Newton Raphson Method - Edexcel Past Exam Questions

1.

Figure 1

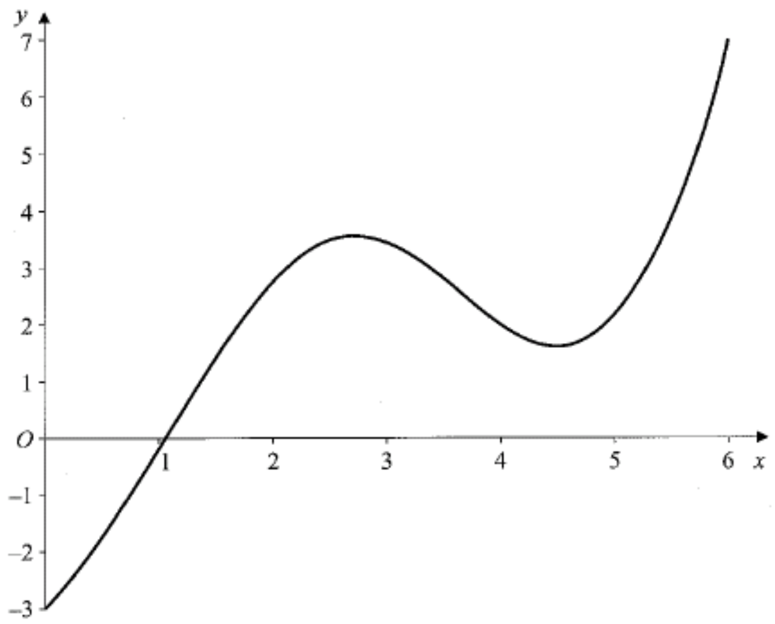

Figure 1 shows part of the graph of $y = f(x)$, where

$$f(x) = x \sin x + 2x - 3.$$

The equation $f(x) = 0$ has a single root α.

(*a*) Taking $x_1 = 1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, to 3 significant figures. **(5)**

(*b*) Given instead that $x_1 = 5$ is taken as a first approximation to α in the Newton-Raphson procedure,

(i) use Figure 1 to produce a rough sketch of $y = f(x)$ for $3 \leq x \leq 6$,

and by drawing suitable tangents, and without further calculation,

(ii) show the approximate positions of x_2 and x_3, the second and third approximations to α.

(2)

Jan 2005 Q4

2.

$$f(x) = 1 - e^x + 3 \sin 2x$$

The equation $f(x) = 0$ has a root α in the interval $1.0 < x < 1.4$.

(*b*) Taking your answer to part (*a*) as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α.

(4)

(*c*) By considering the change of sign of $f(x)$ over an appropriate interval, show that your answer to part (*b*) is accurate to 2 decimal places.

(2)

June 2005 Q4

3.

$$f(x) = 0.25x - 2 + 4 \sin \sqrt{x}.$$

(*a*) Show that the equation $f(x) = 0$ has a root α between $x = 0.24$ and $x = 0.28$. **(2)**

The equation $f(x) = 0$ also has a root β between $x = 10.75$ and $x = 11.25$.

(*c*) Taking 11 as a first approximation to β, use the Newton-Raphson process on $f(x)$ once to obtain a second approximation to β. Give your answer to 2 decimal places. **(6)**

June 2006 Q6

4.

$$f(x) = \ln x + x - 3, \quad x > 0.$$

(*a*) Find f(2.0) and f(2.5), each to 4 decimal places, and show that the root α of the equation $f(x) = 0$ satisfies $2.0 < \alpha < 2.5$. **(3)**

(*b*) Taking 2.25 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α, giving your answer to 3 decimal places. **(5)**

(*c*) Show your answer in part (*c*) gives α correct to 3 decimal places. **(2)**

Jan 2007 Q6 ***(edited)***

5.

$$f(x) = x^3 + 8x - 19.$$

(*a*) Show that the equation $f(x) = 0$ has only one real root. **(3)**

(*b*) Show that the real root of $f(x) = 0$ lies between 1 and 2. **(2)**

(*c*) Obtain an approximation to the real root of $f(x) = 0$ by performing two applications of the Newton-Raphson procedure to $f(x)$, using $x = 2$ as the first approximation. Give your answer to 3 decimal places. **(4)**

(*d*) By considering the change of sign of $f(x)$ over an appropriate interval, show that your answer to part (*c*) is accurate to 3 decimal places. **(2)**

June 2007 Q4

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6. $f(x) = 3x^2 + x - \tan\left(\frac{x}{2}\right) - 2, \qquad -\pi < x < \pi.$

The equation $f(x) = 0$ has a root α in the interval [0.7, 0.8].

(*b*) Taking 0.75 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places. **(4)**

Jan 2008 Q4

7.

$$f(x) = 4\cos x + e^{-x}.$$

(*a*) Show that the equation $f(x) = 0$ has a root α between 1.6 and 1.7 **(4)**

(*b*) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures. **(4)**

June 2008 Q2

8.

$$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20.$$

(*a*) Show that the equation f(x) = 0 has a root α in the interval [1.1, 1.2]. **(2)**

(*b*) Find f′(x). **(3)**

(*c*) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 significant figures.
(4)
Jan 2009 Q5

9.

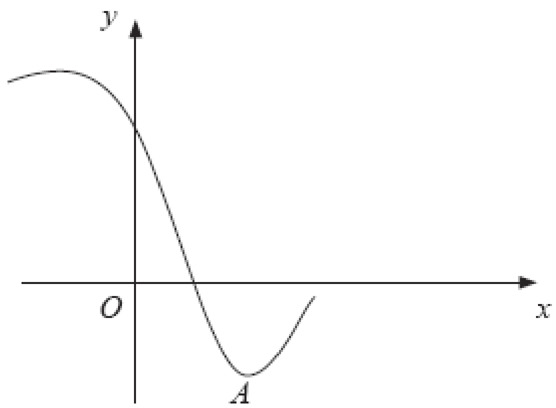

Figure 1

Figure 1 shows part of the curve with equation $y = f(x)$, where

$$f(x) = 1 - x - \sin(x^2).$$

The point A, with x-coordinate p, is a stationary point on the curve.

The equation f(x) = 0 has a root α in the interval $0.6 < \alpha < 0.7$.

(*a*) Explain why $x_0 = p$ is not suitable to use as a first approximation to α when applying the Newton-Raphson procedure to f(x). **(1)**

(*b*) Using $x_0 = 0.6$ as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 decimal places. **(5)**

(*c*) By considering the change of sign of f(x) over an appropriate interval, show that your answer to part (*b*) is accurate to 3 decimal places. **(2)**
Jan 2009 Q4

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10. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

(*a*) show that $2.2 < \alpha < 2.3$ **(2)**

(*b*) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places. **(5)**

June 2009 Q4 *(edited)*

11. Given that α is the only real root of the equation

$$\sin 2x - \ln 3x = 0,$$

(*a*) show that $0.8 < \alpha < 0.9$ **(2)**

(*b*) Taking 0.9 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = \sin 2x - \ln 3x$ to obtain a second approximation to α, giving your answer to 3 decimal places. **(5)**

June 2009 Q5

12.

$$f(x) = x \cos x - 2x + 5.$$

(*a*) Show that $f(x) = 0$ has a root α in the interval [2, 2.1]. **(2)**

(*b*) Taking 2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α, giving your answer to 2 decimal places. **(5)**

(*c*) Show that your answer to part (b) gives α **correct** to 2 decimal places. **(2)**

Jan 2010 Q2

13.

$$f(x) = 3x^2 - \frac{11}{x^2}.$$

(*a*) Write down, to 3 decimal places, the value of f(1.3) and the value of f(1.4). **(1)**

The equation $f(x) = 0$ has a root α between 1.3 and 1.4

(*c*) Taking 1.4 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α, giving your answer to 3 decimal places. **(5)**

Jan 2010 Q2

14.

$$f(x) = x^3 - \frac{7}{x} + 2, x > 0.$$

(*a*) Show that f $(x) = 0$ has a root α between 1.4 and 1.5. **(2)**

(*c*) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - \frac{7}{x} + 2, x > 0$ to obtain a second approximation to α, giving your answer to 3 decimal places. **(5)**

June 2010 Q3

15.

$$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geq 0.$$

The root α of the equation f $(x) = 0$ lies in the interval [1.6,1.8].

(*b*) Differentiate f(x) to find f $'(x)$. **(2)**

(*c*) Taking 1.7 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places. **(4)**

Jan 2011 Q3

16.

$$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0.$$

(*a*) Use differentiation to find f $'(x)$.

(2)

The root α of the equation f(x) = 0 lies in the interval [0.7, 0.9].

(*b*) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to f(x) to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

June 2011 Q4
