

Parametric Equations 2 - Edexcel Past Exam Questions **MARK SCHEME**

## Question 1

Question Number	Scheme	Marks
	$x = \sqrt{3} \sin 2t = \sqrt{3} \times 2 \sin t \cos t$ $x^2 = 12 \sin^2 t \cos^2 t = 12(1 - \cos^2 t) \cos^2 t$ $x^2 = 12 \left(1 - \frac{y}{4}\right) \frac{y}{4}$ <p style="text-align: center;"><i>or equivalent</i></p>	<input type="checkbox"/> M1 <input type="checkbox"/> M1 A1 (3) <b>[12]</b>
	<i>Alternative to (c)</i> $y = 2 \cos 2t + 2$ $\sin^2 2t + \cos^2 2t = 1$ $\frac{x^2}{3} + \frac{(y-2)^2}{4} = 1$	<input type="checkbox"/> M1 <input type="checkbox"/> M1 A1 (3)

**Question 2**

Question Number	Scheme	Marks
(a)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$ So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ Either $k = 2$ or $-2 \leq x \leq 2$	M1  A1 cso isw  B1
(b)	Range: $0 \leq f(x) \leq 2$ or $0 \leq y \leq 2$ or $0 \leq f \leq 2$	See notes [3] B1 B1 [2]

**Question 3**

Question Number	Scheme	Marks
	$\{1 + \tan^2 t = \sec^2 t\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\frac{x}{27}}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$ $\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} *$ $a = 27 \text{ and } b = 216 \text{ or } 27 \leq x \leq 216$	M1 A1 * cso B1 [3]

**Question 4**

Question Number	Scheme	Marks
	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$	
(a)	<p><b>Main Scheme</b></p> $x = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right)$ $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ <p>So, <math>\{x + y\} = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right) + 2\sin t</math></p> $= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$ $= 2\sqrt{3}\cos t *$	<span style="float: right;">M1 oe</span> <span style="float: right;">dM1</span> <span style="float: right;">Correct proof A1 * [3]</span>
(a)	<p><b>Alternative Method 1</b></p> $x = 4\left(\cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right)\right)$ $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ $= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$ <p>So, <math>x = 2\sqrt{3}\cos t - y</math></p> <p><math>x + y = 2\sqrt{3}\cos t *</math></p>	<span style="float: right;">Forms an equation in x, y and t. dM1</span> <span style="float: right;">Correct proof A1 * [3]</span>
(b)	<p><b>Main Scheme</b></p> $\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ $\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$ $\Rightarrow (x+y)^2 + 3y^2 = 12$	<span style="float: right;">Applies <math>\cos^2 t + \sin^2 t = 1</math> to achieve an equation containing <b>only</b> x's and y's. M1</span> <span style="float: right;">(x+y)<sup>2</sup> + 3y<sup>2</sup> = 12 A1 {a = 3, b = 12} [2]</span>
(b)	<p><b>Alternative Method 1</b></p> $(x+y)^2 = 12\cos^2 t = 12(1-\sin^2 t) = 12 - 12\sin^2 t$ <p>So, <math>(x+y)^2 = 12 - 3y^2</math></p> $\Rightarrow (x+y)^2 + 3y^2 = 12$	<span style="float: right;">Applies <math>\cos^2 t + \sin^2 t = 1</math> to achieve an equation containing <b>only</b> x's and y's. M1</span> <span style="float: right;">(x+y)<sup>2</sup> + 3y<sup>2</sup> = 12 A1 [2]</span>
(b)	<p><b>Alternative Method 2</b></p> $(x+y)^2 = 12\cos^2 t$ <p>As <math>12\cos^2 t + 12\sin^2 t = 12</math></p> <p>then <math>(x+y)^2 + 3y^2 = 12</math></p>	<span style="float: right;">M1, A1 [2]</span>
		5

Question Notes		
(a)	M1 Note	$\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \left(\frac{\sqrt{3}}{2}\right) \cos t \pm \left(\frac{1}{2}\right) \sin t$ <b>If a candidate states</b> $\cos(A + B) = \cos A \cos B \pm \sin A \sin B$ , but there is an error <i>in its application</i> then give M1. <u>Awarding the dM1 mark which is dependent on the first method mark</u>
Main	dM1 Note	Adds their expanded $x$ (which is in terms of $t$ ) to $2 \sin t$ Writing $x + y = \dots$ is not needed in the Main Scheme method.
Alt 1	dM1	Forms an equation in $x$ , $y$ and $t$ .
(b)	A1* Note M1 A1 SC Note Note Note	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors. $\{x + y\} = 4 \cos\left(t + \frac{\pi}{6}\right) + 2 \sin t$ , by itself is M0M0A0. Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing <b>only</b> $x$ 's and $y$ 's. leading $(x + y)^2 + 3y^2 = 12$ Award Special Case B1B0 for a candidate who writes down either <ul style="list-style-type: none"> <li>• <math>(x + y)^2 + 3y^2 = 12</math> from no working</li> <li>• <math>a = 3, b = 12</math>, but <u>does not provide a correct proof</u></li> </ul> Alternative method 2 is fine for M1 A1 Writing $(x + y)^2 = 12 \cos^2 t$ followed by $12 \cos^2 t + a(4 \sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0 Writing $(x + y)^2 = 12 \cos^2 t$ followed by $12 \cos^2 t + a(4 \sin^2 t) = b$ <ul style="list-style-type: none"> <li>• states <math>a = 3, b = 12</math></li> <li>• and refers to either <math>\cos^2 t + \sin^2 t = 1</math> or <math>12 \cos^2 t + 12 \sin^2 t = 12</math></li> <li>• and there is no incorrect working</li> </ul> would get M1A1

**Question 5**

Question Number	Scheme	Marks
	$\left\{ t = \frac{x-3}{4} \Rightarrow \right\} y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates $t$ to achieve an equation in only $x$ and $y$ M1
	$y = x - 3 + 8 + \frac{10}{x-3}$	
	$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3} \quad \text{or} \quad y(x-3) = (x-3)(x-3) + 8(x-3) + 10$	See notes dM1
	$\text{or } y = \frac{(x+5)(x-3) + 10}{x-3} \quad \text{or} \quad y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$	
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3}, \quad \{a=2 \text{ and } b=-5\}$	Correct algebra leading to $y = \frac{x^2 + 2x - 5}{x-3} \quad \text{or } a=2 \text{ and } b=-5$ A1 eso
		[3]
		6

Question Number	Scheme	Marks
	<u>Alternative Method 1 of Equating Coefficients</u> $y = \frac{x^2 + ax + b}{x-3} \Rightarrow y(x-3) = x^2 + ax + b$	
	$y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2 + 32t + 10$	
	$x^2 + ax + b = (4t+3)^2 + a(4t+3) + b$	
	$(4t+3)^2 + a(4t+3) + b = 16t^2 + 32t + 10$	Correct method of obtaining an equation in only $t$ , $a$ and $b$ M1
	$t: \quad 24 + 4a = 32 \Rightarrow a = 2$	Equates their coefficients in $t$ and finds both $a = \dots$ and $b = \dots$ dM1
	$\text{constant: } 9 + 3a + b = 10 \Rightarrow b = -5$	$a = 2$ and $b = -5$ A1
		[3]
	<u>Alternative Method 2 of Equating Coefficients</u> $\left\{ t = \frac{x-3}{4} \Rightarrow \right\} y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates $t$ to achieve an equation in only $x$ and $y$ M1
	$y = x - 3 + 8 + \frac{10}{x-3} \Rightarrow y = x + 5 + \frac{10}{(x-3)}$	
	$y(x-3) = (x+5)(x-3) + 10 \Rightarrow x^2 + ax + b = (x+5)(x-3) + 10$	dM1
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3} \quad \text{or equating coefficients to give } a=2 \text{ and } b=-5$	Correct algebra leading to $y = \frac{x^2 + 2x - 5}{x-3} \quad \text{or } a=2 \text{ and } b=-5$ A1 eso
		[3]

**Question 6**

Question Number	Scheme	Notes	Marks
	$\left\{ t = \frac{1}{2} \Rightarrow \right\} P\left(-\frac{5}{2}, -7\right)$	$x = -\frac{5}{2}, y = -7$ or $P\left(-\frac{5}{2}, -7\right)$ seen or implied.	B1
	$\frac{dy}{dx} = \frac{2}{\left(\frac{1}{2}\right)^2}$ and either <ul style="list-style-type: none"> <li>• <math>y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)</math></li> <li>• <math>"-7" = ("8")\left("-\frac{5}{2}"\right) + c</math> So, <math>y = (\text{their } m_r)x + "c"</math></li> </ul>	Some attempt to substitute $t = 0.5$ into their $\frac{dy}{dx}$ which contains $t$ in order to find $m_r$ and either applies $y - (\text{their } y_p) = (\text{their } m_r)(x - \text{their } x_p)$ or finds $c$ from $(\text{their } y_p) = (\text{their } m_r)(\text{their } x_p) + c$ and uses their numerical $c$ in $y = (\text{their } m_r)x + c$	M1
	<b>T:</b> $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	<b>Note:</b> their $x_p$ , their $y_p$ and their $m_r$ must be numerical values in order to award M1		[3]