Partial Fractions in Integration 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question Number	Scheme	Marks
	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $(1 = A)$	M1
	$x \to \frac{1}{3}$ $1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct	A1
	Coefficients of x^2 $0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1 (4)
	(b)(i) $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	
	(ii) $\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 (6)
		[10]

Question 2

Question Number	Scheme	Notes	Marks
	$\int \frac{3y-4}{y(3y+2)}\,dy,\ y>0$, (ii) $\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx,\ x=4\sin^2\theta$		
Way 1	$\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y}+\frac{B}{(3y+2)} \Rightarrow 3y-4 = A(3y+2)+By$	See notes	M1
	$y=0 \Rightarrow -4=2A \Rightarrow A=-2$	At least one of their $A=-2$ **or** their $B=9$	A1
	$y=-\frac{2}{3} \Rightarrow -6=-\frac{2}{3}B \Rightarrow B=9$	Both their $A=-2$ **and** their $B=9$	A1
	$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{-2}{y}+\frac{9}{(3y+2)}\,dy$	Integrates to give at least one of **either** $\frac{A}{y}\to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)}\to\pm\mu\ln(3y+2)$ $A\neq 0,\ B\neq 0$	M1
		At least one term correctly followed through from their A or from their B	A1 ft
	$= -2\ln y + 3\ln(3y+2)\ \{+c\}$	$-2\ln y + 3\ln(3y+2)$ **or** $-2\ln y + 3\ln(y+\frac{2}{3})$ with correct bracketing, simplified or un-simplified. Can apply isw.	A1 cao
			[6]

		Question Notes
	1st M1	Writing $\frac{3y-4}{y(3y+2)} \equiv \frac{A}{y}+\frac{B}{(3y+2)}$ and a complete method for finding the value of at least one of their A or their B.
	Note	M1A1 can be implied *for writing down* either $\frac{3y-4}{y(3y+2)} \equiv \frac{-2}{y}+\frac{\text{their } B}{(3y+2)}$ or $\frac{3y-4}{y(3y+2)} \equiv \frac{\text{their } A}{y}+\frac{9}{(3y+2)}$ with no working.
	Note	Correct bracketing is not necessary for the penultimate A1ft, but is required for the final A1 in (i)
	Note	**Give** 2nd M0 for $\frac{3y-4}{y(3y+2)}$ going directly to $\pm\alpha\ln(3y^2+2y)$
	Note	**…but allow** 2nd M1 for either $\frac{M(6y+2)}{3y^2+2y}\to\pm\alpha\ln(3y^2+2y)$ or $\frac{M(3y+1)}{3y^2+2y}\to\pm\alpha\ln(3y^2+2y)$

<table>
<tr><th></th><th colspan="2">Scheme</th><th>Notes</th><th>Marks</th></tr>
<tr><td rowspan="7">Way 2</td><td colspan="2">$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3y+6}{y(3y+2)}\,dy$</td><td></td><td></td></tr>
<tr><td colspan="2" rowspan="3">$\frac{3y+6}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 3y+6 = A(3y+2) + By$
$y = 0 \Rightarrow 6 = 2A \Rightarrow A = 3$
$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$</td><td>See notes</td><td>M1</td></tr>
<tr><td>At least one of their $A = 3$ or their $B = -6$</td><td>A1</td></tr>
<tr><td>Both their $A = 3$ and their $B = -6$</td><td>A1</td></tr>
<tr><td rowspan="3">$\int \frac{3y-4}{y(3y+2)}\,dy$
$= \int \frac{6y+2}{3y^2+2y}\,dy - \int \frac{3}{y}\,dy + \int \frac{6}{(3y+2)}\,dy$
$= \ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)\ \{+c\}$</td><td colspan="2">Integrates to give at least one of either $\frac{M(6y+2)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
or $\frac{A}{y} \to \pm\lambda\ln y$ or $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$
$M \neq 0, A \neq 0, B \neq 0$</td><td>M1</td></tr>
<tr><td colspan="2">At least one term correctly followed through</td><td>A1 ft</td></tr>
<tr><td colspan="2">$\ln(3y^2+2y) - 3\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified</td><td>A1 cao</td></tr>
<tr><td></td><td colspan="3"></td><td>[6]</td></tr>
<tr><td rowspan="8">Way 3</td><td colspan="2">$\int \frac{3y-4}{y(3y+2)}\,dy = \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{5}{y(3y+2)}\,dy$</td><td></td><td></td></tr>
<tr><td colspan="2" rowspan="3">$\frac{5}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 5 = A(3y+2) + By$
$y = 0 \Rightarrow 5 = 2A \Rightarrow A = \frac{5}{2}$
$y = -\frac{2}{3} \Rightarrow 5 = -\frac{2}{3}B \Rightarrow B = -\frac{15}{2}$</td><td>See notes</td><td>M1</td></tr>
<tr><td>At least one of their $A = \frac{5}{2}$ or their $B = -\frac{15}{2}$</td><td>A1</td></tr>
<tr><td>Both their $A = \frac{5}{2}$ and their $B = -\frac{15}{2}$</td><td>A1</td></tr>
<tr><td rowspan="3">$\int \frac{3y-4}{y(3y+2)}\,dy$
$= \int \frac{3y+1}{3y^2+2y}\,dy - \int \frac{\frac{5}{2}}{y}\,dy + \int \frac{\frac{15}{2}}{(3y+2)}\,dy$
$= \frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)\ \{+c\}$</td><td colspan="2">Integrates to give at least one of either $\frac{M(3y+1)}{3y^2+2y} \to \pm\alpha\ln(3y^2+2y)$
or $\frac{A}{y} \to \pm\lambda\ln y$ or $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$
$M \neq 0, A \neq 0, B \neq 0$</td><td>M1</td></tr>
<tr><td colspan="2">At least one term correctly followed through</td><td>A1 ft</td></tr>
<tr><td colspan="2">$\frac{1}{2}\ln(3y^2+2y) - \frac{5}{2}\ln y + \frac{5}{2}\ln(3y+2)$ with correct bracketing, simplified or un-simplified</td><td>A1 cao</td></tr>
<tr><td colspan="3"></td><td>[6]</td></tr>
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	Scheme	Notes	
Way 4	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y = \int \frac{3y}{y(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$= \int \frac{3}{(3y+2)}\,\mathrm{d}y - \int \frac{4}{y(3y+2)}\,\mathrm{d}y$		
	$\frac{4}{y(3y+2)} \equiv \frac{A}{y} + \frac{B}{(3y+2)} \Rightarrow 4 = A(3y+2) + By$	See notes	M1
	$y = 0 \Rightarrow 4 = 2A \Rightarrow A = 2$	At least one of their $A = 2$ **or** their $B = -6$	A1
	$y = -\frac{2}{3} \Rightarrow 4 = -\frac{2}{3}B \Rightarrow B = -6$	Both their $A = 2$ **and** their $B = -6$	A1
	$\int \frac{3y-4}{y(3y+2)}\,\mathrm{d}y$	Integrates to give at least one of **either** $\frac{C}{(3y+2)} \to \pm\alpha\ln(3y+2)$ **or** $\frac{A}{y} \to \pm\lambda\ln y$ **or** $\frac{B}{(3y+2)} \to \pm\mu\ln(3y+2)$, $A \neq 0,\ B \neq 0,\ C \neq 0$	M1
	$= \int \frac{3}{3y+2}\,\mathrm{d}y - \int \frac{2}{y}\,\mathrm{d}y + \int \frac{6}{(3y+2)}\,\mathrm{d}y$	At least one term correctly followed through	A1 ft
	$= \ln(3y+2) - 2\ln y + 2\ln(3y+2)\ \{+c\}$	$\ln(3y+2) - 2\ln y + 2\ln(3y+2)$ with correct bracketing, simplified or un-simplified	A1 cao
			[6]