Newton Rhapson Method - Edexcel Past Exam Questions MARK SCHEME

Question 1

(a)	Correct method for $f'(x)$; $x\cos x + \sin x + 2$	M1A1
	$f(1) = -0.1585$, $f'(1) = 3.382$ or better seen	A1
	Using N-R correctly: $u_1 = 1 - \dfrac{"-0.1585"}{"3.382"}$; $= 1.05$ (3 s.f)	M1A1 (5)
	[Notes: Answer 1.047, 1.05 implies second A mark}	
(b)	Two tangents drawn, one at $\{5, f(5)\}$, the other at $\{x_2, f(x_2)\}$	M1
	x_2, x_3 marked in appropriate positions	A1 (2) [7]

Question 2

NB. $f(1) = 1 \cdot 0 \cdots$, $f(1 \cdot 1) = 0 \cdot 42 \cdots$, $f(1 \cdot 2) = -0 \cdot 2937 \cdots$

$f(1 \cdot 15) = 0 \cdot 078 \cdots$, $f(1 \cdot 4) = -2 \cdot 05$.

(a) $f(1 \cdot 2) = -0 \cdot 2937 \cdots$

$f(1 \cdot 1) = 0 \cdot 42 \cdots$ and $f(1 \cdot 15) = 0 \cdot 078 \cdots$

$\therefore \quad \underline{\alpha = 1 \cdot 2}$

$f(1 \cdot 2)$ to 1sf or better   B1

Attempt $f(1 \cdot 1), f(1 \cdot 15)$ M1

A1 c.a.o (3)

(b) $f'(x) = 6\cos 2x - e^x$ M1 A1

$x_2 = 1 \cdot 2 - \dfrac{-0 \cdot 2937 \cdots}{f'(1 \cdot 2)}$ M1

$= 1 \cdot 16(2 \cdots)$ A.W.R.T. $1 \cdot 16$ A1 (4)

(c) $f(1 \cdot 155) = 0 \cdot 04 \cdots$
$f(1 \cdot 165) = -0 \cdot 029 \cdots$, $\Big\}$ (change of sign) $\therefore \underline{\underline{\alpha = 1 \cdot 16}}$ M1,
A1

(2)

N.B. $f'(1 \cdot 2) = -7 \cdot 744 \cdots$ ⑨
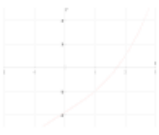
Question 3

Question Number	Scheme	Marks
	(a) $\quad$ $f(0.24) \approx -0.058,\ f(0.28) = 0.089$ $\qquad$ accept 1sf	M1
	$\qquad$ Change of sign (and continuity) $\Rightarrow \alpha \in (0.24, 0.28)$	A1 $\quad$ (2)
	(b) $\quad$ $f(0.26) \approx 0.017$ $\ (\Rightarrow\ \alpha \in (0.24, 0.26))$ $\quad$ accept 1sf	M1
	$\qquad$ $f(0.25) \approx -0.020$ $\ (\Rightarrow\ \alpha \in (0.25, 0.26))$	
	$\qquad$ $f(0.255) \approx -0.001$ $\ \Rightarrow\ \alpha \in (0.255, 0.26)$	M1 A1 (3)
	(c) $\qquad\qquad$ $f(11) \approx 0.0534$ $\qquad$ at least 3sf	B1
	$\qquad\qquad$ $f'(x) = \dfrac{2\cos\sqrt{x}}{\sqrt{x}} + \dfrac{1}{4}$	M1 A1
	$\qquad\qquad$ $f'(11) \approx -0.3438$ $\qquad$ at least 2sf	A1
	$\qquad\qquad$ $\beta \approx 11 + \dfrac{0.0534}{0.3438} \approx 11.16$ $\qquad$ cao	M1 A1 (6)
		[11]
	If $f'(11) \approx -0.3438$ is produced without working, this is to be	
	accepted for three marks M1 A1 A1.	

Question 4

(a) $f(2.0) = -0.30685....... = -0.3069$ AWRT 3 d.p.	M1
$f(2.5) = 0.41629......... = 0.4163$ both correct 4 d.p.	A1
States change of sign, so root (between 2 and 2.5)	B1 **(3)**

Note:
B1 gained if candidate's 2 values do show a change of sign and statement made

(b) $f(2.25) = 0.06093.......(\geq 3$ d.p.$)$ [Allow $\ln.2.25 + 2.25 - 3$]	M1,A1
$f'(x) = \dfrac{1}{x} + 1,$ $f'(2.25) = 1.\dot{4}$ or $1\dfrac{4}{9}$ or $\dfrac{13}{9}$ $(allow\ 1.444)$	M1A1 **(5)**
$\alpha = 2.25 - \dfrac{f(2.25)}{f'(2.25)},$ $= 2.20781.... = 2.208$ AWRT	

(c) $f(2.2075) =,$ $\{ -6.3.... \times 10^{-4} \}$ $f(2.2085) =,$ $\{ 8.1.... \times 10^{-4} \}$	M1
$\therefore$ Correct values $(\geq 1$ s.f.$)$, (root in interval) so root is 2.208 to 3 d.p.	A1 **(2)**

Notes:

Question 5

Question number	Scheme	Marks
	(a) $f'(x) = 3x^2 + 8$ $\qquad$ $3x^2 + 8 = 0$....... or $3x^2 + 8 > 0$....... $\quad$ Correct derivative and, e.g., 'no turning points' or 'increasing function'. $\qquad\qquad$ Simple sketch, (increasing, crossing positive x-axis) _____ $\qquad$ (or, if the M1 A1 has been scored, a <u>reason</u> such as 'crosses $\qquad\qquad$ x-axis only once'). (b) Calculate $f(1)$ and $f(2)$ $\quad$ (<u>Values</u> must be seen) $\qquad$ $f(1) = -10$, $f(2) = 5$, $\qquad$ Sign change, $\therefore$ Root (c) $x_1 = 2 - \dfrac{f(2)}{f'(2)},$ $\qquad = 2 - \dfrac{5}{20}$ $\quad (=1.75)$ $\qquad x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)},$ $\quad \left(= 1.75 - \dfrac{0.359375}{17.1875}\right) = 1.729$ (ONLY) (α) (d) Calculate $f(\alpha - 0.0005)$ and $f(\alpha + 0.0005)$ $\qquad\qquad$ (or a 'tighter' interval that gives a sign change). $\qquad f(1.7285) = -0.0077...$ and $f(1.7295) = 0.0092...,$ $\therefore$ Accurate to 3 d.p.	M1 A1 B1 $\quad$ (3) M1 A1 $\quad$ (2) M1, A1 M1, A1 $\quad$ (4) M1 A1 $\quad$ (2) **11**

Question 6

(a)	$f(0.7) = -0.195028497$ and $f(0.8) = 0.297206781$		B1, B1
	Use $\dfrac{0.8 - \alpha}{\alpha - 0.7} = \dfrac{f(0.8)}{-f(0.7)}$ to obtain $\alpha = \dfrac{-0.8 f(0.7) + 0.7 f(0.8)}{f(0.8) - f(0.7)}$		M1
	$(=0.739620991) = 0.740$ $\quad$ Answer required to 3 dp or better		A1 $\quad$ (4)
(b)	$f'(x) = 6x + 1 - \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$		M1 A1
	Use $x_2 = 0.75 - \dfrac{f(0.75)}{f'(0.75)}$ $(= 0.741087218) = 0.741$ $\quad$ Answer required to 3 dp or better		M1 A1 $\quad$ (4)
			[8]

Question 7

(a)	$f(1.6) = ...$ $f(1.7) = ...$ (Evaluate both)		M1
	$0.08...$ (or 0.09), $-0.3...$ One +ve, one –ve or sign change, $\therefore$ root		A1 (2)
(b)	$f'(x) = -4\sin x - e^{-x}$		B1
	$1.6 - \dfrac{f(1.6)}{f'(1.6)}$		M1
	$= 1.6 - \dfrac{4\cos 1.6 + e^{-1.6}}{(-4\sin 1.6 - e^{-1.6})}$ $\left(= 1.6 - \dfrac{0.085...}{-4.2...}\right)$		A1
			A1 (4)
			(6 marks)

Question 8

Question Number	Scheme	Marks
(a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
	$f(1.1) = 0.30875$, $f(1.2) = -0.28199$ Change of sign in $f(x) \Rightarrow$ root in the interval	A1 (2)
(b)	$f'(x) = \dfrac{3}{2}x^{-\frac{1}{2}} - 9x^{-1\frac{1}{2}}$	M1 A1 A1 (3)
(c)	$f(1.1) = 0.30875..$ $f'(1.1) = -6.37086...$	B1 B1
	$x_1 = 1.1 - \dfrac{0.30875...}{-6.37086..}$	M1
	$= 1.15$ (to 3 sig.figs.)	A1 (4) [9]

Question 9

Question Number	Scheme	Marks
(a)	At st. pt $f'(x) = 0$, $\therefore x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$ is undefined	B1 (1)
	or at st. pt, **tan.** // to x-axis, or **tan.** does not cross x-axis, o.e.	
(b)	$f'(x) = -1 - 2x\cos(x^2)$ (may be seen in body of work)	M1, A1
	$f(0.6) = 0.0477\ldots$, $f'(0.6) = -2.123\ldots$ (may be implied by correct answer)	A1
	Attempt to use $(x_1) = 0.6 \; - \; \dfrac{f(0.6)}{f'(0.6)}$ $[0.6 - \dfrac{0.0477\ldots}{-2.123\ldots}]$	M1
	$= 0.622$ (3 dp) $(0.6224795\ldots)$	A1 (5)
(c)	$f(0.6215) = 1.77\ldots\times10^{-3} > 0$, $f(0.6225) = -3.807\times\ldots\times10^{-4} < 0$	M1
	Change of sign in $f(x)$ in $(0.6215, 0.6225)$ "so 0.622 correct"	A1 (2)
		[8]
Notes	(b) 2ndM: If the N-R statement applied to 0.6 **not** seen, can be implied if answer correct; otherwise M0 If no values for $f(0.6)$, $f'(0.6)$ seen, they can be implied if final answer correct. (c) M: For candidates x_1, calculate $f(x_1 - 0.0005)$ and $f(x_1 + 0.0005)$ (or a tighter interval) A: Requires correct values of $f(0.6215)$ and $f(0.6225)$ (or their acceptable values) [may be rounded, e.g. 2×10^{-3}, or truncated, e.g -3.80×10^{-4}], sign change stated or >0, <0 seen, and conclusion.	

Question 10

Question Number	Scheme	Marks
(a)	$f(2.2) = 2.2^3 - 2.2^2 - 6 \qquad (= -0.192)$ $f(2.3) = 2.3^3 - 2.3^2 - 6 \qquad (= 0.877)$ Change of sign $\Rightarrow$ Root $\qquad$ need numerical values correct (to 1 s.f.).	M1 A1 (2)
(b)	$f'(x) = 3x^2 - 2x$ $f'(2.2) = 10.12$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2.2 - \dfrac{-0.192}{10.12}$ $\qquad = 2.219$	B1 B1 M1 A1ft A1cao (5)

Question 11

Question Number	Scheme	Marks
(a)	$f(0.8) = \sin 1.6 - \ln 2.4 \qquad (= 0.1241...)$ $f(0.9) = \sin 1.8 - \ln 2.7 \qquad (= -0.0194...)$ Values correct (to 1 s.f.). Change of sign $\Rightarrow$ Root	1M1 1A1 (2)
(b)	$f'(x) = 2\cos 2x, \ -\dfrac{1}{x}|$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 0.9 - \dfrac{-0.0194}{-1.5655}, \qquad = 0.888$	1B1, 2B1 1M1 1A1 2A1 (5)
(c)	$\dfrac{0.1241}{k} = \dfrac{0.0194}{0.1 - k} \qquad$ (where root is approx. 0.8 + k) $k = 0.086... \qquad\qquad \alpha \approx 0.886 \qquad\qquad$ (Allow awrt)	M1 1A1ft 2A1 (3) [10]

Question 12

Question Number	Scheme	Marks
	(a) $f(2) = 2\cos 2 - 4 + 5 \qquad (= 0.1677...)$ $f(2.1) = 2.1\cos 2.1 - 4.2 + 5 \quad (= -0.2601...)$ Values correct (to 1 s.f.). $\qquad$ Change of sign $\Rightarrow$ Root (b) $f'(x) = \cos x - x\sin x - 2$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2 - \dfrac{0.1677}{-4.2347}, \qquad = 2.04$ (c) $f(2.035) = ...$ and $f(2.045) = ...$ $\quad 0.0189...$ and $-0.0238...$ $\qquad$ Change of sign $\Rightarrow$ Correct to 2 d.p.	 M1 A1 $\qquad$ (2) M1 A1 M1 A1, A1 $\quad$ (5) M1 A1 $\qquad$ (2) **[9]**
	(c) The M1 is also given for evaluating f at the ends of a 'tighter' interval.	

Question 13

Question Number	Scheme	Marks
	(a) $f(1.3) = -1.439$ and $f(1.4) = 0.268$ $\qquad$ (allow awrt)	B1 (1)
	(b) $f(1.35) < 0 \; (-0.568...) \qquad \Rightarrow \qquad 1.35 < \alpha < 1.4$ $\quad f(1.375) < 0 \; (-0.146...) \qquad \Rightarrow \qquad 1.375 < \alpha < 1.4$	M1 A1 A1 (3)
	(c) $f'(x) = 6x + 22x^{-3}$ $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1.4 - \dfrac{0.268}{16.417}, \qquad = 1.384$	M1 A1 M1 A1, A1 (5) **[9]**

Question 14

Question Number	Scheme	Marks
	(a) $f(1.4) = ...$ and $f(1.5) = ...$ Evaluate both $f(1.4) = -0.256$ (or $-\dfrac{32}{125}$), $f(1.5) = 0.708...$ (or $\dfrac{17}{24}$) **Change of sign, $\therefore$ root** **Alternative method:** **Graphical method** could earn M1 if 1.4 and 1.5 are both indicated A1 then needs correct graph and conclusion, i.e. change of sign $\therefore$ root	M1 A1 (2)
	(b) $f(1.45) = 0.221...$ or 0.2 [$\therefore$ root is in [1.4, 1.45]] $f(1.425) = -0.018...$ or -0.019 or -0.02 $\therefore$ root is in [1.425, 1.45]	M1 M1 A1cso (3)
	(c) $f'(x) = 3x^2 + 7x^{-2}$ $f'(1.45) = 9.636...$ (Special case: $f'(x) = 3x^2 + 7x^{-2} + 2$ then $f'(1.45) = 11.636...$) $x_1 = 1.45 - \dfrac{f(1.45)}{f'(1.45)} = 1.45 - \dfrac{0.221...}{9.636...} = 1.427$	M1 A1 A1ft M1 A1cao (5) **10 marks**

Question 15

Question Number	Scheme	Marks
	(a) $f(1.4) = ...$ and $f(1.5) = ...$ Evaluate both $f(1.4) = -0.256$ (or $-\dfrac{32}{125}$), $f(1.5) = 0.708...$ (or $\dfrac{17}{24}$) **Change of sign, $\therefore$ root** **Alternative method:** **Graphical method** could earn M1 if 1.4 and 1.5 are both indicated A1 then needs correct graph and conclusion, i.e. change of sign $\therefore$ root	M1 A1 (2)
	(b) $f(1.45) = 0.221...$ or 0.2 [$\therefore$ root is in $[1.4, 1.45]$] $f(1.425) = -0.018...$ or -0.019 or -0.02 $\therefore$ root is in $[1.425, 1.45]$	M1 M1 A1cso (3)
	(c) $f'(x) = 3x^2 + 7x^{-2}$ $f'(1.45) = 9.636...$ (Special case: $f'(x) = 3x^2 + 7x^{-2} + 2$ then $f'(1.45) = 11.636...$) $x_1 = 1.45 - \dfrac{f(1.45)}{f'(1.45)} = 1.45 - \dfrac{0.221...}{9.636...} = 1.427$	M1 A1 A1ft M1 A1cao (5) **10 marks**

Question 16

Question Number	Scheme	Notes	Marks
(a)	$f(x) = x^2 + \dfrac{5}{2x} - 3x - 1, \quad x \neq 0$ $f(x) = x^2 + \dfrac{5}{2}x^{-1} - 3x - 1$ $f'(x) = 2x - \dfrac{5}{2}x^{-2} - 3\{+ 0\}$ $\left\{ f'(x) = 2x - \dfrac{5}{2x^2} - 3 \right\}$	 At least two of the four terms differentiated correctly. Correct differentiation. (Allow any correct unsimplified form)	 M1 A1 (2)
(b)	$f(0.8) = 0.8^2 + \dfrac{5}{2(0.8)} - 3(0.8) - 1 (= 0.365)\left(= \dfrac{73}{200} \right)$	A correct numerical expression for f(0.8)	B1
	$f'(0.8) = -5.30625\left(= \dfrac{-849}{160} \right)$	Attempt to insert $x = 0.8$ into their f′(x). Does not require an evaluation. (If f′(0.8) is incorrect for their derivative and there is no working score M0)	M1
	$\alpha_2 = 0.8 - \left(\dfrac{"0.365"}{"-5.30625"} \right)$	Correct application of Newton-Raphson using their values. Does not require an evaluation.	M1
	$= 0868786808...$		
	$= 0.869$ (3dp)	0.869	A1 cao
	A correct answer only with no working scores no marks. N-R must be seen. **Ignore any further applications of N-R**		(4)
	A derivative of $2x - 5(2x)^{-2} - 3$ is quite common and leads to f′(0.8) = -3.353125 and a final answer of 0.909. This would normally score M1A0B1M1M1A0 (4/6) Similarly for a derivative of $2x - 10x^{-2} - 3$ where the corresponding values are f′(0.8) = -17.025 and answer 0.821		
			6