Parametric Differentiation 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question Number	Scheme	Marks
	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}$	B1$\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0$, $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2}$, $x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}$, $y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi$, $x = 4\sin\left(\frac{7\pi}{6}\right) = -2$, $y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}$, $x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}$, $y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] 8
(a)	B1: Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified. B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. B1: Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. Note: This is a follow through mark. *Alternative differentiation in part (a)* $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$	

Question 2

Question Number	Scheme	Marks
	(a) $\frac{dx}{dt} = 2\sqrt{3}\cos 2t$	B1
	$\frac{dy}{dt} = -8\cos t\sin t$	M1 A1
	$\frac{dy}{dx} = \frac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\frac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\frac{dy}{dx} = -\frac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\frac{2}{3}\right)$	A1 (5)
	(b) When $t = \frac{\pi}{3}$ $\qquad x = \frac{3}{2},\ y = 1$ $\qquad$ can be implied	B1
	$m = -\frac{2}{3}\sqrt{3}\tan\left(\frac{2\pi}{3}\right) \quad (=2)$	M1
	$y - 1 = 2\left(x - \frac{3}{2}\right)$	M1
	$y = 2x - 2$	A1 (4)
	(c) $x = \sqrt{3}\sin 2t = \sqrt{3}\times 2\sin t\cos t$	M1
	$x^2 = 12\sin^2 t\cos^2 t = 12\left(1-\cos^2 t\right)\cos^2 t$	
	$x^2 = 12\left(1 - \frac{y}{4}\right)\frac{y}{4}$ $\qquad$ or equivalent	M1 A1 (3)
		[12]
	Alternative to (c)	
	$y = 2\cos 2t + 2$	M1
	$\sin^2 2t + \cos^2 2t = 1$	
	$\frac{x^2}{3} + \frac{(y-2)^2}{4} = 1$	M1 A1 (3)

Question 3

Question Number	Scheme		Marks
	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t \text{ or } \frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t}\left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
		Correct value for $\frac{dy}{dx}$ of 1	A1 cao cso
			[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$		M1
	$= 2\sin^2 t$		
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 cso isw
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			[3]
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$	See notes	B1 B1
			[2]
			9

Notes for Question

(a)

B1: At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.

B1: Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.

M1: Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$.

This mark may be implied by their final answer.

Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied).

A1: For an answer of 1 *by correct solution only.*

Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.

Note: Applying $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$.

Special Case: Award SC: B0B0M1A1 for $\frac{dx}{dt} = -2\cos t$, $\frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$

which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$

Note: It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

Notes for Question Continued		
(b)	**M1:** Uses the **correct** double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1. **A1:** Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form** $y = \mathrm{f}(x)$. For example: $y = \frac{2x^2}{4}$ or $y = 2\left(\frac{x}{2}\right)^2$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ or $y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$ and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation. **IMPORTANT: Please check working as this result can be fluked from an incorrect method.** **Award A0** if there is a $+c$ added to their answer. **B1:** Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.	
(c)	**Note: The values of 0 and/or 2 need to be evaluated in this part** **B1:** Achieves an inclusive upper or lower limit, using acceptable notation. Eg: $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ **B1:** $0 \leqslant \mathrm{f}(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant \mathrm{f} \leqslant 2$ **Special Case: SC: B1B0** for either $0 < \mathrm{f}(x) < 2$ or $0 < \mathrm{f} < 2$ or $0 < y < 2$ or $(0, 2)$ **Special Case: SC: B1B0** for $0 \leqslant x \leqslant 2$. **IMPORTANT: Note that:** Therefore candidates can use either y or f in place of $\mathrm{f}(x)$ **Examples:** $0 \leqslant x \leqslant 2$ is SC: B1B0 — $0 < x < 2$ is B0B0 $x \geqslant 0$ is B0B0 — $x \leqslant 2$ is B0B0 $\mathrm{f}(x) > 0$ is B0B0 — $\mathrm{f}(x) < 2$ is B0B0 $x > 0$ is B0B0 — $x < 2$ is B0B0 $0 \geqslant \mathrm{f}(x) \geqslant 2$ is B0B0 — $0 < \mathrm{f}(x) \leqslant 2$ is B1B0 $0 \leqslant \mathrm{f}(x) < 2$ is B1B0. — $\mathrm{f}(x) \geqslant 0$ is B1B0 $\mathrm{f}(x) \leqslant 2$ is B1B0 — $\mathrm{f}(x) \geqslant 0$ and $\mathrm{f}(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$ $2 \leqslant \mathrm{f}(x) \leqslant 2$ is B0B0 — $\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ is B1B0. $\lvert\mathrm{f}(x)\rvert \leqslant 2$ is B1B0 — $\lvert\mathrm{f}(x)\rvert \geqslant 2$ is B0B0 $1 \leqslant \mathrm{f}(x) \leqslant 2$ is B1B0 — $1 < \mathrm{f}(x) < 2$ is B0B0 $0 \leqslant \mathrm{f}(x) \leqslant 4$ is B1B0 — $0 < \mathrm{f}(x) < 4$ is B0B0 $0 \leqslant$ Range $\leqslant 2$ is B1B0 — Range is in between 0 and 2 is B1B0 $0 <$ Range < 2 is B0B0. — Range $\geqslant 0$ is B1B0 Range $\leqslant 2$ is B1B0 — Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0. $[0, 2]$ is B1B1 — $(0, 2)$ is SC B1B0	
Aliter (a) Way 2	$\frac{dx}{dt} = 2\cos t$, $\frac{dy}{dt} = 2\sin 2t$, At $t = \frac{\pi}{6}$, $\frac{dx}{dt} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$, $\frac{dy}{dt} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$ Hence $\frac{dy}{dx} = 1$	So B1, B1. So implied M1, A1.

Notes for Question Continued

Aliter (a) Way 3	$y = \frac{1}{2}x^2 \Rightarrow \frac{dy}{dx} = x$	Correct differentiation of their Cartesian equation. B1ft
		Finds $\frac{dy}{dx} = x$, using the correct Cartesian equation only. B1
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = 2\sin\left(\frac{\pi}{6}\right)$	Finds the value of "x" when $t = \frac{\pi}{6}$ and substitutes this into their $\frac{dy}{dx}$ M1
	$= 1$	Correct value for $\frac{dy}{dx}$ of 1 A1
Aliter (b) Way 2	$y = 1 - \cos 2t = 1 - (2\cos^2 t - 1)$	M1
	$y = 2 - 2\cos^2 t \Rightarrow \cos^2 t = \frac{2-y}{2} \Rightarrow 1 - \sin^2 t = \frac{2-y}{2}$	
	$1 - \left(\frac{x}{2}\right)^2 = \frac{2-y}{2}$	(Must be in the form $y = f(x)$).
	$y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	A1
Aliter (b) Way 3	$x = 2\sin t \Rightarrow t = \sin^{-1}\left(\frac{x}{2}\right)$	
	So, $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	Rearranges to make t the subject and substitutes the result into y. M1
		$y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ A1 oe
Aliter (b) Way 4	$y = 1 - \cos 2t \Rightarrow \cos 2t = 1 - y \Rightarrow t = \frac{1}{2}\cos^{-1}(1-y)$	
	So, $x = \pm 2\sin\left(\frac{1}{2}\cos^{-1}(1-y)\right)$	Rearranges to make t the subject and substitutes the result into y. M1
	So, $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	$y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$ A1 oe
Aliter (b) Way 5	$\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2 + c$	$\frac{dy}{dx} = x \Rightarrow y = \frac{1}{2}x^2 + c$ M1
	Eg: when eg: $t = 0$ (nb: $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$), $x = 0, y = 1 - 1 = 0 \Rightarrow c = 0 \Rightarrow y = \frac{1}{2}x^2$	Full method of finding $y = \frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$ A1
	Note: $\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2$, with no attempt to find c is M1A0.	

Question 4

Question Number	Scheme		Marks
	$x = 27\sec^3 t, \quad y = 3\tan t, \quad 0 \leqslant t \leqslant \frac{\pi}{3}$		
(a)	$\frac{dx}{dt} = 81\sec^2 t\sec t\tan t, \quad \frac{dy}{dt} = 3\sec^2 t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	$\frac{dy}{dx} = \frac{3\sec^2 t}{81\sec^3 t\tan t}\left\{= \frac{1}{27\sec t\tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72}\left\{= \frac{3}{54} = \frac{1}{18}\right\}$	$\frac{4}{72}$	A1 cao cso
			[4]
(b)	$\left\{1 + \tan^2 t = \sec^2 t\right\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * cso
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			[3]

Question 5

Question Number	Scheme		Marks
	$x = 4\cos\left(t + \dfrac{\pi}{6}\right), \quad y = 2\sin t$		
(a)	**Main Scheme**		
	$x = 4\left(\cos t\cos\left(\dfrac{\pi}{6}\right) - \sin t\sin\left(\dfrac{\pi}{6}\right)\right)$	$\cos\left(t+\dfrac{\pi}{6}\right) \to \cos t\cos\left(\dfrac{\pi}{6}\right) \pm \sin t\sin\left(\dfrac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\dfrac{\pi}{6}\right) - \sin t\sin\left(\dfrac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\dfrac{\sqrt{3}}{2}\right)\cos t - \left(\dfrac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**Alternative Method 1**		
	$x = 4\left(\cos t\cos\left(\dfrac{\pi}{6}\right) - \sin t\sin\left(\dfrac{\pi}{6}\right)\right)$	$\cos\left(t+\dfrac{\pi}{6}\right) \to \cos t\cos\left(\dfrac{\pi}{6}\right) \pm \sin t\sin\left(\dfrac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\dfrac{\sqrt{3}}{2}\right)\cos t - \left(\dfrac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(b)	**Main Scheme**		
	$\left(\dfrac{x+y}{2\sqrt{3}}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \dfrac{(x+y)^2}{12} + \dfrac{y^2}{4} = 1$		
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
		$\{a = 3, b = 12\}$	[2]
(b)	**Alternative Method 1**		
	$(x+y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x+y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**Alternative Method 2**		
	$(x+y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x+y)^2 + 3y^2 = 12$		M1, A1
			[2]
			5

		Question Notes
(a)	M1	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1. **<u>Awarding the dM1 mark which is dependent on the first method mark</u>**
Main	dM1	Adds their expanded x (which is in terms of t) to $2\sin t$
	Note	Writing $x + y = ...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x+y\} = 4\cos\left(t+\frac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.
	A1	leading $(x+y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x+y)^2 + 3y^2 = 12$ from no working • $a = 3, b = 12$, but **<u>does not provide a correct proof</u>**.
	Note	Alternative method 2 is fine for M1 A1
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a = 3, b = 12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

Question 6

Question Number	Scheme		Marks
	$x = 3\tan\theta$, $y = 4\cos^2\theta$ or $y = 2 + 2\cos 2\theta$, $0 \leqslant \theta < \frac{\pi}{2}$. $\frac{dx}{d\theta} = 3\sec^2\theta$, $\frac{dy}{d\theta} = -8\cos\theta\sin\theta$ **or** $\frac{dy}{d\theta} = -4\sin 2\theta$		
	$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ $\left\{= -\frac{8}{3}\cos^3\theta\sin\theta = -\frac{4}{3}\sin 2\theta\cos^2\theta\right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$	M1 A1 oe
	At $P(3, 2)$, $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{8}{3}\cos^3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$ $\left\{= -\frac{2}{3}\right\}$ So, $m(\mathbf{N}) = \frac{3}{2}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1 M1
	Either **N:** $y - 2 = \frac{"3"}{2}(x - 3)$ **or** $2 = \left(\frac{"3"}{2}\right)(3) + c$	**see notes**	M1
	$\{$At Q, $y = 0$, so, $-2 = \frac{3}{2}(x-3)\}$ **giving** $\underline{x = \frac{5}{3}}$	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67	A1 **cso** **[6]**

Question 7

Question Number	Scheme		Marks
(a)	$x = t - 4\sin t, \quad y = 1 - 2\cos t, \quad -\frac{2\pi}{3} \leqslant t \leqslant \frac{2\pi}{3} \quad A(k, 1)$ lies on the curve, $k > 0$		
	$\{\text{When } y = 1,\}\ 1 = 1 - 2\cos t \Rightarrow t = -\frac{\pi}{2}, \frac{\pi}{2}$ $k \text{ (or } x) = \frac{\pi}{2} - 4\sin\left(\frac{\pi}{2}\right)$ or $x = -\frac{\pi}{2} - 4\sin\left(-\frac{\pi}{2}\right)$	Sets $y = 1$ to find t and uses their t to find x.	M1
	$\left\{\text{When } t = -\frac{\pi}{2}, k > 0,\right\}$ so $k = 4 - \frac{\pi}{2}$ or $\frac{8-\pi}{2}$	x or $k = 4 - \frac{\pi}{2}$	A1
			[2]
(b)	$\frac{dx}{dt} = 1 - 4\cos t, \quad \frac{dy}{dt} = 2\sin t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\quad \frac{dy}{dx} = \frac{2\sin t}{1 - 4\cos t}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes their t into their $\frac{dy}{dx}$.	M1;
	At $t = -\frac{\pi}{2}, \quad \frac{dy}{dx} = \frac{2\sin\left(-\frac{\pi}{2}\right)}{1 - 4\cos\left(-\frac{\pi}{2}\right)}\ ; = -2$	Correct value for $\frac{dy}{dx}$ of -2	A1 cao cso
			[4]
(c)	$\frac{2\sin t}{1 - 4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	gives $\quad 4\sin t - 4\cos t = -1$	See notes	A1
	So $\ 4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right); = -1$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right); = -1$	See notes	M1; A1
	$t = \sin^{-1}\left(\frac{-1}{4\sqrt{2}}\right) + \frac{\pi}{4}$ or $t = \cos^{-1}\left(\frac{1}{4\sqrt{2}}\right) - \frac{\pi}{4}$	See notes	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6] 12

	Question Notes	
(c)	NOTE	**VERY IMPORTANT NOTE FOR PART (c)** Candidates who state $t = 0.6077$ with no intermediate working from $4\sin t - 4\cos t = -1$ will get 2nd M0, 2nd A0, 3rd M0, 3rd A0. They will not express $4\sin t - 4\cos t$ as either $4\sqrt{2}\sin\left(t - \frac{\pi}{4}\right)$ or $-4\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$. OR use any acceptable alternative method to achieve $t = 0.6077$
	NOTE	Alternative methods for part (c) are given on the next page.

	Question	Alternative Methods for Part (c)	
(c)	**Alternative Method 1:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $\left(\frac{2\sin t}{1-4\cos t}\right)^2 = \frac{1}{4}$ or $(4\sin t)^2 = (4\cos t - 1)^2$ or $(4\sin t + 1)^2 = (4\cos t)^2$ etc.	Squaring to give a correct equation. This mark can be implied by a "squared" correct equation.	A1
		Note: You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	
	or $\pm a\sin^2 t \pm b\sin t \pm c = 0$ or eg. $\pm a\cos^2 t \pm b\cos t = \pm c$ where $a \neq 0$, $b \neq 0$ and $c \neq 0$.	Squares their equation, applies $\sin^2 t + \cos^2 t = 1$ and achieves a **three term quadratic equation** of the form $\pm a\cos^2 t \pm b\cos t \pm c = 0$	M1
	• **Either** $32\cos^2 t - 8\cos t - 15 = 0$ • **or** $32\sin^2 t + 8\sin t - 15 = 0$	For a correct **three term quadratic equation.**	A1
	• **Either** $\cos t = \frac{8 \pm \sqrt{1984}}{64} = \frac{1+\sqrt{31}}{8} \Rightarrow t = \cos^{-1}(...)$ • **or** $\sin t = \frac{-8 \pm \sqrt{1984}}{64} = \frac{-1 \pm \sqrt{31}}{8} \Rightarrow t = \sin^{-1}(...)$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]
(c)	**Alternative Method 2:**		
	$\frac{2\sin t}{1-4\cos t} = -\frac{1}{2}$	Sets their $\frac{dy}{dx} = -\frac{1}{2}$	M1
	eg. $(4\sin t - 4\cos t)^2 = (-1)^2$	Squaring to give a correct equation. This mark can be implied by a correct equation. **Note:** You can also give 1st A1 in this method for $4\sin t - 4\cos t = -1$ as in the main scheme.	A1
	So $16\sin^2 t - 32\sin t\cos t + 16\cos^2 t = 1$		
	leading to $16 - 16\sin 2t = 1$	Squares their equation, applies both $\sin^2 t + \cos^2 t = 1$ and $\sin 2t = 2\sin t\cos t$ and then achieves an equation of the form $\pm a \pm b\sin 2t = \pm c$	M1
		$16 - 16\sin 2t = 1$ or equivalent.	A1
	$\left\{\sin 2t = \frac{15}{16} \Rightarrow\right\}$ $t = \frac{\sin^{-1}(....)}{2}$	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t = ...$	dM1
	$t = 0.6076875626... = 0.6077$ (4 dp)	anything that rounds to 0.6077	A1
			[6]

	Question	Notes
(a)	M1	Sets $y=1$ to find t and uses their t to find x.
	Note	M1 can be implied by either x or $k = 4-\frac{\pi}{2}$ or 2.429... or $\frac{\pi}{2}-4$ or $-2.429...$
	A1	x or $k = 4-\frac{\pi}{2}$ or $\frac{8-\pi}{2}$
	Note	A decimal answer of 2.429... (without a correct **exact** answer) is A0.
	Note	Allow A1 for a candidate using $t=\frac{\pi}{2}$ to find $x=\frac{\pi}{2}-4$ **and** then stating that k must be $4-\frac{\pi}{2}$ o.e.
(b)	B1	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working.
	B1	Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working.
	M1	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute their t into their expression for $\frac{dy}{dx}$.
	Note	This mark may be implied by their final answer.
		i.e. $\frac{dy}{dx}=\frac{2\sin t}{1-4\cos t}$ followed by an answer of -2 (from $t=-\frac{\pi}{2}$) or 2 (from $t=\frac{\pi}{2}$)
	Note	**Applying** $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$.
	A1	Using $t=-\frac{\pi}{2}$ $\left(\text{and not } t=\frac{3\pi}{2}\right)$ to find a correct $\frac{dy}{dx}$ of -2 *by correct solution only.*
(c)	NOTE	If a candidate uses an incorrect $\frac{dy}{dx}$ expression in part (c) then the accuracy marks are not obtainable.
	1st M1	Sets their $\frac{dy}{dx}=-\frac{1}{2}$
	1st A1	Rearranges to give the correct equation with $\sin t$ and $\cos t$ **on the same side.**
		eg. $4\sin t-4\cos t=-1$ or $4\cos t-4\sin t=1$ or $\sin t-\cos t=-\frac{1}{4}$ or $\cos t-\sin t=\frac{1}{4}$
		or $4\sin t-4\cos t+1=0$ or $4\cos t-4\sin t-1=0$ or $\sin t-\cos t+\frac{1}{4}=0$ etc. are fine for A1.
	2nd M1	Rewrites $\pm\lambda\sin t\pm\mu\cos t$ in the form of either $R\cos(t\pm\alpha)$ or $R\sin(t\pm\alpha)$ where $R\neq 1$ or 0 and $\alpha\neq 0$
	2nd A1	Correct equation. Eg. $4\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-1$ or $-4\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=-1$
		or $\sqrt{2}\sin\left(t-\frac{\pi}{4}\right)=-\frac{1}{4}$ or $\sqrt{2}\cos\left(t+\frac{\pi}{4}\right)=\frac{1}{4}$, etc.
	Note	Unless recovered, give A0 for $4\sqrt{2}\sin(t-45^\circ)=-1$ or $-4\sqrt{2}\cos(t+45^\circ)=-1$, etc.
	3rd M1	**which is dependent on the 2nd M1 mark.** Uses correct algebraic processes to give $t=...$
	4th A1	anything that rounds to 0.6077
	Note	**Do not give** the final A1 mark in (c) if there any extra solutions given in the range $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.
	Note	You can give the final A1 mark in (c) if extra solutions are given outside of $-\frac{2\pi}{3}\leqslant t\leqslant\frac{2\pi}{3}$.

Question 8

Question Number	Scheme		Marks
	Note: You can mark parts (a) and (b) together.		
(a)	$x = 4t + 3,\ y = 4t + 8 + \frac{5}{2t}$		
	$\frac{dx}{dt} = 4,\ \frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	Both $\frac{dx}{dt} = 4$ or $\frac{dt}{dx} = \frac{1}{4}$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	B1
	So, $\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \frac{5}{8}t^{-2} = 1 - \frac{5}{8t^2}\right\}$	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$	M1 o.e.
	$\{\text{When } t = 2,\}\ \frac{dy}{dx} = \frac{27}{32}$	$\frac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 2: Cartesian Method		
	$\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$	$\frac{dy}{dx} = 1 - \frac{10}{(x-3)^2}$, simplified or un-simplified.	B1
		$\frac{dy}{dx} = \pm\lambda \pm \frac{\mu}{(x-3)^2},\ \lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2, x = 11\}\ \frac{dy}{dx} = \frac{27}{32}$	$\frac{27}{32}$ or 0.84375 cao	A1
			[3]
	Way 3: Cartesian Method		
	$\frac{dy}{dx} = \frac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\frac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \frac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\frac{dy}{dx} = \frac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) = \text{their } "x^2 + ax + b"$, $g(x) = x - 3$	M1
	$\{\text{When } t = 2, x = 11\}\ \frac{dy}{dx} = \frac{27}{32}$	$\frac{27}{32}$ or 0.84375 cao	A1
			[3]
(b)	$\left\{t = \frac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \frac{10}{x-3}$		
	$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ *or* $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ *or* $y = \frac{(x+5)(x-3) + 10}{x-3}$ *or* $y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3},\ \{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \frac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 cso
			[3] 6

Question Number	Scheme	Marks
(b)	**<u>Alternative Method 1 of Equating Coefficients</u>**	
	$y = \dfrac{x^2+ax+b}{x-3} \Rightarrow y(x-3) = x^2+ax+b$	
	$y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2+32t+10$	
	$x^2+ax+b = (4t+3)^2 + a(4t+3) + b$	
	$(4t+3)^2 + a(4t+3) + b = 16t^2+32t+10$ — Correct method of obtaining an equation in only t, a and b	M1
	t: $24+4a=32 \Rightarrow a=2$ constant: $9+3a+b=10 \Rightarrow b=-5$ — Equates their coefficients in t and finds both $a = \ldots$ and $b = \ldots$	dM1
	$a=2$ and $b=-5$	A1
		[3]
(b)	**<u>Alternative Method 2 of Equating Coefficients</u>**	
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$ — Eliminates t to achieve an equation in only x and y	M1
	$y = x-3+8+\dfrac{10}{x-3} \Rightarrow y = x+5+\dfrac{10}{(x-3)}$ $\underline{y(x-3)} = (x+5)(x-3)+10 \Rightarrow x^2+ax+b = \underline{(x+5)(x-3)+10}$	dM1
	$\Rightarrow y = \dfrac{x^2+2x-5}{x-3}$ or equating coefficients to give $a=2$ and $b=-5$ — Correct algebra leading to $y = \dfrac{x^2+2x-5}{x-3}$ or $a=2$ and $b=-5$	A1 cso
		[3]

		Question Notes
(a)	B1	$\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$ or $\frac{dy}{dt} = \frac{8t^2 - 5}{2t^2}$ or $\frac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\frac{dy}{dt}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\frac{dy}{dt}$ divided by a candidate's $\frac{dx}{dt}$ or $\frac{dy}{dt}$ multiplied by a candidate's $\frac{dt}{dx}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\frac{dy}{dt}$ and their $\frac{dx}{dt}$ and then dividing their values the correct way round.
	A1	$\frac{27}{32}$ or 0.84375 cao
(b)	M1	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that k can be 1)** • Combining all three parts of their $\underline{x-3} + \overline{\overline{8}} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $\underline{x+5} + \underline{\left(\frac{10}{x-3}\right)}$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right) + 8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = \underline{x-3} + \overline{\overline{8}} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ or their $y = \underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x - 3 + 8 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 8 + 10}{x-3}$. Should be $\ldots + 8(x-3) + \ldots$ **dM0** for $y = x - 3 + \frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3) + 10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x(x-3) + 5 + 10}{x-3}$. Should be $\ldots + 5(x-3) + \ldots$ **dM0** for $y = x + 5 + \frac{10}{x-3} \rightarrow y(x-3) = x(x-3) + 5(x-3) + 10(x-3)$. Should be just 10.
	Note	$y = x + 5 + \frac{10}{x-3} \rightarrow y = \frac{x^2 + 2x - 5}{x-3}$ with no intermediate working is dM1A1.

Question 9

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \qquad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t} \quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t or their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** *Some evidence* of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y = \text{awrt } 8.7$	M1
	Coordinates are $(4, 5\sqrt{3})$	$(4, 5\sqrt{3})$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
			[2]
			6

		Question Notes
(a)	**1st A1**	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin(2(45))$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4,\ y = 5\sqrt{3}$ followed by $(5\sqrt{3}, 4)$ is A0.

Question Number	Scheme	Notes	Marks
	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \ \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{matrix} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{matrix}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right) \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** *Some evidence* of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]

Question 10

Question Number	Scheme	Notes	Marks
	$x = 3t - 4,\ y = 5 - \dfrac{6}{t},\quad t > 0$		
(a)	$\dfrac{dx}{dt} = 3,\ \dfrac{dy}{dt} = 6t^{-2}$		
	$\dfrac{dy}{dx} = \dfrac{6t^{-2}}{3}\ \left\{= \dfrac{6}{3t^2} = 2t^{-2} = \dfrac{2}{t^2}\right\}$	their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$ to give $\dfrac{dy}{dx}$ in terms of t or their $\dfrac{dy}{dt}$ multiplied by their $\dfrac{dt}{dx}$ to give $\dfrac{dy}{dx}$ in terms of t	M1
		$\dfrac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1** if **both** $\dfrac{dx}{dt}$ **and** $\dfrac{dy}{dt}$ are stated **correctly** and **explicitly**.		[2]
	Note: You can recover the work for part (a) in part (b).		
(a) **Way 2**	$y = 5 - \dfrac{18}{x+4} \Rightarrow \dfrac{dy}{dx} = \dfrac{18}{(x+4)^2} = \dfrac{18}{(3t)^2}$	Writes $\dfrac{dy}{dx}$ in the form $\dfrac{\pm\lambda}{(x+4)^2}$, ***and*** writes $\dfrac{dy}{dx}$ as a function of t.	M1
		Correct un-simplified or simplified answer, in terms of t. **See note.**	A1 isw
			[2]
(b)	$\left\{t = \dfrac{1}{2} \Rightarrow\right\} P\left(-\dfrac{5}{2}, -7\right)$	$x = -\dfrac{5}{2},\ y = -7$ **or** $P\left(-\dfrac{5}{2}, -7\right)$ seen or implied.	B1
	$\dfrac{dy}{dx} = \dfrac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_T)x + "c"$	**Some** attempt to substitute $t = 0.5$ into their $\dfrac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_P) = (\text{their } m_T)(x - \text{their } x_P)$ **or** finds c from $(\text{their } y_P) = (\text{their } m_T)(\text{their } x_P) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		[3]

(c) **Way 1**	$\left\{t = \dfrac{x+4}{3} \Rightarrow\right\} y = 5 - \dfrac{6}{\left(\frac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow y = 5 - \dfrac{18}{x+4} \Rightarrow y = \dfrac{5(x+4) - 18}{x+4}$		
	So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
(c) **Way 2**	$\left\{t = \dfrac{6}{5-y} \Rightarrow\right\} x = \dfrac{18}{5-y} - 4$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	$\Rightarrow (x+4)(5-y) = 18 \Rightarrow 5x - xy + 20 - 4y = 18$		
	$\{\Rightarrow 5x + 2 = y(x+4)\}$ So, $y = \dfrac{5x+2}{x+4},\ \{x > -4\}$	$y = \dfrac{5x+2}{x+4}$ (or implied equation)	A1 cso
			[3]
	Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)		8

Question Number	Scheme	Notes	Marks
(c) Way 3	$y = \frac{3at-4a+b}{3t-4+4} = \frac{3at}{3t} - \frac{4a-b}{3t} = a - \frac{4a-b}{3t} \Rightarrow a = 5$	A full method leading to the value of a being found	M1
		$y = a - \frac{4a-b}{3t}$ and $a = 5$	A1
	$\frac{4a-b}{3} = 6 \Rightarrow b = 4(5) - 6(3) = 2$	**Both** $a = 5$ **and** $b = 2$	A1
			[3]

		Question Notes
(a)	**Note**	Condone $\frac{dy}{dx} = \frac{\left(\frac{6}{t^2}\right)}{3}$ for A1
	Note	You can ignore subsequent working following on from a correct expression for $\frac{dy}{dx}$ in terms of t.
(b)	**Note**	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{dy}{dx}}$ or $\frac{1}{\text{their } \frac{dy}{dx}}$ or $-\left(\text{their } \frac{dy}{dx}\right)$) is M0.
	Note	**Final A1:** A correct solution is required from a correct $\frac{dy}{dx}$.
	Note	**Final A1:** You can ignore subsequent working following on from a correct solution.
(c)	**Note**	**1st M1:** A full attempt to eliminate t is defined as either • rearranging one of the parametric equations to make t the subject and substituting for t in the other parametric equation (only the RHS of the equation required for M mark) • rearranging both parametric equations to make t the subject and putting the results equal to each other.
	Note	Award M1A1 for $\frac{6}{5-y} = \frac{x+4}{3}$ or equivalent.