Trapezium Rule - Edexcel Past Exam Questions MARK SCHEME

Question 1: June 05 Q5

Question Number	Scheme	Marks
	(a) $\int x e^{2x} dx = \frac{1}{2}x e^{2x} - \frac{1}{2}\int e^{2x}\, dx$ Attempting parts in the right direction	M1 A1
	$= \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x}$	A1
	$\left[\frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}e^2$	M1 A1
		(5)
	(b) $x = 0.4 \Rightarrow y \approx 0.89022$	
	$x = 0.8 \Rightarrow y \approx 3.96243$ Both are required to 5 d.p	B1
		(1)
	(c) $I \approx \frac{1}{2} \times 0.2 \times [\ \dots\ ]$	B1
	$\approx \dots \times [0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243)]$	M1 A1ft
	ft their answers to (b)	
	$\approx 0.1 \times 21.67522$	
	≈ 2.168 cao	A1 **(4)**
		[10]
	Note $\frac{1}{4} + \frac{1}{4}e^2 \approx 2.097 \dots$	

Question 2: Jan 06 Q2

2. (a)			
		x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
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y	1	**1.01959**	**1.08239**	1.20269	**1.41421**		M1 A1	
			(2)
	M1 for one correct, A1 for all correct		
(b)	Integral = $\frac{1}{2}\times\frac{\pi}{16}\times\{1+1.4142+2(1.01959+\ldots+1.20269)\}$	M1 A1√	
	$\left(=\frac{\pi}{32}\,x\,9.02355\right)$ = 0.8859	A1 cao	(3)
(c)	Percentage error = $\frac{approx-0.88137}{0.88137}\times100$ = 0.51 % (allow 0.5% to 0.54% for A1)	M1 A1	(2)
			[7]
	M1 gained for $(\pm)\ \frac{approx-\ln(1+\sqrt{2})}{\ln(1+\sqrt{2})}$		

Question 3: Jan 07 Q8

Question Number	Scheme	Marks
(a)	x: 0, 1, 2, 3, 4, 5 y: e^1, e^2, $e^{\sqrt{7}}$, $e^{\sqrt{10}}$, $e^{\sqrt{13}}$, e^4 or y: 2.71828…, 7.38906…, 14.09403…, 23.62434…, 36.80197…, 54.59815…	
	Either $e^{\sqrt{7}}$, $e^{\sqrt{10}}$ and $e^{\sqrt{13}}$ or awrt 14.1, 23.6 and 36.8 or e to the power awrt 2.65, 3.16, 3.61 (or mixture of decimals and e's) ***At least*** two correct All three correct	B1 B1 **[2]**
(b)	$I \approx \frac{1}{2}\times 1 ; \times \underline{\left\{ e^1 + 2\left(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}\right) + e^4 \right\}}$	
	Outside brackets $\frac{1}{2}\times 1$	B1;
	<u>For structure of trapezium rule</u> $\underline{\{............\}}$;	M1$\sqrt{}$
	$= \frac{1}{2}\times 221.1352227... = 110.5676113... = \underline{110.6}$ (4sf) <u>110.6</u>	A1 **cao** **[3]**

Question Number	Scheme	Marks
(c)	$t = (3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx} = \frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$ $A(3x+1)^{-\frac{1}{2}}$ or $t\frac{dt}{dx} = A$	M1
	… or $t^2 = 3x+1 \Rightarrow \underline{2t\frac{dt}{dx} = 3}$ $\underline{\tfrac{3}{2}(3x+1)^{-\frac{1}{2}}}$ or $\underline{2t\frac{dt}{dx} = 3}$	A1
	so $\frac{dt}{dx} = \frac{3}{2.(3x+1)^{\frac{1}{2}}} = \frac{3}{2t} \Rightarrow \frac{dx}{dt} = \frac{2t}{3}$ $\therefore I = \int e^{\sqrt{(3x+1)}}\, dx = \int e^t \frac{dx}{dt}.dt = \int e^t.\frac{2t}{3}.dt$ Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t … … and moves on to substitute this into I to convert an integral wrt x to an integral wrt t.	dM1
	$\therefore I = \underline{\int \tfrac{2}{3} t e^t \, dt}$ $\underline{\int \tfrac{2}{3} t e^t}$	A1
	change limits: when $x = 0$, $t = 1$ & when $x = 5$, $t = 4$ changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$	B1
	Hence $I = \int_1^4 \tfrac{2}{3} t e^t \, dt$; where $a = 1$, $b = 4$, $k = \tfrac{2}{3}$	**[5]**

(d)	$\left\{\begin{array}{ll} u = t & \Rightarrow \frac{du}{dt} = 1 \\ \frac{dv}{dt} = e^t & \Rightarrow v = e^t \end{array}\right\}$	Let k be any constant for the first three marks of this part.	
	$k\int t e^t \, dt = k\left(t e^t - \int e^t.1 \, dt\right)$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression with a constant factor k.	A1
	$= k\left(\underline{t e^t - e^t}\right) + c$	<u>Correct integration</u> with/without a constant factor k	A1
	$\therefore \int_1^4 \frac{2}{3} t e^t \, dt = \frac{2}{3}\left\{\left(4e^4 - e^4\right) - \left(e^1 - e^1\right)\right\}$	Substitutes their changed limits into the integrand and subtracts oe.	dM1 **oe**
	$= \frac{2}{3}(3e^4) = \underline{2e^4} = 109.1963...$	either $2e^4$ or awrt 109.2	A1
			[5]
			15 marks

Question 4: June 06 Q6

Question Number	Scheme		Marks
(a)	<table><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td></tr><tr><td>y</td><td>0</td><td>0.5 ln 1.5</td><td>ln 2</td><td>1.5 ln 2.5</td><td>2 ln 3</td></tr><tr><td>or y</td><td>0</td><td>0.2027325541…</td><td>ln2</td><td>1.374436098…</td><td>2 ln 3</td></tr></table>	Either 0.5 ln 1.5 and 1.5 ln 2.5 or awrt 0.20 and 1.37 (or mixture of decimals and ln's)	B1 **[1]**
(b)(i)	$I_1 \approx \frac{1}{2} \times 1 \times \underline{\{0 + 2(\ln 2) + 2\ln 3\}}$	<u>For structure of trapezium rule</u> $\underline{\{.............\}}$;	M1;
	$= \frac{1}{2} \times 3.583518938... = 1.791759... = 1.792$ (4sf)	1.792	A1 cao
(ii)	$I_2 \approx \frac{1}{2} \times 0.5 \; ; \times \underline{\{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3\}}$	Outside brackets $\frac{1}{2} \times 0.5$ <u>For structure of trapezium rule</u> $\underline{\{.............\}}$;	B1; M1 $\sqrt{}$
	$= \frac{1}{4} \times 6.737856242... = 1.684464...$	awrt 1.684	A1 **[5]**
(c)	With increasing ordinates, <u>the line segments at the top of the trapezia are closer to the curve.</u>	<u>Reason</u> or an appropriate diagram elaborating the correct reason.	B1 **[1]**

Question 5: June 07 Q7

Question Number	Scheme		Marks
(a)	$\begin{array}{c\|c\|c\|c\|c\|c} x & 0 & \frac{\pi}{16} & \frac{\pi}{8} & \frac{3\pi}{16} & \frac{\pi}{4} \\ \hline y & 0 & 0.445995927... & 0.643594252... & 0.817421946... & 1 \end{array}$		
	Enter marks into ePEN in the correct order.	0.446 or awrt 0.44600 awrt 0.64359 awrt 0.81742	B1 B1 B1 **[3]**
(b) **Way 1**	0 can be implied $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{16}; \times \underline{\{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\}}$	Outside brackets $\frac{1}{2} \times \frac{\pi}{16}$ or $\frac{\pi}{32}$ <u>For structure of trapezium rule</u> $\underline{\{............\}}$; Correct expression inside brackets which all must be multiplied by $\frac{h}{2}$.	B1 <u>M1</u>$\sqrt{}$ <u>A1</u>$\sqrt{}$
	$= \frac{\pi}{32} \times 4.81402... = 0.472615308... = \underline{0.4726}$ (4dp)	for seeing <u>0.4726</u>	A1 **cao** **[4]**
Aliter (b) **Way 2**	$\text{Area} \approx \frac{\pi}{16} \times \left\{ \frac{0+0.44600}{2} + \frac{0.44600+0.64359}{2} + \frac{0.64359+0.81742}{2} + \frac{0.81742+1}{2} \right\}$ which is equivalent to: $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{16}; \times \underline{\{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\}}$	$\frac{\pi}{16}$ and a divisor of 2 on all terms inside brackets. One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	B1 <u>M1</u>$\sqrt{}$ <u>A1</u>$\sqrt{}$
	$= \frac{\pi}{16} \times 2.40701... = 0.472615308... = \underline{0.4726}$	<u>0.4726</u>	A1 **cao** **[4]**

$\textit{Area} = \frac{1}{2} \times \frac{\pi}{20} \times \{0 + 2(0.44600 + 0.64359 + 0.81742) + 1\} = 0.3781$, gains B0M1A1A0

In (a) for $x = \frac{\pi}{16}$ writing 0.4459959... then 0.45600 gains B1 for awrt 0.44600 even though 0.45600 is incorrect.

In (b) you can follow though a candidate's values from part (a) to award M1 ft, A1 ft

Question 6: Jan 08 Q1

Question Number	Scheme		Marks
(a)	x: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π y: 0, 1.844321332…, 4.810477381…, 8.87207, 0	awrt 1.84432 awrt 4.81048 or 4.81047	B1 B1 **[2]**
(b) **Way 1**	0 can be implied Area $\approx \frac{1}{2} \times \frac{\pi}{4}$; $\times \underline{\left\{0 + 2\left(1.84432 + 4.81048 + 8.87207\right) + 0\right\}}$	Outside brackets awrt 0.39 or $\frac{1}{2} \times$ awrt 0.79 $\frac{1}{2} \times \frac{\pi}{4}$ or $\frac{\pi}{8}$	B1
		<u>For structure of trapezium rule</u> $\underline{\{............\}}$;	<u>M1</u> $\sqrt{}$
		Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant".	<u>A1</u> $\sqrt{}$
	$= \frac{\pi}{8} \times 31.05374... = 12.19477518... = \underline{12.1948}$ (4dp)	<u>12.1948</u>	A1 **cao** **[4]**
Aliter (b) **Way 2**	Area $\approx \frac{\pi}{4} \times \left\{ \frac{0+1.84432}{2} + \frac{1.84432+4.81048}{2} + \frac{4.81048+8.87207}{2} + \frac{8.87207+0}{2} \right\}$	$\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	<u>M1</u> $\sqrt{}$
	Area $\approx \frac{1}{2} \times \frac{\pi}{4}$; $\times \underline{\left\{0 + 2\left(1.84432 + 4.81048 + 8.87207\right) + 0\right\}}$	Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	<u>A1</u> $\sqrt{}$
	$= \frac{\pi}{4} \times 15.52687... = 12.19477518... = \underline{12.1948}$ (4dp)	<u>12.1948</u>	A1 **cao** **[4]**
			6 marks

Note an expression like Area $\approx \frac{1}{2} \times \frac{\pi}{4} + 2\left(1.84432 + 4.81048 + 8.87207\right)$ would score B1M1A0A0

Question 7: June 08 Q1

Question Number	Scheme	Marks
(a)	x: 0, 0.4, 0.8, 1.2, 1.6, 2 y: e^0, $e^{0.08}$, $e^{0.32}$, $e^{0.72}$, $e^{1.28}$, e^2 or y: 1, 1.08329…, 1.37713…, 2.05443…, 3.59664…, 7.38906…	B1 (1)
(b)	Area $\approx \frac{1}{2}\times 0.4\,;\times\underline{\left[\ e^0+2\left(e^{0.08}+e^{0.32}+e^{0.72}+e^{1.28}\right)+e^2\ \right]}$	B1; M1
	$=0.2\times 24.61203164\ldots\ =\ 4.922406\ldots\ =\underline{4.922}$ (4sf)	A1 (3)
		(4 marks)

Question 8: June 09 Q2

Question Number	Scheme		Marks	
Q (a)	1.14805	awrt 1.14805	B1	(1)
(b)	$A\approx\frac{1}{2}\times\frac{3\pi}{8}(\ \ldots\ )$		B1	
	$=\ \ldots\left(3+2\left(2.77164+2.12132+1.14805\right)+0\right)$	0 can be implied	M1	
	$=\frac{3\pi}{16}\left(3+2\left(2.77164+2.12132+1.14805\right)\right)$	ft their (a)	A1ft	
	$=\frac{3\pi}{16}\times 15.08202\ \ldots\ =8.884$	cao	A1	(4)
(c)	$\int 3\cos\left(\frac{x}{3}\right)dx=\frac{3\sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$		M1 A1	
	$=9\sin\left(\frac{x}{3}\right)$			
	$A=\left[9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}}=9-0=9$	cao	A1	(3)
				[8]

Question 9: Jan 10 Q2

Question Number	Scheme	Marks	
	(a) 1.386, 2.291 awrt 1.386, 2.291	B1 B1	(2)
	(b) $A \approx \frac{1}{2} \times 0.5(\ ... \ )$	B1	
	$= \ ... \ \left(0+2(0.608+1.386+2.291+3.296+4.385)+5.545\right)$	M1	
	$= 0.25\left(0+2(0.608+1.386+2.291+3.296+4.385)+5.545\right)$ ft their (a)	A1ft	
	$= 0.25 \times 29.477 \ ... \ \approx 7.37$ cao	A1	(4)
	(c)(i) $\int x \ln x \, dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \times \frac{1}{x} dx$	M1 A1	
	$= \frac{x^2}{2}\ln x - \int \frac{x}{2} dx$		
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4} \quad (+C)$	M1 A1	
	(ii) $\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^4 = (8\ln 4 - 4) - \left(-\frac{1}{4}\right)$	M1	
	$= 8\ln 4 - \frac{15}{4}$		
	$= 8(2\ln 2) - \frac{15}{4}$ $\ln 4 = 2\ln 2$ seen or implied	M1	
	$= \frac{1}{4}(64\ln 2 - 15)$ $a = 64, b = -15$	A1	(7)
			[13]

Question 10: June 10 Q1

Question Number	Scheme		Marks	
	(a) $y\left(\frac{\pi}{6}\right) \approx 1.2247,\ y\left(\frac{\pi}{4}\right) = 1.1180$	accept awrt 4 d.p.	B1 B1	(2)
	(b)(i) $I \approx \left(\frac{\pi}{12}\right)(1.3229 + 2\times 1.2247 + 1)$	B1 for $\frac{\pi}{12}$	B1 M1	
	≈ 1.249	cao	A1	
	(ii) $I \approx \left(\frac{\pi}{24}\right)(1.3229 + 2\times(1.2973 + 1.2247 + 1.1180) + 1)$	B1 for $\frac{\pi}{24}$	B1 M1	
	≈ 1.257	cao	A1	(6) [8]

Question 11: Jan 11 Q7

Question Number	Scheme		Marks	
(a)	$x = 3 \Rightarrow y = 0.1847$	awrt	B1	
	$x = 5 \Rightarrow y = 0.1667$	awrt or $\frac{1}{6}$	B1	(2)
(b)	$I \approx \underline{\underline{\frac{1}{2}}}[0.2 + 0.1667 + 2(0.1847 + 0.1745)]$		$\underline{\underline{\text{B1}}}$ M1 A1ft	
	≈ 0.543	0.542 or 0.543	A1	(4)
(c)	$\frac{\mathrm{d}x}{\mathrm{d}u} = 2(u-4)$		B1	
	$\int \frac{1}{4+\sqrt{(x-1)}}\,\mathrm{d}x = \int \frac{1}{u}\times 2(u-4)\,\mathrm{d}u$		M1	
	$= \int \left(2 - \frac{8}{u}\right)\mathrm{d}u$		A1	
	$= 2u - 8\ln u$		M1 A1	
	$x = 2 \Rightarrow u = 5,\ x = 5 \Rightarrow u = 6$		B1	
	$[2u - 8\ln u]_5^6 = (12 - 8\ln 6) - (10 - 8\ln 5)$		M1	
	$= 2 + 8\ln\left(\frac{5}{6}\right)$		A1	(8) [14]

Question 12: June 11 Q4

Question Number	Scheme	Marks
	(a) 0.0333, 1.3596 awrt 0.0333, 1.3596	B1 B1 (2)
	(b) $\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4}[\ ...]$	B1
	$\approx\ ...\ \left[0+2(0.0333+0.3240+1.3596)+3.9210\right]$	M1
	≈ 1.30 Accept 1.3	A1 (3)
	(c) $u = x^2+2 \Rightarrow \frac{du}{dx} = 2x$	B1
	$\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2+2)\,dx$	B1
	$\int x^3 \ln(x^2+2)\,dx = \int x^2 \ln(x^2+2)\,x\,dx = \int (u-2)(\ln u)\tfrac{1}{2}\,du$	M1
	Hence $\text{Area}(R) = \frac{1}{2}\int_2^4 (u-2)\ln u\,du$ ✱ cso	A1 (4)
	(d) $\int (u-2)\ln u\,du = \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u^2}{2}-2u\right)\frac{1}{u}\,du$	M1 A1
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \int \left(\frac{u}{2}-2\right)du$	
	$= \left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\quad(+C)$	M1 A1
	$\text{Area}(R) = \frac{1}{2}\left[\left(\frac{u^2}{2}-2u\right)\ln u - \left(\frac{u^2}{4}-2u\right)\right]_2^4$	
	$= \tfrac{1}{2}\left[(8-8)\ln 4 - 4 + 8 - ((2-4)\ln 2 - 1 + 4)\right]$	M1
	$= \tfrac{1}{2}(2\ln 2 + 1)$ $\ln 2 + \tfrac{1}{2}$	A1 (6) [15]