Trapezium Rule 2 - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question Number	Scheme		Marks
(a)	0.73508		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2} \times \frac{\pi}{8}$; $\times \underline{\left[0 + 2\left(\text{their } 0.73508 + 1.17157 + 1.02280\right) + 0\right]}$		B1 <u>M1</u>
	$= \frac{\pi}{16} \times 5.8589... = 1.150392325... = 1.1504 \text{ (4 dp)}$	awrt 1.1504	A1 **[3]**
(c)	$\underline{\{u = 1 + \cos x\} \Rightarrow \frac{du}{dx} = -\sin x}$		<u>B1</u>
	$\left\{\int \frac{2\sin 2x}{(1+\cos x)} dx = \right\} \int \frac{2(2\sin x \cos x)}{(1+\cos x)} dx$	$\sin 2x = 2\sin x \cos x$	B1
	$= \int \frac{4(u-1)}{u}.(-1) du \quad \left\{= 4\int \frac{(1-u)}{u} du\right\}$		M1
	$= 4\int \left(\frac{1}{u} - 1\right) du = 4(\ln u - u) + c$		dM1
	$= 4\ln(1+\cos x) - 4(1+\cos x) + c \quad = 4\ln(1+\cos x) - 4\cos x + k$	**AG**	A1 **cso** **[5]**
(d)	$= \left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - \left[4\ln(1+\cos 0) - 4\cos 0\right]$	Applying limits $x = \frac{\pi}{2}$ and $x = 0$ either way round.	M1
	$= [4\ln 1 - 0] - [4\ln 2 - 4]$		
	$= 4 - 4\ln 2 \quad \{= 1.227411278...\}$	$\pm 4(1 - \ln 2)$ or $\pm(4 - 4\ln 2)$ or awrt ± 1.2, ***however*** found.	A1
	Error $= \lvert(4 - 4\ln 2) - 1.1504...\rvert$ $= 0.0770112776... = 0.077 \text{ (2sf)}$	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 **cso** **[3]**
			12

(a)	**B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2} \times \frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196 **M1:** <u>For structure of trapezium rule</u>$\left[\underline{.............}\right]$; (0 can be implied). **A1:** anything that rounds to 1.1504 <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly** Award B1M0A0 for $\frac{1}{2} \times \frac{\pi}{8} + 2(\text{their } 0.73508 + 1.17157 + 1.02280)$ (nb: answer of 6.0552). Award B1M0A0 for $\frac{1}{2} \times \frac{\pi}{8}$ $(0 + 0) + 2(\text{their } 0.73508 + 1.17157 + 1.02280)$ (nb: answer of 5.8589). <u>*Alternative method for part (b): Adding individual trapezia*</u> Area $\approx \frac{\pi}{8} \times \left[\frac{0 + 0.73508}{2} + \frac{0.73508 + 1.17157}{2} + \frac{1.17157 + 1.02280}{2} + \frac{1.02280 + 0}{2}\right] = 1.150392325...$ B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets. M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. A1: anything that rounds to 1.1504

Question 2

Question Number	Scheme	Marks
	(a)	
		x	1	2	3	4		M1
	y	$\ln 2$	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	$2\ln 8$	
		0.6931	1.9605	3.1034	4.1589	
	Area $=\frac{1}{2}\times 1(\ \dots)$	B1
	$\approx \dots\ \left(0.6931+2(1.9605+3.1034)+4.1589\right)$	M1
	$\approx \frac{1}{2}\times 14.97989\ \dots\ \approx 7.49$ 7.49 cao	A1 (4)
	(b) $\int x^{\frac{1}{2}}\ln 2x\,\mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}}\ln 2x - \int \frac{2}{3}x^{\frac{3}{2}}\times\frac{1}{x}\mathrm{d}x$	M1 A1
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x - \int \frac{2}{3}x^{\frac{1}{2}}\,\mathrm{d}x$	
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x - \frac{4}{9}x^{\frac{3}{2}}\quad (+C)$	M1 A1 (4)
	(c) $\left[\frac{2}{3}x^{\frac{3}{2}}\ln 2x - \frac{4}{9}x^{\frac{3}{2}}\right]_1^4 = \left(\frac{2}{3}4^{\frac{3}{2}}\ln 8 - \frac{4}{9}4^{\frac{3}{2}}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$	M1
	$=(16\ln 2 - \ \dots) - \ \dots$ Using or implying $\ln 2^n = n\ln 2$	M1
	$=\frac{46}{3}\ln 2 - \frac{28}{9}$	A1 (3)
		[11]

Question 3

Question Number	Scheme		Marks
(a)	1.0981		B1 **cao**
			[1]
(b)	Area $\approx \frac{1}{2} \times 1\,; \times \underline{\left[0.5 + 2\left(0.8284 + \text{their } 1.0981\right) + 1.3333\right]}$		B1; $\underline{\text{M1}}$
	$= \frac{1}{2} \times 5.6863 \;=\; 2.84315 \;=\; 2.843 \;\;(3 \text{ dp})$	2.843 or awrt 2.843	A1
			[3]
(c)	$\left\{u = 1 + \sqrt{x}\right\} \Rightarrow \underline{\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2}x^{-\frac{1}{2}}}$ or $\underline{\frac{\mathrm{d}x}{\mathrm{d}u} = 2(u-1)}$		$\underline{\text{B1}}$
	$\left\{\int \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = \right\} \int \frac{(u-1)^2}{u}.\,2(u-1)\;\mathrm{d}u$	$\int \frac{(u-1)^2}{u}\ldots\ldots$	M1
		$\int \frac{(u-1)^2}{u}.\,2(u-1)$	A1
	$= 2\int \frac{(u-1)^3}{u}\,\mathrm{d}u \;=\{2\}\int \frac{(u^3 - 3u^2 + 3u - 1)}{u}\,\mathrm{d}u$	Expands to give a "four term" cubic in u. Eg: $\pm Au^3 \pm Bu^2 \pm Cu \pm D$	M1
	$= \{2\}\int\left(u^2 - 3u + 3 - \frac{1}{u}\right)\mathrm{d}u$	An attempt to divide at least three terms in ***their cubic*** by u. See notes.	M1
	$= \{2\}\left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	$\int \frac{(u-1)^3}{u} \rightarrow \left(\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \ln u\right)$	A1
	$\text{Area}(R) = \left[\frac{2u^3}{3} - 3u^2 + 6u - 2\ln u\right]_2^3$		
	$= \left(\frac{2(3)^3}{3} - 3(3)^2 + 6(3) - 2\ln 3\right) - \left(\frac{2(2)^3}{3} - 3(2)^2 + 6(2) - 2\ln 2\right)$	Applies limits of 3 and 2 in u or 4 and 1 in x and subtracts either way round.	M1
	$= \frac{11}{3} + 2\ln 2 - 2\ln 3$ or $\frac{11}{3} + 2\ln\left(\frac{2}{3}\right)$ or $\frac{11}{3} - \ln\left(\frac{9}{4}\right)$, etc	Correct exact answer or equivalent.	A1
			[8]
			12

Question 4

Question Number	Scheme		Marks
(a)	$6.248046798... = 6.248$ (3dp)	6.248 or awrt 6.248	B1 [1]
(b)	Area $\approx \frac{1}{2}\times 2 ;\times \underline{\left[3+2\left(7.107+7.218+\text{their } 6.248\right)+5.223\right]}$		B1; <u>M1</u>
	$= 49.369 = 49.37$ (2 dp)	49.37 or awrt 49.37	A1 [3]
(c)	$\left\{\int (4te^{-\frac{1}{3}t}+3)\,dt\right\} = -12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}\{dt\}$	$\pm Ate^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{dt\}$, $A \neq 0, B \neq 0$	M1
		See notes.	A1
	$+3t$	$3 \to 3t$	B1
	$= -12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}\{+3t\}$	$-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$	A1
	$\left[-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} + 3t\right]_0^8 =$		
	$= \left(-12(8)e^{-\frac{1}{3}(8)} - 36e^{-\frac{1}{3}(8)} + 3(8)\right) - \left(-12(0)e^{-\frac{1}{3}(0)} - 36e^{-\frac{1}{3}(0)} + 3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.	dM1
	$= \left(-96e^{-\frac{8}{3}} - 36e^{-\frac{8}{3}} + 24\right) - (0 - 36 + 0)$		
	$= 60 - 132e^{-\frac{8}{3}}$	$60 - 132e^{-\frac{8}{3}}$	A1 [6]
(d)	Difference $= \left\lvert 60 - 132e^{-\frac{8}{3}} - 49.37\right\rvert = 1.458184439... = 1.46$ (2 dp)	1.46 or awrt 1.46	B1 [1]
			11

Notes for Question

(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	**B1:** Outside brackets $\frac{1}{2}\times 2$ or 1
	M1: <u>For structure of trapezium rule</u>$[$ .... $]$. Allow one miscopy of their values.
	A1: 49.37 or anything that rounds to 49.37
	Note: It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37)
	<u>**Note:**</u> Working must be seen to demonstrate the use of the trapezium rule. <u>**Note**</u>: actual area is 50.828…
	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,**
	Award B1M0A0 for $1 + 3 + 2\left(7.107 + 7.218 + \text{their } 6.248\right) + 5.223$ (nb: answer of 50.369).

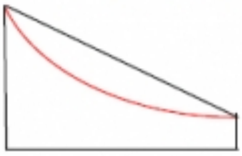
Notes for Question Continued	
(b) ctd	*Alternative method for part (b): Adding individual trapezia* Area $\approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right]=49.369$ **B1:** 2 and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 49.37
(c)	**M1:** For $4te^{-\frac{1}{3}t} \to \pm Ate^{-\frac{1}{3}t} \pm B\int e^{-\frac{1}{3}t}\{dt\}$, $A \neq 0, B \neq 0$ **A1:** For $te^{-\frac{1}{3}t} \to \left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ (some candidates lose the 4 and this is fine for the first A1 mark). or $4te^{-\frac{1}{3}t} \to 4\left(-3te^{-\frac{1}{3}t} - \int -3e^{-\frac{1}{3}t}\right)$ or $-12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}$ or $12\left(-te^{-\frac{1}{3}t} - \int -e^{-\frac{1}{3}t}\right)$ These results can be implied. They can be simplified or un-simplified. **B1:** $3 \to 3t$ or $3 \to 3x$ (bod) . **Note:** Award B0 for 3 integrating to $12t$ (implied), which is a common error when taking out a factor of 4. **Be careful** some candidates will factorise out 4 and have $4\left(....+\frac{3}{4}\right) \to 4\left(....+\frac{3}{4}t\right)$ which would then be fine for B1. **Note:** Allow B1 for $\int_0^8 3\,dt = 24$ **A1:** For correct integration of $4te^{-\frac{1}{3}t}$ to give $-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t}$ or $4\left(-3te^{-\frac{1}{3}t} - 9e^{-\frac{1}{3}t}\right)$ or equivalent. This can be simplified or un-simplified. **dM1:** Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t}$ or $\pm\lambda te^{-\frac{1}{3}t} \pm \mu e^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round. **Note:** Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dM1. So, just subtracting zero is M0. **A1:** An exact answer of $60 - 132e^{-\frac{8}{3}}$. A decimal answer of 50.82818444... without a correct answer is A0. **Note:** A decimal answer of 50.82818444... without a correct exact answer is A0. **Note:** If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0. **IMPORTANT:** that is fine for candidates to work in terms of x rather than t in part (c). **Note:** The "$3t$" is needed for B1 and the final A1 mark.
(d)	**B1:** 1.46 or awrt 1.46 or -1.46 or awrt -1.46. Candidates may give correct decimal answers of 1.458184439... or 1.459184439... **Note:** You can award this mark whether or not the candidate has answered part (c) correctly.

Question 5

Question Number	Scheme		Marks
	x: 1, 2, 3, 4; y: 1.42857, 0.90326, ***0.682116...***, 0.55556	$y = \dfrac{10}{2x+5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y = 0.68212$ (5 dp)	0.68212	B1 cao
			[1]
(b)	$\frac{1}{2}\times 1\times\left[1.42857+0.55556+2(0.90326+\text{their } 0.68212)\right]$	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$	B1 aef
		For structure of $[.............]$	M1
	$\{=\frac{1}{2}(5.15489)\} = 2.577445 = 2.5774$ (4 dp)	anything that rounds to 2.5774	A1
			[3]
(c)	• Overestimate and a reason such as • {top of} trapezia lie above the curve • a diagram which gives reference to the extra area • concave or convex • $\dfrac{d^2y}{dx^2} > 0$ (can be implied) • bends inwards • curves downwards		B1
			[1]
(d)	$\{u=\sqrt{x}\Rightarrow\}\ \dfrac{du}{dx}=\dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{dx}{du}=2u$		B1
	$\displaystyle\int\frac{10}{2u^2+5u}.\,2u\,du$	Either $\left\{\displaystyle\int\right\}\dfrac{\pm ku}{\alpha u^2\pm\beta u}\{du\}$ or $\left\{\displaystyle\int\right\}\dfrac{\pm k}{u(\alpha u^2\pm\beta u)}\{du\}$	M1
	$\left\{=\displaystyle\int\frac{20}{2u+5}\,du\right\} = \dfrac{20}{2}\ln(2u+5)$	$\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\dfrac{5}{2}\right)$, $\lambda\neq 0$ **with no other terms.**	M1
		$\dfrac{20}{2u+5}\to\dfrac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\dfrac{5}{2}\right)$	A1 cso
	$\left\{\left[\dfrac{20}{2}\ln(2u+5)\right]_1^2\right\} = 10\ln(2(2)+5)-10\ln(2(1)+5)$	Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9-10\ln 7$ or $10\ln\left(\dfrac{9}{7}\right)$ or $20\ln 3-10\ln 7$		A1 oe cso
			[6]
			11

Question Notes

(a)	B1	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	B1	Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$ or equivalent.
	M1	For structure of trapezium rule$[\ ....\ ]$
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 2.5774
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428…)

(b) contd	Note	Award B1M1A1 for $\frac{1}{2}(1.42857 + 0.55556) + (0.90326 + \text{their } 0.68212) = 2.577445$
		Bracketing mistake: **Unless the final answer implies that the calculation has been done correctly**
		award B1M0A0 for $\frac{1}{2}\times 1\ + 1.42857 + 2(0.90326 + \text{their } 0.68212) + 0.55556$ (nb: answer of 5.65489).
		award B1M0A0 for $\frac{1}{2}\times 1\ (1.42857 + 0.55556) + 2(0.90326 + \text{their } 0.68212)$ (nb: answer of 4.162825).
		Alternative method: Adding individual trapezia
		Area $\approx 1\times\left[\frac{1.42857+0.90326}{2} + \frac{0.90326+\text{"}0.68212\text{"}}{2} + \frac{\text{"}0.68212\text{"}+0.55556}{2}\right] = 2.577445$
	B1	**B1**: 1 and a divisor of 2 on all terms inside brackets.
	M1	**M1**: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.
	A1	**A1**: anything that rounds to 2.5774
(c)	B1	Overestimate **and** either trapezia lie above curve **or** a diagram that gives reference to the extra area
		eg. This diagram is sufficient. It must show the top of a trapezium lying above the curve. 
		or concave **or** convex **or** $\frac{d^2y}{dx^2} > 0$ (can be implied) **or** bends inwards **or** curves downwards.
	Note	Reason of "gradient is negative" by itself is B0.
(d)	B1	$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ or $du = \frac{1}{2\sqrt{x}}\,dx$ or $2\sqrt{x}\,du = dx$ or $dx = 2u\,du$ or $\frac{dx}{du} = 2u$ o.e.
	M1	Applying the substitution and achieving $\left\{\int\right\}\frac{\pm ku}{\alpha u^2 \pm \beta u}\{du\}$ or $\left\{\int\right\}\frac{\pm k}{u(\alpha u^2 \pm \beta u)}\{du\}$, $k, \alpha, \beta \neq 0$. Integral sign and du not required for this mark.
	M1	Cancelling u and integrates to achieve $\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right)$, $\lambda \neq 0$ **with no other terms.**
	A1	cso. Integrates $\frac{20}{2u+5}$ to give $\frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$, un-simplified or simplified.
	Note	BE CAREFUL! Candidates must be integrating $\frac{20}{2u+5}$ or equivalent.
		So $\int\frac{10}{2u+5}\,du = 10\ln(2u+5)$ WOULD BE A0 and final A0.
	M1	Applies limits of 2 and 1 in u or 4 and 1 in x in their (i.e. any) changed function and subtracts the correct way round.
	A1	Exact answers of either $10\ln 9 - 10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$ or $20\ln\left(\frac{3}{\sqrt{7}}\right)$ or $\ln\left(\frac{9^{10}}{7^{10}}\right)$ or equivalent. **Correct solution only.**
	Note	You can ignore subsequent working which follows from a correct answer.
	Note	A decimal answer of 2.513144283... (without a correct **exact** answer) is A0.

Question 6

Question Number	Scheme		Marks
(a)	Area $\approx \frac{1}{2}\times 0.5\,;\times\left[\underline{2+2(4.077+7.389+10.043)+0}\right]$		B1; M1
	$=\frac{1}{4}\times 45.018\ =\ 11.2545\ = 11.25\,(2\text{ dp})$	11.25	A1 cao
			[3]
(b)	Any one of • Increase the number of strips • Use more trapezia • Make h smaller • Increase the **number of** x and/or y values used • Shorter /smaller intervals for x • More values of y. • More intervals of x • Increase n		B1
			[1]
(c)	$\left\{\int (2-x)e^{2x}\,dx\right\},\ \left\{\begin{array}{l} u=2-x\ \Rightarrow \frac{du}{dx}=-1 \\ \frac{dv}{dx}=e^{2x}\ \Rightarrow\ v=\frac{1}{2}e^{2x}\end{array}\right\}$		
	$=\frac{1}{2}(2-x)e^{2x}-\int -\frac{1}{2}e^{2x}\,\{dx\}$	Either $(2-x)e^{2x}\to \pm\lambda(2-x)e^{2x}\pm\int \mu e^{2x}\,\{dx\}$ or $\pm xe^{2x}\to\pm\lambda xe^{2x}\pm\int\mu e^{2x}\,\{dx\}$	M1
		$(2-x)e^{2x}\to \frac{1}{2}(2-x)e^{2x}-\int -\frac{1}{2}e^{2x}\,\{dx\}$	A1
	$=\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}$	$\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}$	A1 oe
	Area $=\left\{\left[\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}\right]_0^2\right\}$		
	$=\left(0+\frac{1}{4}e^4\right)-\left(\frac{1}{2}(2)e^0+\frac{1}{4}e^0\right)$	Applies limits of 2 and 0 *to all terms* and subtracts the correct way round.	dM1
	$=\frac{1}{4}e^4-\frac{5}{4}$	$\frac{1}{4}e^4-\frac{5}{4}$ or $\frac{e^4-5}{4}$ **cao**	A1 oe
			[5] 9

Question Notes

(a)	B1	Outside brackets $\frac{1}{2}\times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$.
	M1	<u>For structure of trapezium rule</u>$[\ \underline{\ldots\ldots\ldots\ldots}\ ]$. Condone missing 0.
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	11.25 **cao**
	Note	Working must be seen to demonstrate the use of the trapezium rule. The actual area is 12.39953751…
	Note	Award B1M1A1 for $\frac{0.5}{2}(2+0)+\frac{1}{2}(4.077+7.389+10.043)=11.25$

(a) contd	**Bracketing mistake: Unless the final answer implies that the calculation has been done correctly.**	
	Award B1M0A0 for $\frac{1}{2}\times 0.5+2+2(4.077+7.389+10.043)+0$ (nb: answer of 45.268).	
	Alternative method for part (a): Adding individual trapezia	
	Area $\approx 0.5\times\left[\frac{2+4.077}{2}+\frac{4.077+7.389}{2}+\frac{7.389+10.043}{2}+\frac{10.043+0}{2}\right]=11.2545=11.25$ (2 dp) cao	
	B1	0.5 and a divisor of 2 on all terms inside brackets.
	M1	First and last ordinates once and the middle ordinates twice inside brackets ignoring the 2.
	A1	11.25 **cao**
(b)	**B0**	Give B0 for • smaller values of x and/or y. • use more decimal places
(c)	**M1**	Either $(2-x)e^{2x}\to\pm\lambda(2-x)e^{2x}\pm\int\mu e^{2x}\{dx\}$ or $\pm xe^{2x}\to\pm\lambda xe^{2x}\pm\int\mu e^{2x}\{dx\}$
	A1	$(2-x)e^{2x}\to\frac{1}{2}(2-x)e^{2x}-\int-\frac{1}{2}e^{2x}\{dx\}$ either un-simplified or simplified.
	A1	Correct expression, i.e. $\frac{1}{2}(2-x)e^{2x}+\frac{1}{4}e^{2x}$ or $\frac{5}{4}e^{2x}-xe^{2x}$ (or equivalent)
	dM1	**which is dependent on the 1st M1 mark being awarded.** **Complete method** of applying limits of 2 and 0 **to all terms** and subtracting the correct way round.
	Note	Evidence of a proper consideration of the limit of 0 is needed for M1. So, just subtracting zero is M0.
	A1	$\frac{1}{4}e^4-\frac{5}{4}$ or $\frac{e^4-5}{4}$. Do not allow $\frac{1}{4}e^4-\frac{5}{4}e^0$ unless simplified to give $\frac{1}{4}e^4-\frac{5}{4}$
	Note	12.39953751... **without seeing** $\frac{1}{4}e^4-\frac{5}{4}$ is A0.
	Note	12.39953751... from **NO** working is M0A0A0M0A0.

Question 7

Question Number	Scheme		Marks
(a)	x: 1, 1.2, 1.4, 1.6, 1.8, 2 y: 0, 0.2625, *0.659485...*, 1.2032, 1.9044, 2.7726	$y = x^2 \ln x$	
	$\{\text{At } x = 1.4\}\ y = 0.6595\ (4\text{ dp})$	0.6595	B1 cao
			[1]
(b)	$\frac{1}{2} \times (0.2) \times \underline{\left[0 + 2.7726 + 2\left(0.2625 + \text{their } 0.6595 + 1.2032 + 1.9044\right)\right]}$	Outside brackets $\frac{1}{2} \times (0.2)$ or $\frac{1}{10}$	B1 o.e.
	{**Note:** The "0" does not have to be included in [……]}	For structure of $[\ldots\ldots\ldots\ldots]$	M1
	$\left\{= \frac{1}{10}(10.8318)\right\} = 1.08318 = 1.083\ (3\text{ dp})$	anything that rounds to 1.083	A1
			[3]
(c) **Way 1**	$\left\{\mathrm{I} = \int x^2 \ln x \, \mathrm{d}x\right\}$, $\left\{\begin{array}{l} u = \ln x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = x^2 \Rightarrow v = \frac{1}{3}x^3 \end{array}\right\}$		
	$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\mathrm{d}x\}$	Either $x^2 \ln x \to \pm \lambda x^3 \ln x - \int \mu x^3 \left(\frac{1}{x}\right)\{\mathrm{d}x\}$ or $\pm \lambda x^3 \ln x - \int \mu x^2 \{\mathrm{d}x\}$, where $\lambda, \mu > 0$	M1
		$x^2 \ln x \to \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\left(\frac{1}{x}\right)\{\mathrm{d}x\}$, simplified or un-simplified	A1
	$= \frac{x^3}{3}\ln x - \frac{x^3}{9}$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
	$\text{Area}(R) = \left\{\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_1^2\right\} = \left(\frac{8}{3}\ln 2 - \frac{8}{9}\right) - \left(0 - \frac{1}{9}\right)$	**dependent on the previous M mark.** Applies limits of 2 and 1 and subtracts the correct way round	dM1
	$= \frac{8}{3}\ln 2 - \frac{7}{9}$	$\frac{8}{3}\ln 2 - \frac{7}{9}$ or $\frac{1}{9}(24\ln 2 - 7)$	A1 oe cso
			[5]

(c) **Way 2**	$\mathrm{I} = x^2(x\ln x - x) - \int 2x(x\ln x - x)\mathrm{d}x$	$\left\{\begin{array}{l} u = x^2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \ln x \Rightarrow v = x\ln x - x \end{array}\right\}$	
	So, $3\mathrm{I} = x^2(x\ln x - x) + \int 2x^2\{\mathrm{d}x\}$		
	and $\mathrm{I} = \frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{\mathrm{d}x\}$	A full method of applying $u = x^2$, $v' = \ln x$ to give $\pm\lambda x^2(x\ln x - x) \pm \mu\int x^2\{\mathrm{d}x\}$	M1
		$\frac{1}{3}x^2(x\ln x - x) + \frac{1}{3}\int 2x^2\{\mathrm{d}x\}$ simplified or un-simplified	A1
	$= \frac{1}{3}x^2(x\ln x - x) + \frac{2}{9}x^3$	$\frac{x^3}{3}\ln x - \frac{x^3}{9}$, simplified or un-simplified	A1
		Then award dM1A1 in the same way as above	M1 A1
			[5]
			9

		Question Notes
(a)	**B1**	0.6595 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{2}\times\frac{1}{5}$ or $\frac{1}{10}$ or equivalent.
	M1 **Note**	<u>For structure of trapezium rule</u> $\left[\text{.............}\right]$ No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1 **Note**	anything that rounds to 1.083 Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.070614704…)
	Note	Full marks can be gained in part (b) for using an incorrect part (a) answer of 0.6594
	Note	Award B1M1A1 for $\frac{1}{10}(2.7726)+\frac{1}{5}(0.2625+\text{their } 0.6595+1.2032+1.9044)=\text{awrt } 1.083$
	<u>Bracketing mistake:</u> Unless the final answer implies that the calculation has been done correctly Award B1M0A0 for $\frac{1}{2}(0.2)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)+2.7726$ (answer of 10.9318) Award B1M0A0 for $\frac{1}{2}(0.2)(2.7726)+2(0.2625+\text{their } 0.6595+1.2032+1.9044)$ (answer of 8.33646)	
	<u>Alternative method: Adding individual trapezia</u> $\text{Area}\approx 0.2\times\left[\frac{0+0.2625}{2}+\frac{0.2625+\text{"}0.6595\text{"}}{2}+\frac{\text{"}0.6595\text{"}+1.2032}{2}+\frac{1.2032+1.9044}{2}+\frac{1.9044+2.7726}{2}\right]=1.08318\ldots$	
	B1	0.2 and a divisor of 2 on all terms inside brackets
	M1	First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2
	A1	anything that rounds to 1.083
(c)	**A1** **Note** **Note**	Exact answer needs to be a two term expression in the form $a\ln b+c$ Give A1 e.g. $\frac{8}{3}\ln 2-\frac{7}{9}$ or $\frac{1}{9}(24\ln 2-7)$ or $\frac{4}{3}\ln 4-\frac{7}{9}$ or $\frac{1}{3}\ln 256-\frac{7}{9}$ or $-\frac{7}{9}+\frac{8}{3}\ln 2$ or $\ln 2^{\frac{8}{3}}-\frac{7}{9}$ or equivalent. Give final A0 for a final answer of $\frac{8\ln 2-\ln 1}{3}-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{1}{3}\ln 1-\frac{7}{9}$ or $\frac{8\ln 2}{3}-\frac{8}{9}+\frac{1}{9}$ or $\frac{8}{3}\ln 2-\frac{7}{9}+c$
	Note	$\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2$ followed by awrt 1.07 with no correct answer seen is dM1A0
	Note	Give dM0A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2\to\left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\frac{1}{9}$ (adding rather than subtracting)
	Note	Allow dM1A0 for $\left[\frac{x^3}{3}\ln x-\frac{x^3}{9}\right]_1^2\to\left(\frac{8}{3}\ln 2-\frac{8}{9}\right)-\left(0+\frac{1}{9}\right)$
	SC	A candidate who uses $u=\ln x$ and $\frac{dv}{dx}=x^2$, $\frac{du}{dx}=\frac{\alpha}{x}$, $v=\beta x^3$, writes down the correct "by parts" formula but makes only one error when applying it can be awarded Special Case 1st M1.

Question 8

Question Number	Scheme	Notes	Marks
	x: 0, 0.2, 0.4, 0.6, 0.8, 1; y: 2, ***1.8625426...***, 1.71830, 1.56981, 1.41994, 1.27165	$y = \dfrac{6}{(2+e^x)}$	
(a)	$\{\text{At } x = 0.2,\}\ y = 1.86254$ (5 dp)	1.86254	B1 **cao**
	Note: Look for this value on the given table or in their working.		**[1]**
(b)	$\frac{1}{2}(0.2)\left[2 + 1.27165 + 2\left(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994\right)\right]$	Outside brackets $\frac{1}{2}\times(0.2)$ or $\frac{1}{10}$ or $\frac{1}{2}\times\frac{1}{5}$	B1 o.e.
		For structure of $\left[\ldots\ldots\right]$	M1
	$\left\{= \frac{1}{10}(16.41283)\right\} = 1.641283 = 1.6413$ (4 dp)	anything that rounds to 1.6413	A1
			[3]
(c)	$\{u = e^x \text{ or } x = \ln u \Rightarrow\}$		
	$\frac{du}{dx} = e^x$ **or** $\frac{du}{dx} = u$ **or** $\frac{dx}{du} = \frac{1}{u}$ **or** $du = u\,dx$ etc., **and** $\int \frac{6}{(e^x+2)}\,dx = \int \frac{6}{(u+2)u}\,du$	**See notes**	B1 *
	$\{x = 0\} \Rightarrow a = e^0 \Rightarrow \underline{a = 1}$ $\{x = 1\} \Rightarrow b = e^1 \Rightarrow \underline{b = e}$	$a = 1$ and $b = e$ or $b = e^1$ or evidence of $0 \to 1$ and $1 \to e$	B1
	NOTE: 1st B1 mark CANNOT be recovered for work in part (d) **NOTE: 2nd B1 mark CAN be recovered for work in part (d)**		**[2]**

(d) Way 1	$\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$ $\Rightarrow 6 \equiv A(u+2) + Bu$	Writing $\frac{6}{u(u+2)} \equiv \frac{A}{u} + \frac{B}{(u+2)}$, o.e. **or** $\frac{1}{u(u+2)} \equiv \frac{P}{u} + \frac{Q}{(u+2)}$, o.e., and a complete method for finding the value of at least one of **their** A or **their** B (or **their** P or **their** Q)	M1
	$u = 0 \Rightarrow A = 3$ $u = -2 \Rightarrow B = -3$	**Both their** $A = 3$ **and their** $B = -3$. (Or **their** $P = \frac{1}{2}$ **and their** $Q = -\frac{1}{2}$ with the factor of 6 in front of the integral sign)	A1
	$\int \frac{6}{u(u+2)}\,du = \int \left(\frac{3}{u} - \frac{3}{(u+2)}\right) du$ $= 3\ln u - 3\ln(u+2)$ or $= 3\ln 2u - 3\ln(2u+4)$	Integrates $\frac{M}{u} \pm \frac{N}{u \pm k}$, $M, N, k \neq 0$; (i.e. ***a two term partial fraction***) to obtain either $\pm\lambda\ln(\alpha u)$ **or** $\pm\mu\ln(\beta(u \pm k))$; $\lambda, \mu, \alpha, \beta \neq 0$	M1
		Integration of both terms is **correctly followed through** from **their** M and from **their** N.	A1 ft
	$\left\{\text{So}\left[3\ln u - 3\ln(u+2)\right]_1^{e}\right\}$ $= \left(3\ln(e) - 3\ln(e+2)\right) - \left(3\ln 1 - 3\ln 3\right)$ [Note: A proper consideration of the limit of $u = 1$ is required for this mark]	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b > 0, b \neq 1, a > 0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$= 3 - 3\ln(e+2) + 3\ln 3$ **or** $3(1 - \ln(e+2) + \ln 3)$ **or** $3 + 3\ln\left(\frac{3}{e+2}\right)$ **or** $3\ln\left(\frac{e}{e+2}\right) - 3\ln\left(\frac{1}{3}\right)$ **or** $3 - 3\ln\left(\frac{e+2}{3}\right)$ **or** $3\ln\left(\frac{3e}{e+2}\right)$ **or** $\ln\left(\frac{27e^3}{(e+2)^3}\right)$	see notes	A1 cso
	Note: Allow e^1 in place of e for the final A1 mark.		**[6]**
	Note: Give final A0 for $3 - 3\ln e + 2 + 3\ln 3$ (i.e. bracketing error) unless recovered.		**12**
	Note: Give final A0 for $3 - 3\ln(e+2) + 3\ln 3 - 3\ln 1$, where $3\ln 1$ has not been simplified to 0		
	Note: Give final A0 for $3\ln e - 3\ln(e+2) + 3\ln 3$, where $3\ln e$ has not been simplified to 3		

	Question Notes	
(b)	**Note**	**M1:** Do not allow an extra y-value ***or*** a repeated y value in their […] Do not allow an omission of a y-ordinate in their […] for M1 **unless** they give the correct answer of awrt 1.6413, in which case both M1 and A1 can be scored.
	Note	**A1:** Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 1.64150274…)
	Note	Full marks can be gained in part (b) for awrt 1.6413 even if B0 is given in part (a)
	Note	Award B1M1A1 for $\frac{1}{10}(2+1.27165) + \frac{1}{5}(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994) = \text{awrt } 1.6413$
	Bracketing mistakes: Unless the final answer implies that the calculation has been done correctly Award B1M0A0 for $\frac{1}{2}(0.2) + 2 + 2(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994) + 1.27165$ $(=16.51283)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2 + 1.27165) + 2(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994)$ $(=13.468345)$ Award B1M0A0 for $\frac{1}{2}(0.2)(2) + 2(\text{their } 1.86254 + 1.71830 + 1.56981 + 1.41994) + 1.27165$ $(=14.61283)$	
	Alternative method: Adding individual trapezia $\text{Area} \approx 0.2 \times \left[\frac{2 + \text{"}1.86254\text{"}}{2} + \frac{\text{"}1.86254\text{"} + 1.71830}{2} + \frac{1.71830 + 1.56981}{2} + \frac{1.56981 + 1.41994}{2} + \frac{1.41994 + 1.27165}{2}\right]$ $= 1.641283$ **B1** 0.2 and a divisor of 2 on all terms inside brackets **M1** First and last ordinates once and two of the middle ordinates inside brackets ignoring the 2 **A1** anything that rounds to 1.6413	

(c)	**1st B1**	Must start from either • $\int y\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(e^x+2)}\,\mathrm{d}x$, with integral sign and $\mathrm{d}x$ • $\int \frac{6}{(e^x+2)}\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u$, with integral sign and $\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ **and** state either $\frac{\mathrm{d}u}{\mathrm{d}x}=e^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$ **and** end at $\int \frac{6}{u(u+2)}\,\mathrm{d}u$, with integral sign and $\mathrm{d}u$, **with no incorrect working.**
	Note	So, just writing $\frac{\mathrm{d}u}{\mathrm{d}x}=e^x$ **and** $\int \frac{6}{(e^x+2)}\,\mathrm{d}x=\int \frac{6}{u(u+2)}\,\mathrm{d}u$ is sufficient for 1st B1
	Note	Give 2nd B0 for $b=2.718...$, without reference to $a=1$ and $b=e$ or $b=e^1$
	Note	You can also give the 1st B1 mark for using a reverse process. i.e. Proceeding from $\int \frac{6}{u(u+2)}\,\mathrm{d}u$ to $\int \frac{6}{(e^x+2)}\,\mathrm{d}x$, **with no incorrect working,** and stating either $\frac{\mathrm{d}u}{\mathrm{d}x}=e^x$ **or** $\frac{\mathrm{d}u}{\mathrm{d}x}=u$ **or** $\frac{\mathrm{d}x}{\mathrm{d}u}=\frac{1}{u}$ **or** $\mathrm{d}u=u\,\mathrm{d}x$
(d)	**Note**	Give final A0 for $3-3\ln(e+2)+3\ln 3$ simplifying to $1-\ln(e+2)+\ln 3$ (i.e. dividing their correct final answer by 3) Otherwise, you can ignore incorrect working (isw) following on from a correct exact value.
	Note	A decimal answer of 1.641502724... (without a correct **exact** answer) is final A0
	Note	$\left[-3\ln(u+2)+3\ln u\right]_1^e$ followed by awrt 1.64 (without a correct **exact** answer) is final M1A0

	Question Notes Continued	
(d)	**Note**	**BE CAREFUL! Candidates will assign their own "A" and "B" for this question.**
	Note	*Writing down* $\frac{6}{(u+2)u}$ in the form $\frac{A}{(u+2)}+\frac{B}{u}$ with at least one of A or B correct is 1st M1
	Note	*Writing down* $\frac{6}{(u+2)u}$ as $\frac{-3}{(u+2)}+\frac{3}{u}$ is 1st M1 1st A1.
	Note	**Condone** $\int\left(\frac{3}{u}-\frac{3}{(u+2)}\right)\mathrm{d}u$ to give $3\ln u-3\ln u+2$ (poor bracketing) for 2nd A1.
	Note	**Award M0A0M1A1ft** for a candidate who writes down e.g. $\int\frac{6}{u(u+2)}\,\mathrm{d}u=\int\left(\frac{6}{u}+\frac{6}{(u+2)}\right)\mathrm{d}u=6\ln u+6\ln(u+2)$ **AS EVIDENCE OF WRITING** $\frac{6}{u(u+2)}$ **AS PARTIAL FRACTIONS.**
	Note	**Award M0A0M0A0** for a candidate who writes down $\int\frac{6}{u(u+2)}\,\mathrm{d}u=6\ln u+6\ln(u+2)$ **or** $\int\frac{6}{u(u+2)}\,\mathrm{d}u=\ln u+6\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{6}{u(u+2)}$ as partial fractions.
	Note	**Award M1A1M1A1** for a candidate who writes down $\int\frac{6}{u(u+2)}\,\mathrm{d}u=3\ln u-3\ln(u+2)$ **WITHOUT ANY EVIDENCE OF WRITING** $\frac{6}{u(u+2)}$ as partial fractions.
	Note	If they lose the "6" and find $\int_1^{e}\frac{1}{u(u+2)}\mathrm{d}u$ we can allow a maximum of M1A0M1A1ftM1A0

	Question Notes Continued		
(d) **Way 2**	$\left\{\int \frac{6}{u^2+2u}\mathrm{d}u = \int \frac{3(2u+2)}{u^2+2u}\mathrm{d}u - \int \frac{6u}{u^2+2u}\mathrm{d}u\right\}$		
	$= \int \frac{3(2u+2)}{u^2+2u}\mathrm{d}u - \int \frac{6}{u+2}\mathrm{d}u$	$\int \frac{\pm\alpha(2u+2)}{u^2+2u}\{\mathrm{d}u\} \pm \int \frac{\delta}{u+2}\{\mathrm{d}u\},\ \alpha, \beta, \delta \neq 0$	M1
		Correct expression	A1
	$= 3\ln(u^2+2u) - 6\ln(u+2)$	Integrates $\frac{\pm M(2u+2)}{u^2+2u} \pm \frac{N}{u \pm k}$, $M, N, k \neq 0$, to obtain any one of $\pm\lambda\ln(u^2+2u)$ **or** $\pm\mu\ln(\beta(u \pm k))$; $\lambda, \mu, \beta \neq 0$	M1
		Integration of both terms is **correctly followed through** from **their** M and from **their** N	A1 ft
	$\left\{\text{So, } \left[3\ln(u^2+2u) - 6\ln(u+2)\right]_1^{e}\right\}$ $= \left(3\ln(e^2+2e) - 6\ln(e+2)\right) - \left(3\ln 3 - 6\ln 3\right)$	**dependent on the 2nd M mark** Applies limits of e and 1 (or their b and their a, where $b > 0, b \neq 1, a > 0$) in u **or** applies limits of 1 and 0 in x and subtracts the correct way round.	dM1
	$= 3\ln(e^2+2e) - 6\ln(e+2) + 3\ln 3$	$3\ln(e^2+2e) - 6\ln(e+2) + 3\ln 3$	A1 o.e.
			[6]
(d) **Way 3**	Applying $u = \theta - 1$		
	$\left\{\int_1^{e} \frac{6}{u(u+2)}\mathrm{d}u = \right\} \int_2^{1+e} \frac{6}{(\theta-1)(\theta+1)}\mathrm{d}\theta = \int_2^{1+e} \frac{6}{\theta^2-1}\mathrm{d}u = \left[3\ln\left(\frac{\theta-1}{\theta+1}\right)\right]_2^{1+e}$		M1A1M1A1
	$= 3\ln\left(\frac{1+e-1}{e+1+1}\right) - 3\ln\left(\frac{2-1}{2+1}\right) = 3\ln\left(\frac{e}{e+2}\right) - 3\ln\left(\frac{1}{3}\right)$	3rd M mark is dependent on 2nd M mark	dM1A1
			[6]