}{x^3}\ln x\,dx = \frac{1}{x^3}(x\ln x - x) - \int (x\ln x - x)\frac{-3}{x^4}\,dx$	
	$k\int \frac{1}{x^3}\ln x\,dx = \frac{1}{x^3}(x\ln x - x) \pm \int \frac{\lambda}{x^3}\,dx$ where $k \neq 1$	M1
	$-2\int \frac{1}{x^3}\ln x\,dx = \frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\,dx$ — Any one of $\frac{1}{x^3}(x\ln x - x)$ or $-\int \frac{3}{x^3}\,dx$	A1
	$\frac{1}{x^3}(x\ln x - x) - \int \frac{3}{x^3}\,dx$ and $k = -2$	A1
	$-2\int \frac{1}{x^3}\ln x\,dx = \frac{1}{x^3}(x\ln x - x) + \frac{3}{2x^2}\ \{+c\}$ — $\pm\int \mu\frac{1}{x^3} \rightarrow \pm\beta x^{-2}$.	dM1
	$\int \frac{1}{x^3}\ln x\,dx = -\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}\ \{+c\}$ — $-\frac{1}{2x^3}(x\ln x - x) - \frac{3}{4x^2}$ or equivalent with/without $+c$.	A1
	$= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\ \{+c\}$	

Question 3

Question Number	Scheme		Marks
(a)	$\int x^2 e^x \, dx$, 1st Application: $\left\{ \begin{array}{l} u = x^2 \Rightarrow \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$, 2nd Application: $\left\{ \begin{array}{l} u = x \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x \Rightarrow v = e^x \end{array} \right\}$		
	$= x^2 e^x - \int 2xe^x dx$	$x^2 e^x - \int \lambda xe^x \{dx\}$, $\lambda > 0$	M1
		$x^2 e^x - \int 2xe^x \{dx\}$	A1 oe
	$= x^2 e^x - 2\left(xe^x - \int e^x dx\right)$	**Either** $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ or for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$	M1
	$= x^2 e^x - 2(xe^x - e^x)$ $\{+ c\}$	$\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$	M1
		Correct answer, with/without $+ c$	A1
			[5]
(b)	$\left\{ \left[x^2 e^x - 2(xe^x - e^x) \right]_0^1 \right\}$ $= \left(1^2 e^1 - 2(1e^1 - e^1)\right) - \left(0^2 e^0 - 2(0e^0 - e^0)\right)$	Applies limits of 1 and 0 to an expression of the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, $A \neq 0$, $B \neq 0$ and $C \neq 0$ and subtracts the correct way round.	M1
	$= e - 2$	$e - 2$ cso	A1 oe
			[2] 7

Notes for Question	
(a)	**M1:** Integration by parts is applied in the form $x^2 e^x - \int \lambda xe^x \{dx\}$, where $\lambda > 0$. (must be in this form).
	A1: $x^2 e^x - \int 2xe^x \{dx\}$ or equivalent.
	M1: **Either** achieving a result in the form $\pm Ax^2 e^x \pm Bxe^x \pm C\int e^x \{dx\}$ (can be implied) (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) **or** for $\pm K\int xe^x \{dx\} \to \pm K\left(xe^x - \int e^x \{dx\}\right)$
	M1: $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$ (where $A \neq 0$, $B \neq 0$ and $C \neq 0$)
	A1: $x^2 e^x - 2(xe^x - e^x)$ or $x^2 e^x - 2xe^x + 2e^x$ or $(x^2 - 2x + 2)e^x$ or equivalent with/without $+ c$.
(b)	**M1:** **Complete method** of applying limits of 1 and 0 to their part (a) answer in the form $\pm Ax^2 e^x \pm Bxe^x \pm Ce^x$, (where $A \neq 0$, $B \neq 0$ and $C \neq 0$) and subtracting the correct way round. Evidence of a proper consideration of the limit of 0 (as detailed above) is needed for M1. So, just subtracting zero is M0.
	A1: $e - 2$ or $e^1 - 2$ or $-2 + e$. Do not allow $e - 2e^0$ unless simplified to give $e - 2$. **Note:** that 0.718... without seeing $e - 2$ or equivalent is A0. **WARNING:** Please note that this A1 mark is for correct solution only. So incorrect $[......]_0^1$ leading to $e - 2$ is A0.
	Note: If their part (a) is correct candidates can get M1A1 in part (b) for $e - 2$ from no working.
	Note: 0.718... from no working is M0A0

Question 4

Question Number	Scheme		Marks
(i)	$\int xe^{4x}\ dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\ \{dx\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\},\quad \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\ \{dx\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\ \{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1