Integration by Substitution- Edexcel Past Exam Questions

1.

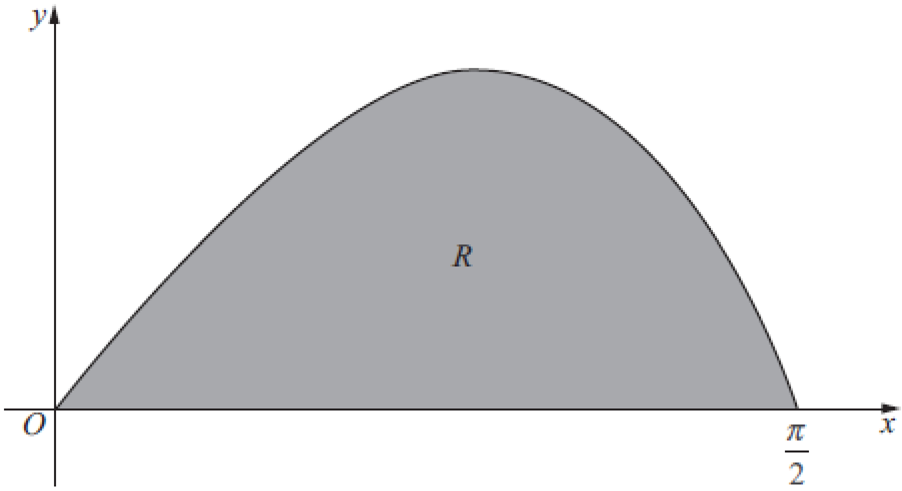

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant. **(5)**

Jan 12 Q6*(edited)*

2.

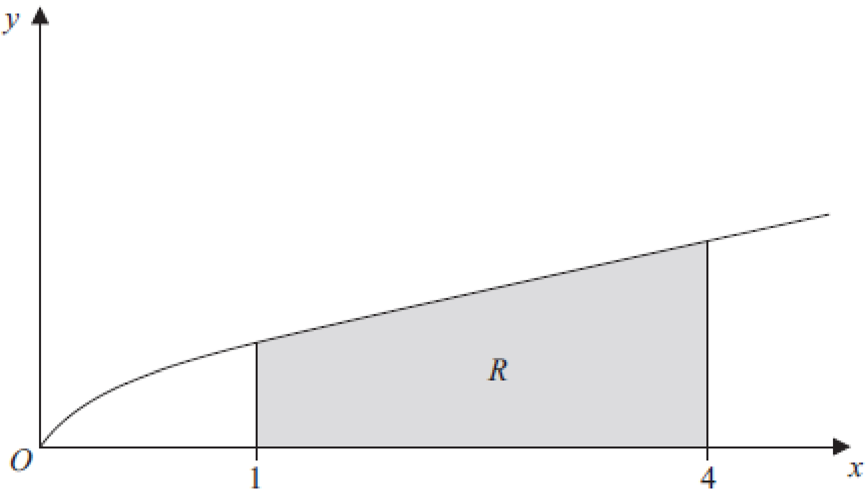

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R. **(8)**

Jan 13 Q4 *(edited)*

3. (*a*) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x}-1)}\,dx = \int \frac{2}{u(2u-1)}\,du$$

(3)

(*b*) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\,dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined. **(7)**

June 13 Q5

4. Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\, dx$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

June 13(R) Q3

5.

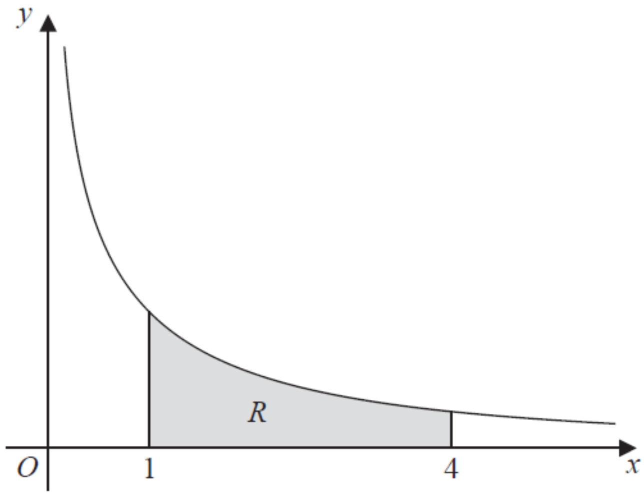

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x+5\sqrt{x}}$, $x > 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.

Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x+5\sqrt{x}}\, dx$$

(6)

June 14 Q3 *(edited)*

6.

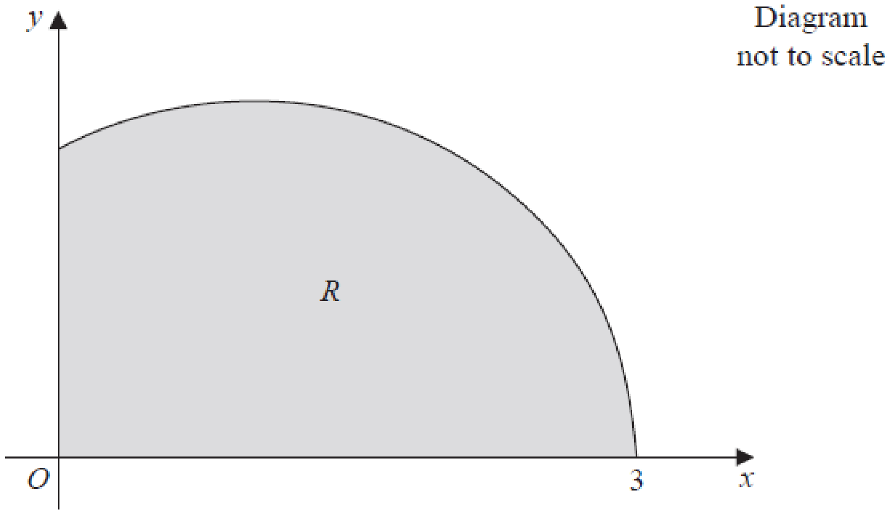

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leq x \leq 3$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta,$$

where k is a constant to be determined. **(5)**

June 15 Q6 *(edited)*

7. (i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y.$$

(6)

(ii) (*a*) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta\,\mathrm{d}\theta,$$

where λ is a constant to be determined. **(5)**

(*b*) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,\mathrm{d}x,$$

giving your answer in the form $a\pi + b$, where a and b are exact constants. **(4)**

June 16 Q6

8.

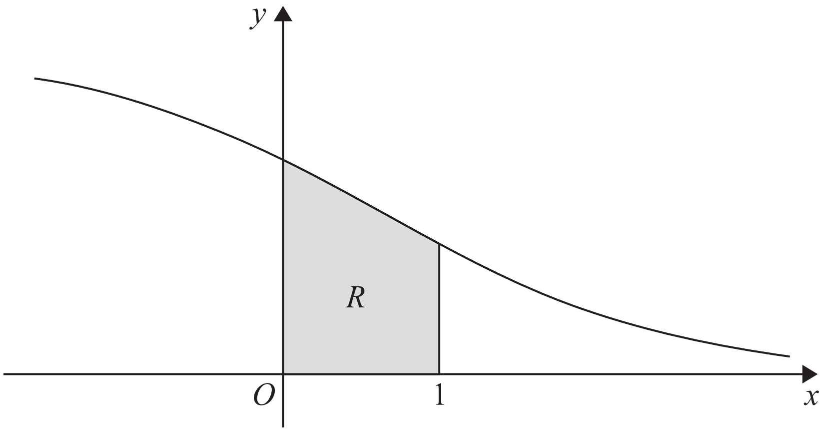

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{6}{(e^x + 2)}$, $x \in \mathbb{R}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 1$

Use the substitution $u = e^x$ to show that the area of R can be given by

$$\int_a^b \frac{6}{u(u+2)}\,du$$

where a and b are constants to be determined.

(2)

June 17 Q3*(edited)*