Numerical Methods: Iterations 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question No	Scheme	Marks
	(a) f(0.8) = 0.082, f(0.9)= -0.089	M1
	Change of sign ⇒ root (0.8,0.9)	A1
		(2)
	(b)	
	$f'(x) = 2x - 3 - \sin(\frac{1}{2}x)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow \quad x = \frac{3+\sin(\frac{1}{2}x)}{2}$	M1A1*
		(4)
	(c) Sub x_0=2 into $x_{n+1} = \frac{3+\sin(\frac{1}{2}x_n)}{2}$	M1
	x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	A1,A1
		(3)
	(d) [1.90775,1.90785]	M1
	f'(1.90775)=-0.00016.. AND f'(1.90785)= 0.0000076..	M1
	Change of sign ⇒ x=1.9078	A1
		(3)
		(12 marks)

Question 2

Question Number	Scheme	Marks
2.	(a) $x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$	
	$\Rightarrow x^2(x+3)=12-4x$	M1
	$\Rightarrow x^2=\dfrac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1A1*
		(3)
	(b) $x_1=1.41$, *awrt* $x_2=1.20$ $x_3=1.31$	M1A1,A1
		(3)
	(c) Choosing (1.2715,1.2725) or tighter containing root 1.271998323	M1
	f(1.2725) = (+)0.00827... f(1.2715) = −0.00821....	M1
	Change of sign⇒α=1.272	A1
		(3) **(9 marks)**

Alternative to (a) working backwards

2(a)

	$x=\sqrt{\dfrac{4(3-x)}{(x+3)}} \Rightarrow x^2=\dfrac{4(3-x)}{(x+3)} \Rightarrow x^2(x+3)=4(3-x)$	M1
	$x^3+3x^2=12-4x \Rightarrow x^3+3x^2+4x-12=0$	dM1
	States that this is f(x)=0	A1*
		(3)

Question 3

Question Number	Scheme	Marks
	(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6-x)+1$	M1A1* (2)
	(b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6-x_n)+1 \Rightarrow x_1 = 2.3863$	M1, A1
	AWRT 4 dp. $x_2 = 2.2847$ $x_3 = 2.3125$	A1 (3)
	(c) Chooses interval [2.3065,2.3075]	M1
	$g(2.3065)$=-0.0002(7), $g(2.3075)$=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1 (3)
		(8 marks)

(a) M1 Sets g(x)=0, and using correct *ln* work, makes the x of the e^{x-1} term the subject of the formula.
Look for $e^{x-1}+x-6=0 \Rightarrow e^{x-1}=\pm 6 \pm x \Rightarrow x=\ln(\pm 6 \pm x) \pm 1$

Do not accept $e^{x-1}=6-x$ without firstly seeing $e^{x-1}+x-6=0$ or a statement that g(x)=0 ⇒

A1* cso. $x=\ln(6-x)+1$ **Note that this is a given answer (and a proof).**

'Invisible' brackets are allowed for the M but not the A

Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.

$0=e^{x-1}+x-6 \Rightarrow \quad 0=x-1+\ln(x-6) \Rightarrow x=\ln(6-x)+1$

(b) M1 Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1$ to produce a numerical value for x_1.
Evidence for the award could be any of $\ln(6-2)+1$, $\ln 4+1$, 2.3….. or awrt 2.4

A1 Answer correct to 4 dp $x_1=2.3863$.
The subscript is not important. Mark as the first value given/found.

A1 Awrt 4 dp. $x_2=2.2847$ *and* $x_3=2.3125$
The subscripts are not important. Mark as the second and third values given/found

(c) M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates $g(2.3065)$ and $g(2.3075)$ with at least one of these correct to 1sf.
The answers can be rounded or truncated
$g(2.3065)$ = -0.0003 rounded, $g(2.3065)$= -0.0002 truncated
$g(2.3075)$ = (+) 0.004 rounded and truncated

A1 Both values correct (rounded or truncated),
A reason which could include change of sign, >0 <0, $g(2.3065)\times g(2.3075)<0$
AND a minimal conclusion such as hence root, α=2.307 or ◘
Do not accept continued iteration as question demands an interval to be chosen.

Alternative solution to (a) working backwards

M1 Proceeds from $x=\ln(6-x)+1$ using correct exp work to …….=0

A1 **Arrives correctly** at $e^{x-1}+x-6=0$ **and** makes a statement to the effect that this is g(x)=0

Alternative solution to (c) using $f(x)=\ln(6-x)+1-x$ **{Similarly** $h(x)=x-1-\ln(6-x)$**}**

M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates f(2.3065) and f(2.3075) with at least 1 correct rounded or truncated
f(2.3065) = 0.000074. Accept 0.00007 rounded or truncated. Also accept 0.0001

f(2.3075) = - 0.0011.. Accept -0.001 rounded or truncated

Question 4

Question Number	Scheme	Marks
(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe.	M1A1
	Puts $f'(x) = 0$ to give $x = -1$ and $x = 0$ or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	A1
		(5)
(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm\frac{4}{5}e^{-x}$	B1*
		(1)
(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$	M1A1
	$\Rightarrow x_2 = \text{awrt } 0.492,\ x_3 = \text{awrt } 0.489$	A1
		(3)
(d)	$\alpha = 0.49$	B1
	$f(0.485) = -0.487,\ f(0.495) = (+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu' + uv'$. If the rule is quoted it must be correct.

It can be implied by their $u = ..., v = ..., u' = ..., v' = ...$ followed by their $vu' + uv'$

If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x} + Bxe^{2x}$

A1 $f'(x) = 50x^2e^{2x} + 50xe^{2x}$.

Allow un simplified forms such as $f'(x) = 25x^2 \times 2e^{2x} + 50x \times e^{2x}$

dM1 Sets $f'(x) = 0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x

This is dependent upon the first M1 being scored.

A1 Both $x = -1$ and $x = 0$ or one complete coordinate . Accept $(0,-16)$ and $(-1, 25e^{-2}-16)$ or $(-1, awrt -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.

$x = -1, 0 \quad y = \frac{25}{e^2} - 16, -16$ or linked together by coordinate pairs $(0,-16)$ and $(-1, 25e^{-2}-16)$ but the 'pairs' must be correct and exact.

Notes for Question Continued

(b)

B1 This is a show that question and all elements must be seen

Candidates must 1) State that f(x)=0 or writes $25x^2e^{2x} - 16 = 0$ or $25x^2e^{2x} = 16$

2) Show at least one intermediate (correct) line with either

x^2 or x the subject. Eg $x^2 = \frac{16}{25}e^{-2x}$, $x = \sqrt{\frac{16}{25}e^{-2x}}$ oe

or square rooting $25x^2e^{2x} = 16 \Rightarrow 5xe^x = \pm 4$

or factorising by DOTS to give $(5xe^x + 4)(5xe^x - 4) = 0$

3) Show the given answer $x = \pm\frac{4}{5}e^{-x}$.

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that f(x)=0

(c)

M1 Substitutes $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = ...$

This can be implied by $x_1 = \frac{4}{5}e^{-0.5}$, or awrt 0.49

A1 x_1 = awrt 0.485 3dp. Mark as the first value given. Don't be concerned by the subscript.

A1 x_2 = awrt 0.492, x_3 = awrt 0.489 3dp. Mark as the second and third values given.

(d)

B1 States $\alpha = 0.49$

B1 Justifies by

either calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$f(0.485) = -0.5, f(0.495) = (+)0.5$ rounded

$f(0.485) = -0.4, f(0.495) = (+)0.4$ truncated

giving a reason – accept change of sign, >0 <0 or $f(0.485) \times f(0.495) < 0$

and giving a minimal conclusion. Eg. Accept hence root or $\alpha = 0.49$

A smaller interval containing the root may be used, eg f (0.49) and f (0.495). Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of x_4= awrt 0.491 and stating (or seeing each value round to) 0.49

Question 5

Question Number	Scheme	Marks
(a)	$f(x)=0 \Rightarrow x^2+3x+1=0$	
	$\Rightarrow x=\dfrac{-3\pm\sqrt{5}}{2}=$ awrt -0.382, -2.618	M1A1
		(2)
(b)	Uses $vu'+uv'$ $\quad f'(x)=e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x$	M1A1A1
		(3)
(c)	$e^{x^2}(2x+3)+(x^2+3x+1)e^{x^2}\times 2x=0$	
	$\Rightarrow e^{x^2}\{2x^3+6x^2+4x+3\}=0$	M1
	$\Rightarrow x(2x^2+4)=-3(2x^2+1)$	M1
	$\Rightarrow x=-\dfrac{3(2x^2+1)}{2(x^2+2)}$	A1*
		(3)
(d)	Sub $x_0=-2.4$ into $x_{n+1}=-\dfrac{3(2x_n^{\ 2}+1)}{2(x_n^{\ 2}+2)}$	
	$x_1=awrt-2.420,\ x_2=awrt-2.427\ \ x_3=awrt-2.430$	M1A1,A1
		(3)
(e)	Sub x= - 2.425 and -2.435 into $f'(x)$ and start to compare signs	
	$f'(-2.425)=+22.4, f'(-2.435)=-15.02$	M1
	Change in sign, hence $f'(x)=0$ in between. Therefore α = - 2.43 (2dp)	A1
		(2)
		(13 marks)
Alt (c)	$x=-\dfrac{3(2x^2+1)}{2(x^2+2)} \Rightarrow 2x(x^2+2)=-3(2x^2+1) \Rightarrow 2x^3+6x^2+4x+3=0$	M1
	$f'(x)=e^{x^2}\{2x^3+6x^2+4x+3\}=0$ when $2x^3+6x^2+4x+3=0$	M1
	Hence the minimum point occurs when $x=-\dfrac{3(2x^2+1)}{(2x^2+4)}$	A1

Question Number	Scheme	Marks
Alt 1 (e)	Sub x= - 2.425 and -2.435 into cubic part of $f'(x) = 2x^3 + 6x^2 + 4x + 3$ and start to compare signs	
	Adapted $f'(-2.425) = +0.06, f'(-2.435) = -0.04$	M1
	Change in sign, hence $f'(x) = 0$ in between. Therefore α = - 2.43 (2dp)	A1
		(2)
Alt 2 (e)	Sub x= - 2.425, -2.43 and -2.435 into $f(x) = (x^2 + 3x + 1)e^{x^2}$ and start to compare sizes	
	$f(-2.425) = -141.2, f(-2.435) = -141.2, f(-2.43) = -141.3$	M1
	$f(-2.43) < f(-2.425), f(-2.43) < f(-2.435)$. Therefore α = - 2.43 (2dp)	A1
		(2)

Notes for Question

(a)

M1 Solves $x^2 + 3x + 1 = 0$ by completing the square or the formula, producing two 'non integer answers. **Do not accept factorisation here** . Accept awrt -0.4 and -2.6 for this mark

A1 Answers correct. Accept awrt -0.382, -2.618.

Accept just the answers for both marks. Don't withhold the marks for incorrect labelling.

(b)

M1 Applies the product rule $vu' + uv'$ to $(x^2 + 3x + 1)e^{x^2}$.

If the rule is quoted it must be correct and there must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, ie. terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') only accept answers of the form

$$\left(\frac{dy}{dx}\right) = f'(x) = e^{x^2}(Ax + B) + (x^2 + 3x + 1)Cxe^{x^2}$$

A1 One term of $f'(x) = e^{x^2}(2x + 3) + (x^2 + 3x + 1)e^{x^2} \times 2x$ correct.

There is no need to simplify

A1 A fully correct (un simplified) answer $f'(x) = e^{x^2}(2x + 3) + (x^2 + 3x + 1)e^{x^2} \times 2x$

(c)

M1 Sets their $f'(x) = 0$ and either factorises out, or cancels by e^{x^2} to produce a polynomial equation in x

M1 Rearranges the cubic polynomial to $Ax^3 + Bx = Cx^2 + D$ and factorises to reach $x(Ax^2 + B) = Cx^2 + D$ or equivalent

A1* Correctly proceeds to $x = -\dfrac{3(2x^2 + 1)}{2(x^2 + 2)}$. This is a given answer

Notes on Question Continued

(c) Alternative to (c) working backwards

M1 Moves correctly from $x=-\frac{3(2x^2+1)}{2(x^2+2)}$ to $2x^3+6x^2+4x+3=0$

M1 States or implies that $f'(x)=0$

A1 Makes a conclusion to tie up the argument

For example, hence the minimum point occurs when $x=-\frac{3(2x^2+1)}{(2x^2+4)}$

(d)

M1 Sub $x_0=-2.4$ into $x_{n+1}=-\frac{3(2x_n{}^2+1)}{2(x_n{}^2+2)}$

This may be implied by awrt -2.42, or $x_{n+1}=-\frac{3(2\times-2.4^2+1)}{2(-2.4^2+2)}$

A1 Awrt. $x_1=-2.420$.

The subscript is not important. Mark as the first value given

A1 awrt $x_2=-2.427$ awrt $x_3=-2.430$

The subscripts are not important. Mark as the second and third values given

(e)

Note that continued iteration is not allowed

M1 Sub x= - 2.425 and -2.435 into $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $f'(a)\times f'(b)<0$

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Alt 1 using adapted $f'(x)$

(e)

M1 Sub x= - 2.425 and -2.435 into cubic part of $f'(x)$, starts to compare signs and gets at least one correct to 1 sf rounded or truncated.

A1 Both values correct of adapted $f'(x)$ correct (1sf rounded or truncated), a reason and a minimal conclusion

Acceptable reasons are change in sign, positive and negative and $f'(a)\times f'(b)<0$

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Alt 2 using $f(x)$

(e)

M1 Sub x= - 2.425, -2.43 and -2.435 into $f(x)$, starts to compare sizes and gets at least one correct to 4sf rounded

A1 All three values correct of $f(x)$ correct (4sf rounded), a reason and a minimal conclusion

Acceptable reasons are $f(-2.43)<f(-2.425)$, $f(-2.43)<f(-2.435)$, a sketch

Minimal conclusions are hence $\alpha=-2.43$, hence shown, hence root

Question 6

Question Number	Scheme	Marks
(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	M1
	Change of sign $\Rightarrow Q$ lies between	A1
		(2)
(b)	At R $\quad \frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow \quad x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
(c)	$x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$	M1
	$x_1 =$ awrt 1.284 $\quad x_2 =$ awrt 1.276	A1
		(2)
		(8 marks)

(a)

M1 Sub both $x = 2.1$ and $x = 2.2$ into y and achieve at least one correct to 1 sig fig

In radians $y_{2.1} = \text{awrt} - 0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

In degrees $y_{2.1} = \text{awrt } 3$ $y_{2.2} = \text{awrt } 4$

A1 Both values correct to 1 sf with a reason and a minimal conclusion.

$y_{2.1} = \text{awrt} - 0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

Accept change of sign, positive and negative, $y_{2.1} \times y_{2.2} = -1$ as reasons and hence root, Q lies between 2.1 and 2.2, QED as a minimal conclussion.

Accept a smaller interval spanning the root of 2.131528, say 2.13 and 2.14, but the A1 can only be scored when the candidate refers back to the question, stating that as root lies between 2.13 and 2.14 it lies between 2.1 and 2.2

(b)

M1 Differentiating to get $\frac{dy}{dx} = ...\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$ where ... is a constant, or a linear function in x.

A1 $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$

M1 Sets their $\frac{dy}{dx} = 0$ and proceeds to make the x of their $3x^2$ the subject of the formula

Alternatively they could state $\frac{dy}{dx} = 0$ and write a line such as

$2x\sin\left(\frac{1}{2}x^2\right) = 3x^2 - 3$, before making the x of $3x^2$ the subject of the formula

A1* Correct given solution. $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$

Watch for missing x's in their formula

(c)

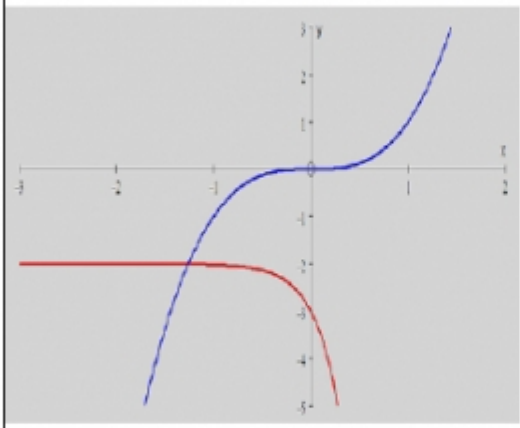
M1 Subs x= 1.3 into the iterative formula to find at least x_1.

This can be implied by $x_1 = \text{awrt } 1.3$ (not just 1.3)

or $x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$ or $x_1 = \text{awrt } 1.006$ (degrees)

A1 Both answers correct (awrt 3 decimal places). The subscripts are not important. Mark as the first and second values seen. $x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$

Question 7

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$	M1, A1
	Puts $\frac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$	A1 *
		(3)
(b)		
	$y = x^3$	B1
	Shape of $y = -2 - e^{4x}$	B1
	$y = -2 - e^{4x}$ cuts y axis at (0,-3)	B1
	$y = -2 - e^{4x}$ has asymptote at $y = -2$	B1
		(4)
(c)	Only one crossing point	B1
		(1)
(d)	−1.26376, -1.26126 Accept answers which round to these answers to 5dp	M1 A1
		(2)
(e)	α = -1.26 and so turning point is at (-1.26, -2.55)	M1 A1cao
		(2)
		12 marks

(a)

M1 Two (of the four) terms differentiated correctly

A1 All correct $\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$

A1* States or sets $\frac{dy}{dx} = 0$, and proceeds correctly to achieve printed answer $x^3 = -2 - e^{4x}$.

(b)

B1 Correct shape and position for $y = x^3$. It must appear to go through the origin.
It must only appear in Quadrants 1 and 3 and have a gradient that is always $\geqslant 0$. The gradient should appear large at either end. Tolerate slips of the pen.See practice and qualification for acceptable curves.

B1 Correct shape for $y = -2 - e^{4x}$, its position is not important for this mark. The gradient must be approximately zero at the left hand end and increase negatively as you move from left to right along the curve. See practice and qualification for acceptable curves.

B1 Score for $y = -2 - e^{4x}$ cutting or meeting the y axis at (0,-3). Its shape is not important.
Accept for the intention of (0,-3), -3 being marked on the y – axis as well as (-3,0)
Do not accept 3 being marked on the negative y axis.

B1 Score for $y = -2 - e^{4x}$ having an asymptote stated as $y = -2$. This is dependent upon the curve appearing to have an asymptote there. Do not accept the asymptote marked as '-2' or indeed $x = -2$. See practice and qualification for acceptable solutions.

(c)

B1 Score for a statement to the effect that the graphs cross at one point. Accept minimal statements such as 'one intersection'. Do not award if their diagram shows more than one intersection. They must have a diagram (which may be incorrect)

(d)

M1 Awarded for applying the iteration formula once. Possible ways in which this can be scored are the sight of $\sqrt[3]{-2 - e^{-4}}$, $\left(-2 - e^{4x-1}\right)^{\frac{1}{3}}$ or awrt -1.264

A1 Both values correct awrt -1.26376, -1.26126 5dps. The subscripts are unimportant for this mark. Score as the first and second values seen.

(e)

M1 Score for EITHER rounding their value in part (c) to 2 dp OR finding turning point of C by substituting a value of x generated from part (d) into $y = e^{4x} + x^4 + 8x + 5$ in order to find the y value. You may accept the appearance of a y value as evidence of finding the turning point (as long as an x value appears to be generated from part (d) and the correct equation is used.)

A1 (-1.26, -2.55) and correct solution only. It is a deduction and you cannot accept the appearance of a correct answer for two marks.

Question 8

Question Number	Scheme	Marks
(a)	$2^{x+1}-3=17-x \Rightarrow 2^{x+1}=20-x$	M1
	$(x+1)\ln 2=\ln(20-x) \Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
(b)	Sub $x_0=3$ into $x_{n+1}=\dfrac{\ln(20-x_n)}{\ln 2}-1, \Rightarrow$ $x_1=3.087$ (awrt)	M1A1
	$x_2=3.080,\ x_3=3.081$ (awrt)	A1
		(3)
(c)	$A=(3.1, 13.9)$ cao	M1,A1
		(2)
		(8 marks)
(a)Alt	$2^{x+1}-3=17-x \Rightarrow 2^{x}=\dfrac{20-x}{2}$	M1
	$x\ln 2=\ln\dfrac{20-x}{2} \Rightarrow x=\ldots$	dM1
	$x=\dfrac{\ln(20-x)}{\ln 2}-1$	A1*
		(3)
(a) backwards	$x=\dfrac{\ln(20-x)}{\ln 2}-1 \Rightarrow (x+1)\ln 2=\ln(20-x)$	M1
	$\Rightarrow 2^{x+1}=20-x$	dM1
	Hence $y=2^{x+1}-3$ meets $y=17-x$	A1*
		(3)

(a)

M1 Setting equations in x equal to each other and proceeding to make 2^{x+1} the subject

dM1 Take ln's or logs of both sides, use the power law and proceed to $x =$..

A1* This is a given answer and all aspects must be correct including ln or $\log_e$ rather than $\log_{10}$

Bracketing on both $(x+1)$ and $\ln(20-x)$ must be correct.

Eg $x+1\ln 2 = \ln(20-x) \Rightarrow x = \dfrac{\ln(20-x)}{\ln 2} - 1$ is A0*

Special case: Students who start from the point $2^{x+1} = 20 - x$ can score M1 dM1A0*

(b)

M1 Sub $x_0 = 3$ into $x_{n+1} = \dfrac{\ln(20-x_n)}{\ln 2} - 1$ to find $x_1 =$..

Accept as evidence $x_1 = \dfrac{\ln(20-3)}{\ln 2} - 1$, awrt $x_1 = 3.1$

Allow $x_0 = 3$ into the miscopied iterative equation $x_1 = \dfrac{\ln(20-3)}{\ln 2}$ to find $x_1 =$..

Note that the answer to this, 4.087, on its own without sight of $\dfrac{\ln(20-3)}{\ln 2}$ is M0

A1 awrt 3 dp $x_1 = 3.087$

A1 awrt $x_2 = 3.080$, $x_3 = 3.081$. Tolerate 3.08 for 3.080

Note that the subscripts are not important, just mark in the order seen

(c) Note that this appears as B1B1 on e pen. It is marked M1A1

M1 For sight of 3.1

Alternatively it can be scored for substituting their value of x or a rounded value of x from (b) into either $2^{x+1} - 3$ or $17 - x$ to find the y coordinate.

A1 $(3.1, 13.9)$

Question 9

Question	Scheme	Marks
(a)	(i) 21	B1
	(ii) $4e^{2x}-25=0 \Rightarrow e^{2x}=\frac{25}{4} \Rightarrow 2x=\ln\left(\frac{25}{4}\right) \Rightarrow x=\frac{1}{2}\ln\left(\frac{25}{4}\right), \Rightarrow x=\ln\left(\frac{5}{2}\right)$	M1A1, A1
	(iii) 25	B1
		(5)
(b)	$4e^{2x}-25=2x+43 \Rightarrow e^{2x}=\frac{1}{2}x+17$	M1
	$\Rightarrow 2x=\ln\left(\frac{1}{2}x+17\right) \Rightarrow x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$	A1*
		(2)
(c)	$x_1=\frac{1}{2}\ln\left(\frac{1}{2}\times 1.4+17\right)=awrt\ 1.44$	M1
	awrt $x_1=1.4368, x_2=1.4373$	A1
		(2)
(d)	Defines a suitable interval 1.4365 and 1.4375	M1
	...and substitutes into a suitable function Eg $4e^{2x}-2x-68$, obtains correct values with both a reason and conclusion	A1
		(2)
		(11 marks)

In part (a) accept points marked on the graph. If they appear on the graph and in the text, the text takes precedence. If they don't mark (a) as (i) (ii) and (iii) mark in the order given. If you feel unsure then please use the review system and your team leader will advise.

(a) (i)

B1 Sight of 21. Accept $(0,21)$

Do not accept just $|4-25|$ or $(21,0)$

(a) (ii)

M1 Sets $4e^{2x}-25=0$ and proceeds via $e^{2x}=\frac{25}{4}$ or $e^{x}=\frac{5}{2}$ to $x=..$

Alternatively sets $4e^{2x}-25=0$ and proceeds via $\left(2e^{x}-5\right)\left(2e^{x}+5\right)=0$ to $e^{x}=..$

A1 $\frac{1}{2}\ln\left(\frac{25}{4}\right)$ or awrt 0.92

A1 cao $\ln\left(\frac{5}{2}\right)$ or $\ln 5-\ln 2$. Accept $\left(\ln\left(\frac{5}{2}\right),0\right)$

(a) (iii)

B1 $k=25$ Accept also 25 or $y=25$

Do not accept just $|-25|$ or $x=25$ or $y=\pm 25$

(b)

M1 Sets $4e^{2x}-25=2x+43$ and makes e^{2x} the subject. Look for $e^{2x}=\frac{1}{4}(2x+43+25)$ condoning sign slips.Condone $|4e^{2x}-25|=2x+43$ and makes $|e^{2x}|$ the subject. Condone for both marks a solution with $x=a/\alpha$

An acceptable alternative is to proceed to $2e^{2x}=x+34 \Rightarrow \ln 2+2x=\ln(x+34)$ using ln laws

A1* Proceeds correctly without errors to the correct solution. This is a given answer and the bracketing must be correct throughout. The solution must have come from $4e^{2x}-25=2x+43$ with the modulus having been taken correctly.

Allow $e^{2x}=\frac{1}{4}(2x+43+25)$ going to $x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ without explanation

Allow $\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ appearing as $\frac{1}{2}\log_e\left(\frac{1}{2}x+17\right)$ but not as $\frac{1}{2}\log\left(\frac{1}{2}x+17\right)$

If a candidate attempts the solution backwards they must proceed from

$x=\frac{1}{2}\ln\left(\frac{1}{2}x+17\right) \Rightarrow e^{2x}=\frac{1}{2}x+17 \Rightarrow 4e^{2x}-25=2x+43$ for the M1

For the A1 it must be tied up with a minimal statement that this is $g(x)=2x+43$

(c)

M1 Subs 1.4 into the iterative formula in an attempt to find x_1

Score for $x_1=\frac{1}{2}\ln\left(\frac{1}{2}\times 1.4+17\right)$ $x_1=\frac{1}{2}\ln(17.7)$ or awrt 1.44

A1 awrt $x_1=1.4368$, $x_2=1.4373$ Subscripts are not important, mark in the order given please.

(d)

M1 For a suitable interval. Accept 1.4365 and 1.4375 (or any two values of a smaller range spanning the root=1.4373) Continued iteration is M0

A1 Substitutes both values into **a suitable function**, which must be defined or implied by their working calculates both values correctly to 1 sig fig (rounded or truncated)

Suitable functions could be $\pm\left(4e^{2x}-2x-68\right)$, $\pm\left(x-\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)\right)$, $\pm\left(2x-\ln\left(\frac{1}{2}x+17\right)\right)$.

Using $4e^{2x}-2x-68$ f (1.4365) = -0.1, f (1.4375) = +0.02 or +0.03

Using $2e^{2x}-x-34$ f (1.4365) = -0.05/-0.06, f (1.4375) = +0.01

Using $x-\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$ f (1.4365) =-0.0007 or -0.0008, f (1.4375) =+ 0.0001 or +0.0002

Using $2x-\ln\left(\frac{1}{2}x+17\right)$ f (1.4365) = -0.001 or -0.002, f (1.4375) =+0.0003 or +0.0004

and states a reason (eg change of sign)

and a gives a minimal conclusion (eg root or tick)

It is valid to compare the two functions. Eg $\begin{array}{l} g(1.4365)=45.7(6)<2\times 1.4365+43=45.8(73) \\ g(1.4375)=45.90>2\times 1.4375+43=45.8(75) \end{array}$

but the conclusion should be $g(x)=2x+43$ in between, hence root .

Similarly candidates can compare the functions x and $\frac{1}{2}\ln\left(\frac{1}{2}x+17\right)$

Question 10

Question Number	Scheme	Marks
(a)	At P $x=-2 \Rightarrow y=3$	B1
	$\frac{dy}{dx}=\frac{4}{2x+5}-\frac{3}{2}$	M1, A1
	$\left.\frac{dy}{dx}\right\|_{x=-2}=\frac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\frac{2}{5}(x-(-2))$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\frac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\frac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)
(c)	Substitutes $x_1=2 \Rightarrow x_2=\frac{20}{11}\ln 9-2$	M1
	Awrt $x_2=1.9950$ and $x_3=1.9929$.	A1
		(2)
		(10 marks)

(a)

B1 $y=3$ at point P. This may be seen embedded within their equation which may be a tangent

M1 Differentiates $\ln(2x+5) \to \frac{A}{2x+5}$ or equivalent. You may see $\ln(2x+5)^2 \to \frac{A(2x+5)}{(2x+5)^2}$

A1 $\frac{dy}{dx} = \frac{4}{2x+5} - \frac{3}{2}$ oe. It need not be simplified.

M1 For using a correct method of finding the equation of the normal using their numerical value of $-\left.\frac{dx}{dy}\right|_{x=-2}$ as the gradient. Allow for $(y-'3') = -\left.\frac{dx}{dy}\right|_{x=-2}(x--2)$, oe.

At least one bracket must be correct for their $(-2,3)$

If the form $y=mx+c$ is used it is scored for proceeding as far as $c=$ ..

A1 $\pm k(5y+2x=11)$ It must be in the form $ax+by=c$ as stated in the question

Score this mark once it is seen. Do not withhold it if they proceed to another form, $y=mx+c$ for example

If a candidate uses a graphical calculator to find the gradient they can score a maximum of B1 M0 A0 M1 A1

(b)

M1 For combining 'their' **linear** $5y+2x=11$ with $y=2\ln(2x+5)-\frac{3x}{2}$ to form equation in just x, condoning slips on the rearrangement of their $5y+2x=11$. Eg $2\ln(2x+5)-\frac{3x}{2}=\frac{11\pm 2x}{5}$ is OK

dM1 Collects the two terms in x and proceeds to $ax=b\ln(2x+5)+c$ Allow numerical slips

A1* This is a given answer. All aspects must be correct including bracketing

(c)

M1 Score for substituting $x_1=2 \Rightarrow x_2=\frac{20}{11}\ln(2\times 2+5)-2$ or exact equivalent

This may implied by $x_2=1.99...$

A1 Both values correct. Allow awrt $x_2=1.9950$ and $x_3=1.9929$ but condone $x_2=1.995$

Ignore subscripts. Mark on the first and second values given.