Partial Fractions in Integration - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q3

Question Number	Scheme	Marks
	(a) $\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$	
	$5x+3 = A(x+2) + B(2x-3)$	
	Substituting $x=-2$ or $x=\frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B.	M1
	$A=3, B=1$	A1, A1 **(3)**
	If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.	
	(b) $\int \frac{5x+3}{(2x-3)(x+2)}\,dx = \frac{3}{2}\ln(2x-3) + \ln(x+2)$	M1 A1ft
	$\left[\ \dots\ \right]_2^6 = \frac{3}{2}\ln 9 + \ln 2$	M1 A1
	$= \ln 54$ cao	A1 **(5)** **[8]**

Question 2: June 07 Q4

Question Number	Scheme		Marks
(a) **Way 1**	A method of long division gives,		
	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \dfrac{4}{(2x+1)(2x-1)}$	$A = 2$	B1
	$\dfrac{4}{(2x+1)(2x-1)} \equiv \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	$4 \equiv B(2x-1) + C(2x+1)$ or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$	Forming any one of these two identities. Can be implied.	M1
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$	**See note below**	
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	either one of $B = -2$ or $C = 2$	A1
		both B and C correct	A1
			[4]
Aliter (a) **Way 2**	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	See below for the award of B1	***decide to award B1 here!! …*** ***…*** *for* $A = 2$	B1
	$2(4x^2+1) \equiv A(2x+1)(2x-1) + B(2x-1) + C(2x+1)$	Forming this identity. Can be implied.	M1
	Equate x^2, $8 = 4A \Rightarrow A = 2$		
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$	**See note below**	
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	either one of $B = -2$ or $C = 2$	A1
		both B and C correct	A1
			[4]
	If a candidate states one of either B or C correctly then the method mark M1 can be implied.		

(b)	$\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$		
	$= 2x - \frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)\ (+c)$	Either $p\ln(2x+1)$ or $q\ln(2x-1)$ or either $p\ln 2x+1$ or $q\ln 2x-1$	M1 *
		$A \to Ax$	B1 $\sqrt{}$
		$-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1)+\ln(2x-1)$ **See note below.**	A1 cso & aef
	$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \left[2x - \ln(2x+1) + \ln(2x-1)\right]_1^2$		
	$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$	Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay.)	depM1 *
	$= 2 + \ln 3 + \ln 3 - \ln 5$		
	$= 2 + \ln\left(\frac{3(3)}{5}\right)$	Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for ***their numerical*** expression.	M1
	$= 2 + \ln\left(\frac{9}{5}\right)$	$2 + \ln\left(\frac{9}{5}\right)$ Or $2 - \ln\left(\frac{5}{9}\right)$ **and** k stated as $\frac{9}{5}$.	A1 [6]
			10 marks

Some candidates may find rational values for B and C. They may combine the denominator of their B or C with (2x +1) or (2x – 1). Hence: Either $\frac{a}{b(2x-1)} \to k\ln(b(2x-1))$ or $\frac{a}{b(2x+1)} \to k\ln(b(2x+1))$ is okay for M1.

Candidates are not allowed to fluke $-\ln(2x+1)+\ln(2x-1)$ for A1. Hence **cso**. If they do fluke this, however, they can gain the final A1 mark for this part of the question.

To award this M1 mark, the candidate must use the appropriate law(s) of logarithms for their ln terms to give a **one single** logarithmic term. Any error ***in applying the laws of logarithms*** would then earn M0.

Note: This is not a dependent method mark.

Question 3: June 09 Q3

Question Number	Scheme	Marks
Q (a)	$f(x)=\frac{4-2x}{(2x+1)(x+1)(x+3)}=\frac{A}{2x+1}+\frac{B}{x+1}+\frac{C}{x+3}$	
	$4-2x=A(x+1)(x+3)+B(2x+1)(x+3)+C(2x+1)(x+1)$	M1
	A method for evaluating one constant	M1
	$x\to-\frac{1}{2},\ \ 5=A(\frac{1}{2})(\frac{5}{2})\ \Rightarrow\ A=4$ any one correct constant	A1
	$x\to-1,\ \ 6=B(-1)(2)\ \Rightarrow\ B=-3$	
	$x\to-3,\ \ 10=C(-5)(-2)\ \Rightarrow\ C=1$ all three constants correct	A1 (4)
(b)	(i) $\int\left(\frac{4}{2x+1}-\frac{3}{x+1}+\frac{1}{x+3}\right)dx$	
	$=\frac{4}{2}\ln(2x+1)-3\ln(x+1)+\ln(x+3)+C$ A1 two ln terms correct	M1 A1ft
	All three ln terms correct and "$+C$" ; ft constants	A1ft (3)
	(ii) $\left[2\ln(2x+1)-3\ln(x+1)+\ln(x+3)\right]_0^2$	
	$=(2\ln 5-3\ln 3+\ln 5)-(2\ln 1-3\ln 1+\ln 3)$	M1
	$=3\ln 5-4\ln 3$	
	$=\ln\left(\frac{5^3}{3^4}\right)$	M1
	$=\ln\left(\frac{125}{81}\right)$	A1 (3)
		[10]

Question 4: Jan 11 Q3

Question Number	Scheme	Marks
(a)	$\frac{5}{(x-1)(3x+2)} = \frac{A}{x-1} + \frac{B}{3x+2}$	
	$5 = A(3x+2) + B(x-1)$	
	$x \to 1$ $\quad 5 = 5A \Rightarrow A = 1$	M1 A1
	$x \to -\frac{2}{3}$ $\quad 5 = -\frac{5}{3}B \Rightarrow B = -3$	A1 (3)
(b)	$\int \frac{5}{(x-1)(3x+2)} \mathrm{d}x = \int \left(\frac{1}{x-1} - \frac{3}{3x+2} \right) \mathrm{d}x$	
	$= \ln(x-1) - \ln(3x+2) \quad (+C)$ ft constants	M1 A1ft A1ft
		(3)