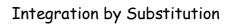


Integration by Substitution 2 - Edexcel Past Exam Questions MARK SCHEME

Question Number	Scheme		Marks	
	$\left\{ u = 1 + \cos x \right\} \implies \frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$		<u>B1</u>	
	$\left\{ \int \frac{2\sin 2x}{(1+\cos x)} \mathrm{d}x = \right\} \int \frac{2(2\sin x \cos x)}{(1+\cos x)} \mathrm{d}x $ $\sin 2x = 2\sin x \cos x $			
	$= \int \frac{4(u-1)}{u} \cdot (-1) du \left\{ = 4 \int \frac{(1-u)}{u} du \right\}$		M1	
	$=4\int \left(\frac{1}{u}-1\right)du=4\left(\ln u-u\right)+c$		dM1	
	$= 4\ln(1 + \cos x) - 4(1 + \cos x) + c = 4\ln(1 + \cos x) - 4\cos x + k$	AG	A1 cso [5]	





Question Number	Scheme	Marks
	$\left\{ u = 1 + \sqrt{x} \right\} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} \text{or} \frac{\mathrm{d}x}{\mathrm{d}u} = 2(u-1)$	<u>B1</u>
	$\left\{ \int \frac{x}{1+\sqrt{x}} dx = \right\} \int \frac{(u-1)^2}{u} \cdot 2(u-1) du$ $\int \frac{(u-1)^2}{u} \cdots$	M1
	$\int \frac{(u-1)^2}{u} \cdot 2(u-1) du$	-1) A1
	$= 2 \int \frac{(u-1)^3}{u} du = \{2\} \int \frac{(u^3 - 3u^2 + 3u - 1)}{u} du$ Expands to give a "four term" cubic in Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm Cu$	I M 1
	$= \{2\} \int \left(u^2 - 3u + 3 - \frac{1}{u}\right) du$ An attempt to divide at least three term their cubic by u. See no	
		nu) A1
	Area(R) = $\left[\frac{2u^3}{3} - 3u^2 + 6u - 2\ln u\right]_2^3$	
	$= \left(\frac{2(3)^3}{3} - 3(3)^2 + 6(3) - 2\ln 3\right) - \left(\frac{2(2)^3}{3} - 3(2)^2 + 6(2) - 2\ln 2\right)$ Applies limits of 3 and 1 in x subtracts either way roughly subtracts either way roughly subtracts.	and M1
	$= \frac{11}{3} + 2\ln 2 - 2\ln 3 \text{or} \frac{11}{3} + 2\ln\left(\frac{2}{3}\right) \text{ or} \frac{11}{3} - \ln\left(\frac{9}{4}\right), \text{ etc} $ Correct exact ans or equival	AI
		[8]



Question Number	Scheme	Marks
(a)	$\left\{x = u^2 \Rightarrow \right\} \frac{\mathrm{d}x}{\mathrm{d}u} = 2u \text{or} \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} \text{or} \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$	В1
	$\left\{ \int \frac{1}{x(2\sqrt{x} - 1)} \mathrm{d}x \right\} = \int \frac{1}{u^2(2u - 1)} 2u \mathrm{d}u$	M1
	$= \int \frac{2}{u(2u-1)} \mathrm{d}u$	A1 * cso
(b)	$2 = A + B \Rightarrow 2 = A(2y-1) + By$	[3]
	$\frac{2}{u(2u-1)} \equiv \frac{A}{u} + \frac{B}{(2u-1)} \Rightarrow 2 \equiv A(2u-1) + Bu$ $u = 0 \Rightarrow 2 = -A \Rightarrow A = -2$ $u = \frac{1}{2} \Rightarrow 2 = \frac{1}{2}B \Rightarrow B = 4$ See notes	M1 A1
	$u = \frac{1}{2} \implies 2 = \frac{1}{2}B \implies B = 4$	
	So $\int \frac{2}{u(2u-1)} du = \int \frac{-2}{u} + \frac{4}{(2u-1)} du$ Integrates $\frac{M}{u} + \frac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ to	M1
	Solution any one of $\pm \lambda \ln u$ of $\pm \mu \ln(2u - 1)$	41.0
	$= -2 \ln u + 2 \ln(2u - 1)$ At least one term correctly followed through $-2 \ln u + 2 \ln(2u - 1).$	A1 ft A1 cao
	So, $[-2\ln u + 2\ln(2u - 1)]_1^3$	711 (40
	$= (-2\ln 3 + 2\ln(2(3) - 1)) - (-2\ln 1 + 2\ln(2(1) - 1))$ Applies limits of 3 and 1 in u or 9 and 1 in x in their integrated function and subtracts the correct way round.	M1
	$= -2\ln 3 + 2\ln 5 - (0)$	
	$= 2\ln\left(\frac{5}{3}\right)$	A1 cso cao
		[7 10
	Notes for Question	
(a)	B1: $\frac{dx}{du} = 2u$ or $dx = 2u du$ or $\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ or $du = \frac{dx}{2\sqrt{x}}$	
	M1: A full substitution producing an integral in u only (including the du) (Integral sign not n	
	The candidate needs to deal with the "x", the " $(2\sqrt{x}-1)$ " and the "dx" and converts fr	
(b)	integral term in x to an integral in u. (Remember the integral sign is not necessary for M A1*: leading to the result printed on the question paper (including the du). (Integral sign is not necessary for M A1*:	
	M1: Writing $\frac{2}{u(2u-1)} \equiv \frac{A}{u} + \frac{B}{(2u-1)}$ or writing $\frac{1}{u(2u-1)} \equiv \frac{P}{u} + \frac{Q}{(2u-1)}$ and a complete	te method for
	finding the value of at least one of their A or their B (or their P or their Q). A1: Both their $A = -2$ and their $B = 4$. (Or their $P = -1$ and their $Q = 2$ with the multiply	ing factor of
	2 in front of the integral sign).	
	M1: Integrates $\frac{M}{u} + \frac{N}{(2u-1)}$, $M \neq 0$, $N \neq 0$ (i.e. a two term partial fraction) to obtain any	one of
	$\pm \lambda \ln u$ or $\pm \mu \ln(2u - 1)$ or $\pm \mu \ln\left(u - \frac{1}{2}\right)$	
	A1ft: At least one term correctly followed through from their A or from their B (or their P and A1: $-2\ln u + 2\ln(2u - 1)$	their Q).
	Notes for Question Continued M1: Applies limits of 3 and 1 in u or 9 and 1 in x in their (i.e. any) changed function and subt	



correct way round.

Note: If a candidate just writes $(-2 \ln 3 + 2 \ln(2(3) - 1))$ oe, this is ok for M1.

A1: $2\ln\left(\frac{5}{3}\right)$ correct answer only. (Note: a = 5, b = 3).

Important note: Award M0A0M1A1A0 for a candidate who writes

$$\int \frac{2}{u(2u-1)} du = \int \frac{2}{u} + \frac{2}{(2u-1)} du = 2\ln u + \ln(2u-1)$$

AS EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ AS PARTIAL FRACTIONS IS GIVEN.

Important note: Award M0A0M0A0A0 for a candidate who writes down either

$$\int \frac{2}{u(2u-1)} du = 2\ln u + 2\ln(2u-1) \text{ or } \int \frac{2}{u(2u-1)} du = 2\ln u + \ln(2u-1)$$

WITHOUT ANY EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ as partial fractions.

Important note: Award M1A1M1A1A1 for a candidate who writes down

$$\int \frac{2}{u(2u-1)} \, \mathrm{d}u = -2\ln u + 2\ln(2u-1)$$

WITHOUT ANY EVIDENCE OF WRITING $\frac{2}{u(2u-1)}$ as partial fractions.

Note: In part (b) if they lose the "2" and find $\int \frac{1}{u(2u-1)} du$ we can allow a maximum of

M1A0 M1A1ftA0 M1A0.



Quootion	•					
Question Number	Scheme	Marks				
	$\int_0^4 \frac{1}{2 + \sqrt{(2x+1)}} \mathrm{d}x \ , \ u = 2 + \sqrt{(2x+1)}$					
	Either $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = u-2$ $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = \pm \lambda(u-2)$	M1				
	Either $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = (u-2)$	A1				
	$\left\{ \int \frac{1}{2 + \sqrt{(2x+1)}} \mathrm{d}x \right\} = \int \frac{1}{u} (u-2) \mathrm{d}u $ (Ignore integral sign and $\mathrm{d}u$).	A1				
	$= \int \left(1 - \frac{2}{u}\right) du$ An attempt to divide each term by u .	dM1				
	$\pm Au \pm B \ln u$	ddM1				
	$= u - 2\ln u$ $u - 2\ln u$	A1 ft				
	$\left\{ \text{So } \left[u - 2 \ln u \right]_3^5 \right\} = \left(5 - 2 \ln 5 \right) - \left(3 - 2 \ln 3 \right)$ Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1				
	$= 2 + 2\ln\left(\frac{3}{5}\right)$ $2 + 2\ln\left(\frac{3}{5}\right)$					
		[8] 8				
Notes for Question						
	M1: Also allow $du = \pm \lambda \frac{1}{(u-2)} dx$ or $(u-2)du = \pm \lambda dx$					
	Note: The expressions must contain du and dx . They can be simplified or un-simplified	.				
	A1: Also allow $du = \frac{1}{(u-2)} dx$ or $(u-2)du = \pm \lambda dx$					
	Note: The expressions must contain du and dx . They can be simplified or un-simplified					
	A1: $\int \frac{1}{u} (u-2) du$. (Ignore integral sign and du).					
	dM1: An attempt to divide each term by u. Note that this mark is dependent on the previous M1 mark being awarded. Note that this mark can be implied by later working.					
	ddM1 : $\pm Au \pm B \ln u$, $A \neq 0$, $B \neq 0$					
	Note that this mark is dependent on the two previous M1 marks being awarded. A1ft: $u - 2 \ln u$ or $\pm Au \pm B \ln u$ being correctly followed through, $A \neq 0$, $B \neq 0$					
	M1: Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the way round.	correct				
	A1: cso and cao. $2 + 2\ln\left(\frac{3}{5}\right)$ or $2 + 2\ln(0.6)$, $\left(= A + 2\ln B$, so $A = 2$, $B = \frac{3}{5}\right)$					
	Note: $2 - 2 \ln \left(\frac{3}{5} \right)$ is A0.					

	Notes for Question Continued				
ctd	Note: $\int \frac{1}{u} (u - 2) du = u - 2 \ln u \text{ with no working is } 2^{\text{nd}} M1, 3^{\text{rd}} M1, 3^{\text{rd}} A1.$				
	but Note: $\int \frac{1}{u} (u-2) du = (u-2) \ln u$ with no working is 2^{nd} M0, 3^{rd} M0, 3^{rd} A0.				



Question Number	Scheme	Marks	
	$\left\{ u = \sqrt{x} \Rightarrow \right\} \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}} \text{ or } \frac{\mathrm{d}x}{\mathrm{d}u} = 2u$	B1	
	$\int \frac{10}{2u^2 + 5u} \cdot 2u du$ Either $\left\{ \int \right\} \frac{\pm k u}{\alpha u^2 \pm \beta u} \left\{ du \right\}$ or $\left\{ \int \right\} \frac{\pm k}{u \left(\alpha u^2 \pm \beta u \right)} \left\{ du \right\}$		
	$\pm \lambda \ln(2u+5) \text{ or } \pm \lambda \ln\left(u+\frac{5}{2}\right), \ \lambda \neq 0$	M1	
	$\left\{ = \int \frac{20}{2u+5} du \right\} = \frac{20}{2} \ln(2u+5)$ with no other terms. $\frac{20}{2u+5} \rightarrow \frac{20}{2} \ln(2u+5) \text{ or } 10 \ln\left(u+\frac{5}{2}\right)$	A1 cso	
	$\left\{ \left[\frac{20}{2} \ln(2u+5) \right]_{1}^{2} \right\} = 10 \ln(2(2)+5) - 10 \ln(2(1)+5)$ Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1	
	$10 \ln 9 - 10 \ln 7$ or $10 \ln \left(\frac{9}{7}\right)$ or $20 \ln 3 - 10 \ln 7$	A1 oe cso	
		[6]	



Question Number	Scheme		Marks	
(a)	$A = \int_0^3 \sqrt{(3-x)(x+1)} dx \ , \ x = 1 + 2\sin\theta$			
	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$	$\frac{dx}{d\theta} = 2\cos\theta \text{ or } 2\cos\theta \text{ used correctly}$ in their working. Can be implied.	В1	
	$\left\{ \int \sqrt{(3-x)(x+1)} dx \text{ or } \int \sqrt{(3+2x-x^2)} dx \right\}$			
	$= \int \sqrt{(3 - (1 + 2\sin\theta))((1 + 2\sin\theta) + 1)} \ 2\cos\theta \ \{d\theta\}$ Substitutes for both x and dx, where $dx \neq \lambda d\theta$. Ignore $d\theta$			
	$= \int \sqrt{(2 - 2\sin\theta)(2 + 2\sin\theta)} \ 2\cos\theta \left\{ d\theta \right\}$			
	$= \int \sqrt{4 - 4\sin^2\theta} 2\cos\theta \left\{ d\theta \right\}$			
	$= \int \sqrt{4 - 4(1 - \cos^2 \theta)} 2 \cos \theta \left\{ d\theta \right\} \text{ or } \int \sqrt{4 \cos^2 \theta} 2 \cos \theta$	$\theta d\theta$ Applies $\cos^2 \theta = 1 - \sin^2 \theta$ see notes	M1	
	$=4\int \cos^2\theta d\theta$, $\{k=4\}$	$4\int \cos^2\theta d\theta \text{ or } \int 4\cos^2\theta d\theta$	A1	
	$0 = 1 + 2\sin\theta$ or $-1 = 2\sin\theta$ or $\sin\theta = -\frac{1}{2}$	Note: $d\theta$ is required here. $\theta = -\frac{\pi}{6}$ See notes	B1	
	and $3 = 1 + 2\sin\theta$ or $2 = 2\sin\theta$ or $\sin\theta = 1 \Rightarrow \theta = \frac{1}{2}$	<u>π</u> <u>2</u>	[5]	



Question Number	Scheme			ı	Votes	Marks
	(i) $\int \frac{3y-4}{y(3y+2)} dy$, $y > 0$, (ii) $\int_0^3 \sqrt{\frac{x}{4x}}$	$\frac{x}{-x}$ dx, x	$=4\sin^2\theta$			
(i)	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2) + By$				See notes	M1
Way 1	$\frac{1}{y(3y+2)} = \frac{1}{y} + \frac{1}{(3y+2)} \longrightarrow 3y - 4 = A(3y+2) + By$ $y = 0 \implies -4 = 2A \implies A = -2$				st one of their their $B = 9$	A1
	$= -\frac{2}{3} \implies -6 = -\frac{2}{3}B \implies B = 9$			A = -2 and	Both their $B = 9$	A1
	Integrates to give at least one of either					
	$\int_{0}^{\infty} 3y - 4 dy = \int_{0}^{\infty} -2 dy = 0$	$\frac{A}{y} \rightarrow$	±λlny or _	$\frac{B}{(3y+2)} \to \pm \mu \ln(3y+2)$		M1
	$\int \frac{3y-4}{y(3y+2)} \mathrm{d}y = \int \frac{-2}{y} + \frac{9}{(3y+2)} \mathrm{d}y$	At lea	st one term co	crectly foll	$A \neq 0$, $B \neq 0$	
					r from their B	A1 ft
	$= -2\ln y + 3\ln(3y + 2) \left\{ + c \right\}$	$-2 \ln y +$	$3\ln(3y + 2)$	or -2lny	$+3\ln(y+\frac{2}{3})$	
			lified or un-sir	with corre	ct bracketing,	A1 cao
						[6]
(ii) (a) Way 1	$\left\{x = 4\sin^2\theta \Longrightarrow\right\} \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta \text{or} \frac{\mathrm{d}x}{\mathrm{d}\theta} =$	$= 4\sin^2\theta \Rightarrow \frac{dx}{d\theta} = 8\sin\theta\cos\theta \text{or} \frac{dx}{d\theta} = 4\sin2\theta \text{or} dx = 8\sin\theta\cos\theta d\theta$				
	$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}} \cdot 8\sin\theta\cos\theta \left\{ d\theta \right\} \text{or} \int \sqrt{\frac{4}{4-4}} d\theta$	$\sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}$. $8\sin\theta\cos\theta \left\{d\theta\right\}$ or $\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}}$. $4\sin2\theta \left\{d\theta\right\}$				M1
	$= \int \underline{\tan \theta} \cdot 8 \sin \theta \cos \theta \left\{ d\theta \right\} \text{ or } \int \underline{\tan \theta} \cdot 4 \sin 2\theta$	$\{d\theta\}$	$\sqrt{\left(\frac{x}{4-x}\right)} \to$	$\pm K \tan \theta$ or	$r \pm K \left(\frac{\sin \theta}{\cos \theta} \right)$	<u>M1</u>
	$= \int 8\sin^2\theta d\theta$		∫8	sin² θ dθ	including $\mathrm{d}\theta$	A1
	$3 = 4\sin^2\theta$ or $\frac{3}{4} = \sin^2\theta$ or $\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta =$	π	Writes	down a co	rrect equation	
	$3 = 4 \sin \theta$ or $- = \sin \theta$ or $\sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{2}$	3	involving $x =$	= 3 leading	to $\theta = \frac{\pi}{3}$ and	B1
	$\{x=0 \rightarrow \theta=0\}$		no incorrect w)	
	•				-	[5]
(ii) (b)	$= \left\{ 8 \right\} \int \left(\frac{1 - \cos 2\theta}{2} \right) d\theta \left\{ = \int \left(4 - 4 \cos 2\theta \right) d\theta \right\}$	θ			$\theta = 1 - 2\sin^2\theta$ I. (See notes)	M1
	()		For :	±αθ±βsii	n2θ, α,β≠0	M1
	$= \left\{ 8 \right\} \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right) \left\{ = 4\theta - 2\sin 2\theta \right\} $ $\sin^2 \theta \rightarrow \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right)$			A1		
	$\left\{ \int_{0}^{\frac{\pi}{3}} 8 \sin^{2} \theta d\theta = 8 \left[\frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \right]_{0}^{\frac{\pi}{3}} \right\} = 8 \left(\left(\frac{\pi}{6} - \frac{1}{4} \left(\frac{\sqrt{3}}{2} \right) \right) - \left(0 + 0 \right) \right)$					
	$= \frac{4}{3}\pi - \sqrt{3}$ "two term"	exact answ	ver of e.g. $\frac{4}{3}\pi$	$-\sqrt{3}$ or $\frac{1}{3}$	$\frac{1}{3}(4\pi - 3\sqrt{3})$	A1 o.e.
						[4] 15
						15



		Question Notes				
(i)	1 st M1	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.				
	Note	M1A1 can be implied for writing down either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y} + \frac{\text{their } B}{(3y+2)}$				
		$\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y} + \frac{9}{(3y+2)} \text{ with no working.}$				
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)				
	Note	Give $2^{\text{nd}} \text{ M0 for } \frac{3y-4}{y(3y+2)}$ going directly to $\pm \alpha \ln(3y^2+2y)$				
	Note	but allow 2 nd M1 for either $\frac{M(6y+2)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$				
(ii)(a)	1st M1	Substitutes $x = 4\sin^2\theta$ and their dx (from their correctly rearranged $\frac{dx}{d\theta}$) into $\sqrt{\left(\frac{x}{4-x}\right)}dx$				
	Note	$dx \neq \lambda d\theta$. For example $dx \neq d\theta$				
	Note	Allow substituting $dx = 4\sin 2\theta$ for the 1 st M1 after a correct $\frac{dx}{d\theta} = 4\sin 2\theta$ or $dx = 4\sin 2\theta d\theta$				
	2 nd M1	Applying $x = 4\sin^2\theta$ to $\sqrt{\left(\frac{x}{4-x}\right)}$ to give $\pm K \tan\theta$ or $\pm K \left(\frac{\sin\theta}{\cos\theta}\right)$				
	Note	Integral sign is not needed for this mark.				
	1st A1	Simplifies to give $\int 8\sin^2\theta \ d\theta$ including $d\theta$				
	2 nd B1	Writes down a correct equation involving $x = 3$ leading to $\theta = \frac{\pi}{3}$ and no incorrect work seen regarding limits				
	Note	Allow 2 nd B1 for $x = 4\sin^2\left(\frac{\pi}{3}\right) = 3$ and $x = 4\sin^2 0 = 0$				
	Note	Allow 2 nd B1 for $\theta = \sin^{-1}\left(\sqrt{\frac{x}{4}}\right)$ followed by $x = 3$, $\theta = \frac{\pi}{3}$; $x = 0$, $\theta = 0$				



(ii)(b)	Ml	Writes down a correct equation involving $\cos 2\theta$ and $\sin^2 \theta$		
(-)(-)		E.g.: $\cos 2\theta = 1 - 2\sin^2 \theta$ or $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ or $K\sin^2 \theta = K\left(\frac{1 - \cos 2\theta}{2}\right)$		
		and applies it to their integral. Note: Allow M1 for a correctly stated formula (via an incorrect rearrangement) being applied to their integral.		
	M1	Integrates to give an expression of the form $\pm \alpha \theta \pm \beta \sin 2\theta$ or $k(\pm \alpha \theta \pm \beta \sin 2\theta)$, $\alpha \neq 0$, $\beta \neq 0$		
		(can be simplified or un-simplified).		
	1 st A1	Integrating $\sin^2 \theta$ to give $\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta$, un-simplified or simplified. Correct solution only.		
		Can be implied by $k \sin^2 \theta$ giving $\frac{k}{2}\theta - \frac{k}{4}\sin 2\theta$ or $\frac{k}{4}(2\theta - \sin 2\theta)$ un-simplified or simplified.		
	2 nd Al A correct solution in part (ii) leading to a "two term" exact answer of			
		e.g. $\frac{4}{3}\pi - \sqrt{3}$ or $\frac{8}{6}\pi - \sqrt{3}$ or $\frac{4}{3}\pi - \frac{2\sqrt{3}}{2}$ or $\frac{1}{3}(4\pi - 3\sqrt{3})$		
	Note	A decimal answer of 2.456739397 (without a correct exact answer) is A0.		
	Note	Candidates can work in terms of λ (note that λ is not given in (ii)) and gain the 1 st three marks (i.e. M1M1A1) in part (b).		
	Note	If they incorrectly obtain $\int_{0}^{\frac{\pi}{3}} 8 \sin^2 \theta d\theta$ in part (i)(a) (or correctly guess that $\lambda = 8$)		
		then the final A1 is available for a correct solution in part (ii)(b).		



	Scheme		Notes	Marks
Way 2	$\int \frac{3y-4}{y(3y+2)} \mathrm{d}y = \int \frac{6y+2}{3y^2+2y} \mathrm{d}y - \int \frac{3y+6y}{y(3y+4y)} \mathrm{d}y$			
	$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$		See notes	M1
	$y = 0 \implies 6 = 2A \implies A = 3$		At least one of their $A = 3$ or their $B = -6$	A1
	$y = -\frac{2}{3} \implies 4 = -\frac{2}{3}B \implies B = -6$ Int $\int \frac{3y - 4}{y(3y + 2)} dy$ $= \int \frac{6y + 2}{2^{\frac{3}{2}} - 2} dy - \int \frac{3}{2} dy + \int \frac{6}{(2x - 2)} dy$		Both their $A = 3$ and their $B = -6$	A1
			Integrates to give at least one of either $\frac{M(6y+2)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ $\pm \lambda \ln y \text{ or } \frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
			ast one term correctly followed through	A1 ft
			$ln(3y^2 + 2y) - 3ln y + 2ln(3y + 2)$ with correct bracketing, simplified or un-simplified	A1 cao
				[6]
Way 3	$\int \frac{3y-4}{y(3y+2)} \mathrm{d}y = \int \frac{3y+1}{3y^2+2y} \mathrm{d}y - \int \frac{5}{y(3y+1)} \mathrm{d}y$	dy		
	$\frac{5}{v(3v+2)} \equiv \frac{A}{v} + \frac{B}{(3v+2)} \Rightarrow 5 = A(3v+2) + \frac{1}{(3v+2)}$	- By	See notes	M1
	$y = 0 \implies 5 = 2A \implies A = \frac{5}{2}$		At least one of their $A = \frac{5}{2}$ or their $B = -\frac{15}{2}$	A1
	$y = -\frac{2}{3} \implies 5 = -\frac{2}{3}B \implies B = -\frac{15}{2}$		Both their $A = \frac{5}{2}$ and their $B = -\frac{15}{2}$	A1
	$\int \frac{3y - 4}{y(3y + 2)} dy$ $= \int \frac{3y + 1}{y(3y + 2)} dy - \int \frac{5}{2} dy + \int \frac{15}{2} dy$	or $\frac{A}{y} \rightarrow$	Integrates to give at least one of either $\frac{M(3y+1)}{3y^2+2y} \rightarrow \pm \alpha \ln(3y^2+2y)$ $\pm \lambda \ln y \text{ or } \frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2)$ $M \neq 0, A \neq 0, B \neq 0$	M1
	$\int 3y^2 + 2y$ $\int y$ $\int (3y + 2)$	At lea	ast one term correctly followed through	A1 ft
	$= \int \frac{3y+1}{3y^2+2y} dy - \int \frac{\frac{5}{2}}{y} dy + \int \frac{\frac{15}{2}}{(3y+2)} dy$ $= \frac{1}{2} \ln(3y^2+2y) - \frac{5}{2} \ln y + \frac{5}{2} \ln(3y+2) \left\{ + c \right\}$		$\frac{1}{2}\ln(3y^2 + 2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y + 2)$ with correct bracketing, simplified or un-simplified	A1 cao
				[6]



	Scheme		Notes	
(i) Way 4	$\int \frac{3y-4}{y(3y+2)} \mathrm{d}y = \int \frac{3y}{y(3y+2)} \mathrm{d}y - \int \frac{4}{y(3y+2)} \mathrm{d}y$			
	$= \int \frac{3}{(3y+2)} \mathrm{d}y - \int \frac{4}{y(3y+2)} \mathrm{d}y$			
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$		See notes	M1
	$y(3y+2) y (3y+2)$ $y=0 \Rightarrow 4=2A \Rightarrow A=2$		At least one of their $A = 2$ or their $B = -6$	A1
	$y = -\frac{2}{3} \implies 4 = -\frac{2}{3}B \implies B = -6$			A1
			Integrates to give at least one of either $ \rightarrow \pm \alpha \ln(3y+2) \text{ or } \frac{A}{y} \rightarrow \pm \lambda \ln y \text{ or } $ $ \frac{B}{(3y+2)} \rightarrow \pm \mu \ln(3y+2), $ $ A \neq 0, B \neq 0, C \neq 0 $	M1
	$\int 3y + 2$ $\int y$ $\int (3y + 2)$	At lea	ast one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2) \left\{+c\right\}$		ln(3y+2) - 2ln y + 2ln(3y+2) with correct bracketing, simplified or un-simplified	A1 cao
		•		[6]

	Alternative methods for B1M1M1A1 in (ii)(a)				
(ii)(a) Way 2	$\left\{ x = 4\sin^2\theta \Longrightarrow \right\} \frac{\mathrm{d}x}{\mathrm{d}\theta} = 8\sin\theta\cos\theta$	As in			B1
	$\int \sqrt{\frac{4\sin^2\theta}{4-4\sin^2\theta}} \cdot 8\sin\theta\cos\theta \left\{ d\theta \right\}$			As before	M1
	$= \int \sqrt{\frac{\sin^2 \theta}{(1-\sin^2 \theta)}} \cdot 8\cos \theta \sin \theta \left\{ d\theta \right\}$				
	$= \int \frac{\sin \theta}{\sqrt{(1-\sin^2 \theta)}} \cdot 8\sqrt{(1-\sin^2 \theta)} \sin \theta \left\{ d\theta \right\}$				
	$= \int \sin \theta . 8 \sin \theta \left\{ d\theta \right\}$		Correct method leading to $\sqrt{(1-\sin^2\theta)}$ being cancelled out		M1
	$= \int 8\sin^2\theta d\theta$		$\int 8\sin^2\theta d\theta$	including $d\theta$	A1 cso
(ii)(a) Way 3	$\left\{x = 4\sin^2\theta \Longrightarrow\right\} \frac{\mathrm{d}x}{\mathrm{d}\theta} = 4\sin 2\theta$			As in Way 1	B1
	$x = 4\sin^2\theta = 2 - 2\cos 2\theta$, $4 - x = 2 + 2\cos 2\theta$				
	$\int \sqrt{\frac{2-2\cos 2\theta}{2+2\cos 2\theta}} \cdot 4\sin 2\theta \left\{ d\theta \right\}$		M1		
	$= \int \frac{\sqrt{2 - 2\cos 2\theta}}{\sqrt{2 + 2\cos 2\theta}} \cdot \frac{\sqrt{2 - 2\cos 2\theta}}{\sqrt{2 - 2\cos 2\theta}} 4\sin 2\theta \left\{ d\theta \right\} = \int \frac{d\theta}{d\theta} d\theta$				
				thod leading to g cancelled out	M1
	$= \int 8\sin^2\theta d\theta \qquad \qquad \int 8\sin^2\theta d\theta$			including $d\theta$	A1 cso



Question Number		Scheme	Notes	Marks				
	$\left\{ u = e^x \text{ or } x = \ln u \Longrightarrow \right\}$							
	$\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x \text{ or } \frac{\mathrm{d}u}{\mathrm{d}x} = u \text{ or } \frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{u} \text{ or } \mathrm{d}u = u \mathrm{d}x \text{ etc., and } \int \frac{6}{(\mathrm{e}^x + 2)} \mathrm{d}x = \int \frac{6}{(u + 2)u} \mathrm{d}u \text{See notes}$		B1 *					
	${x = 0}$	$a=1$ and $b=e$ or $b=e^1$	B1					
	${x = 1}$	$\Rightarrow b = e^1 \Rightarrow \underline{b} = \underline{e}$	or evidence of $0 \rightarrow 1$ and $1 \rightarrow e$	151				
	NOTE: 1 st B1 mark CANNOT be recovered for work in part (d) NOTE: 2 nd B1 mark CAN be recovered for work in part (d)							
	1 st B1 Must start from either							
		• $\int y dx$, with integral sign and dx						
		1						
		• $\int \frac{6}{(e^x + 2)} dx$, with integral sign and dx						
		• $\int \frac{6}{(e^x + 2)} \frac{dx}{du} du$, with integral sign and $\frac{dx}{du} du$						
	and state either $\frac{du}{dx} = e^x$ or $\frac{du}{dx} = u$ or $\frac{dx}{du} = \frac{1}{u}$ or $du = udx$ and end at $\int \frac{6}{u(u+2)} du$, with integral sign and du , with no incorrect working.							
	Note	So, just writing $\frac{du}{dx} = e^x$ and $\int \frac{6}{(e^x + 2)} dx = \int \frac{6}{u(u + 2)} du$ is sufficient for 1 st B1						
	Note Give 2^{nd} B0 for $b = 2.718$, without reference to $a = 1$ and $b = e$ or $b = e^1$							
	Note							
	Proceeding from $\int \frac{6}{u(u+2)} du$ to $\int \frac{6}{(e^x+2)} dx$, with no incorrect working, and stating either $\frac{du}{dx} = e^x$ or $\frac{du}{dx} = u$ or $\frac{dx}{du} = \frac{1}{u}$ or $du = udx$							