Trigonometric Functions 2 (Sec, cosec & cot) - Edexcel Past Exam Qs MARK SCHEME

Question 1

Question No	Scheme	Marks
	Uses the identity $cot^2(3\theta) = cosec^2(3\theta) - 1$ in $2cot^2(3\theta) = 7cosec(3\theta) - 5$	**M1**
	$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$	**A1**
	$(2cosec3\theta - 1)(cosec3\theta - 3) = 0$	**dM1**
	$cosec3\theta = 3$	**A1**
	$\theta = \frac{invsin(\frac{1}{3})}{3}, \frac{19.5°}{3} = awrt\ 6.5°$	**ddM1, A1**
	$\theta = \frac{180° - invsin(\frac{1}{3})}{3}, 53.5°$ Correct 2nd value	**ddM1,A1**
	$\theta = \frac{360° + invsin(\frac{1}{3})}{3}$ Correct 3rd value	**ddM1**
	All 4 correct answers awrt $6.5°, 53.5°, 126.5°$ or $173.5°$	**A1** **(10 marks)**

Question 2

Question Number	Scheme	Marks
(i) (a)	$2\dfrac{\sin x}{\cos x}-\dfrac{\cos x}{\sin x}=\dfrac{5}{\sin x}$	B1
	Uses common denominator to give $2\sin^2 x-\cos^2 x=5\cos x$	M1
	Replaces $\sin^2 x$ by $(1-\cos^2 x)$ to give $2(1-\cos^2 x)-\cos^2 x=5\cos x$	M1
	Obtains $3\cos^2 x+5\cos x-2=0$ (a =3, b = 5, c = -2)	A1 (4)
(b)	Solves $3\cos^2 x+5\cos x-2=0$ to give $\cos x=$	M1
	$\cos x=\frac{1}{3}$ only (rejects $\cos x$ = -2)	A1
	So x = 1.23 or 5.05	dM1A1 (4)
(ii)	Either: $\tan\theta+\cot\theta\equiv\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$ Or: $\tan\theta+\cot\theta\equiv\tan\theta+\dfrac{1}{\tan\theta}$	B1
	Either: $\equiv\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$ Or: $\equiv\dfrac{\tan^2\theta+1}{\tan\theta}$	M1
	Either: $\equiv\dfrac{2}{\sin 2\theta}$ Or: $\equiv\dfrac{1}{\cos^2\theta\times\frac{\sin\theta}{\cos\theta}}\equiv\dfrac{2}{\sin 2\theta}$	M1
	Either: $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$) Or: $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda=2$)	A1 (4)
		12 marks
	Alternatives to Main Scheme	
(i) (a)	$2\tan x-\dfrac{1}{\tan x}=\dfrac{5}{\sin x}$ does not score any marks until	
	$\times\tan x\Rightarrow 2\tan^2 x+1=5\sec x$	B1, M1
	Replaces $\tan^2 x$ by $(\sec^2 x-1)$ to give $2(\sec x^2-1)+1=5\sec x$	M1
	Obtains $3\cos^2 x+5\cos x-2=0$ (a =3, b = 5, c = -2)	A1 (4)
(b)	Solves $3\cos^2 x+5\cos x-2=0$ to give $\cos x=$ or $2\sec^2 x-5\sec x-3=0\Rightarrow\sec x=..$	M1
	$\cos x=\frac{1}{3}$ only (rejects $\cos x$ = -2)	A1
	So x = 1.23 or 5.05	dM1A1 (4)
(ii)	$\tan\theta+\cot\theta=\lambda\operatorname{cosec}2\theta\Rightarrow\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\lambda}{\sin 2\theta}=\left(\cancel{\dfrac{\lambda}{2\sin\theta\cos\theta}}\right)$	B1
	$\times 2\sin\theta\cos\theta\Rightarrow 2\sin^2\theta+2\cos^2\theta=\lambda$	M1
	Factorises $2(\sin^2\theta+\cos^2\theta)=\lambda\Rightarrow 2=\lambda$	M1
	All above correct + a statement like 'hence true', 'QED'	A1 (4)

(i)(a)

B1 Uses definitions $\tan x = \dfrac{\sin x}{\cos x}$, $\cot x = \dfrac{\cos x}{\sin x}$ and $\operatorname{cosec} x = \dfrac{1}{\sin x}$ to write the equation in terms of $\cos x$ and $\sin x$. Condone $5\operatorname{cosec} x = \dfrac{1}{5\sin x}$ as the intention is clear.

Alternatively uses $\cot x = \dfrac{1}{\tan x}$ and $\operatorname{cosec} x = \dfrac{1}{\sin x}$ to write the equation in terms of $\tan x$ and $\sin x$

This may be implied by later work that achieves $A\tan^2 x \pm B = C\sec x$

M1 Either uses common denominator and cross multiples, or multiplies each term by $\sin x \cos x$ to achieve an equation of the form equivalent to $A\sin^2 x \pm B\cos^2 x = C\cos x$. It may be seen on the numerator of a fraction

Alternatively multiplies by $\tan x$ to achieve $A\tan^2 x \pm B = C\sec x$

M1 Uses a correct Pythagorean relationship, usually $\sin^2 x = 1 - \cos^2 x$ to form a quadratic equation in terms of $\cos x$. In the alternative uses $\tan^2 x = \sec^2 x - 1$ to form a quadratic in sec x, followed by $\sec x = \dfrac{1}{\cos x}$ to form a quadratic equation in terms of $\cos x$

A1 Obtains $\pm K\left(3\cos^2 x + 5\cos x - 2\right) = 0$ ($a = 3$, $b = 5$, $c = -2$)

(i)(b)

M1 Uses a standard method to solve their quadratic equation in $\cos x$ from **(i)(a) OR** sec$x$ from an earlier line in (a)
See General Principles for Core Mathematics on how to solve quadratics

A1 $\cos x = \frac{1}{3}$ only Do not need to see -2 rejected

dM1 Uses arcos on their value to obtain at least one answer. It is dependent upon the previous M.
It may be implied by one correct answer

A1 Both values correct awrt 3sf $x = 1.23$ and 5.05.
Ignore any solutions outside the range. Any extra solutions in the range will score A0.
Answers in degrees will score A0.

(ii)

B1 Uses a definition of cot with matching expression for tan. Acceptable answers are

$\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$, $\dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\dfrac{\sin\theta}{\cos\theta}}$, $\tan\theta + \dfrac{1}{\tan\theta}$. Condone a miscopy on the sign. Eg Allow $\tan\theta - \dfrac{1}{\tan\theta}$

M1 Uses common denominator, writing their expression as a single fraction. In the examples given above, example 2 would need to be inverted. The denominator has to be correct and one of the terms must be adapted.

M1 Uses identities $\sin^2\theta + \cos^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\dfrac{\lambda}{\sin 2\theta}$

Alternatively uses $1 + \tan^2\theta = \sec^2\theta = \dfrac{1}{\cos^2\theta}$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ specifically to achieve an expression of the form $\dfrac{\lambda}{\sin 2\theta}$. A line of $\dfrac{1}{\sin\theta\cos\theta}$ achieved on the lhs followed by $\lambda = \dfrac{1}{2}$ or 2 would imply this mark

A1 Achieves printed answer with no errors.

Allow for a different variable as long as it is used consistently.

Question 3

Question Number	Scheme	Marks
(a)	$\tan 2\theta° = \dfrac{2\tan\theta°}{1-\tan^2\theta°} = \dfrac{2p}{1-p^2}$ Final answer	M1A1
		(2)
(b)	$\cos\theta° = \dfrac{1}{\sec\theta°} = \dfrac{1}{\sqrt{1+\tan^2\theta°}} = \dfrac{1}{\sqrt{1+p^2}}$ Final answer	M1A1
		(2)
(c)	$\cot(\theta-45)° = \dfrac{1}{\tan(\theta-45)°} = \dfrac{1+\tan\theta°\tan 45°}{\tan\theta°-\tan 45°} = \dfrac{1+p}{p-1}$ Final answer	M1A1
		(2)
		(6 marks)

(a)

M1 Attempt to use the double angle formula for tangent followed by the substitution $\tan\theta = p$.

For example accept $\tan 2\theta° = \dfrac{2\tan\theta°}{1\pm\tan^2\theta°} = \dfrac{2p}{1\pm p^2}$

Condone unconventional notation such as $\tan 2\theta° = \dfrac{2\tan\theta°}{1\pm\tan\theta^{2\,\circ}}$ followed by an attempt to substitute $\tan\theta = p$ for the M mark. Recovery from this notation is allowed for the A1.

Alternatively use $\tan(A+B) = \dfrac{\tan A+\tan B}{1\pm\tan A\tan B}$ with an attempt at substituting $\tan A = \tan B = p$. The unsimplified answer $\dfrac{p+p}{1-p\times p}$ is evidence

It is possible to use $\tan 2\theta° = \dfrac{\sin 2\theta°}{\cos 2\theta°} = \dfrac{2\sin\theta°\cos\theta°}{2\cos^2\theta°-1} = \dfrac{2\times\dfrac{p}{\sqrt{1\pm p^2}}\times\dfrac{1}{\sqrt{1\pm p^2}}}{2\times\dfrac{1}{1\pm p^2}-1}$ but it is unlikely to succeed.

A1 Correct **simplified** answer of $\tan 2\theta° = \dfrac{2p}{1-p^2}$ or $\dfrac{2p}{(1-p)(1+p)}$.

Do not allow if they "simplify" to $\dfrac{2}{1-p}$

Allow the correct answer for both marks as long as no incorrect working is seen.

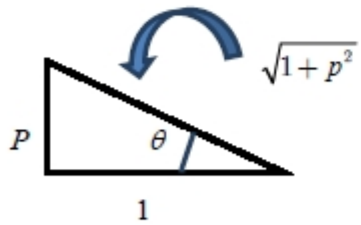
(b)

M1 Attempt to use **both** $\cos\theta = \frac{1}{\sec\theta}$ **and** $1 + \tan^2\theta = \sec^2\theta$ with $\tan\theta = p$ in an attempt to obtain an expression for $\cos\theta$ in terms of p. Condone a slip in the sign of the second identity.

Evidence would be $\cos^2\theta = \frac{1}{\pm 1 \pm p^2}$

Alternatively use a triangle method, attempt Pythagoras' theorem and use $\cos\theta = \frac{adj}{hyp}$

The attempt to use Pythagoras must attempt to use the squares of the lengths.



A1 $\cos\theta^\circ = \frac{1}{\sqrt{1+p^2}}$ Accept versions such as $\cos\theta^\circ = \sqrt{\frac{1}{1+p^2}}$, $\cos\theta^\circ = \pm\frac{1}{\sqrt{1+p^2}}$

Withhold this mark if the candidate goes on to write $\cos\theta^\circ = \frac{1}{1+p}$

(c)

M1 Use the correct identity $\cot(\theta - 45) = \frac{1}{\tan(\theta - 45)}$ and an attempt to use the $\tan(A - B)$ formula with $A=\theta$, $B=45$ and $\tan\theta = p$.

For example accept an unsimplified answer such as $\frac{1}{\frac{\tan\theta \pm \tan 45}{1 \pm \tan\theta \tan 45}} = \frac{1}{\frac{p \pm \tan 45}{1 \pm p\tan 45}}$

It is possible to use $\cot(\theta - 45) = \frac{\cos(\theta - 45)}{\sin(\theta - 45)}$ and an attempt to use the formulae for $\sin(A - B)$ and $\cos(A - B)$ with $A=\theta$, $B=45$. $\sin\theta = \frac{p}{\sqrt{1 \pm p^2}}$ and $\cos\theta = \frac{1}{\sqrt{1 \pm p^2}}$

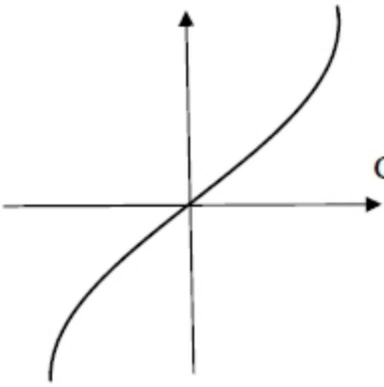
Sight of an expression $\frac{\frac{1}{\sqrt{1\pm p^2}}\cos 45 \pm \frac{p}{\sqrt{1\pm p^2}}\sin 45}{\frac{p}{\sqrt{1\pm p^2}}\cos 45 \pm \frac{1}{\sqrt{1\pm p^2}}\sin 45}$ is evidence.

A1 Uses $\tan 45 = 1$ or $\sin 45 = \cos 45 = \frac{\sqrt{2}}{2}$ *oe* and simplifies answer.

Accept $-\frac{1+p}{1-p}$ or $1 + \frac{2}{p-1}$

Note that there is no isw in any parts of this question.

Question 4

Question	Scheme	Marks
(a)		
	Correct position or curvature	M1
	Correct position **and** curvature	A1
		(2)
(b)	$3\arcsin(x+1)+\pi=0 \Rightarrow \arcsin(x+1)=-\dfrac{\pi}{3}$	M1
	$\Rightarrow (x+1)=\sin\left(-\dfrac{\pi}{3}\right)$	
	$\Rightarrow x=-1-\dfrac{\sqrt{3}}{2}$	dM1A1
		(3) **(5 marks)**

(a) Ignore any scales that appear on the axes

M1 Accept for the method mark
Either one of the two sections with correct curvature passing through (0,0),
Or both sections condoning dubious curvature passing through (0,0) (but do not accept any negative gradients)
Or a curve with a different range or an "extended range"
See the next page for a useful guide for clarification of this mark.

A1 A curve only in quadrants one and three passing through the point (0,0) with a gradient that is always positive. The gradient should appear to be approx ∞ at each end. If you are unsure use review
If range and domain are given then ignore.

(b)

M1 Substitutes $g(x+1)=\arcsin(x+1)$ in $3g(x+1)+\pi=0$ and attempts to make $\arcsin(x+1)$ the subject
Accept $\arcsin(x+1)=\pm\dfrac{\pi}{3}$ or even $g(x+1)=\pm\dfrac{\pi}{3}$. Condone $\dfrac{\pi}{3}$ in decimal form awrt1.047

dM1 Proceeds by evaluating $\sin\left(\pm\dfrac{\pi}{3}\right)$ and making x the subject.

Accept for this mark $\Rightarrow x=\pm\dfrac{\sqrt{3}}{2}\pm 1$. Accept decimal such as -1.866
Do not allow this mark if the candidate works in mixed modes (radians and degrees)
You may condone invisible brackets for both M's as long as the candidate is working correctly with the function

A1 $-1-\dfrac{\sqrt{3}}{2}$ oe with no other solutions. Remember to isw after a correct answer

Be careful with single fractions. $-\dfrac{2-\sqrt{3}}{2}$ and $\dfrac{-2+\sqrt{3}}{2}$ are incorrect but $-\dfrac{2+\sqrt{3}}{2}$ is correct

Note: It is possible for a candidate to change $\dfrac{\pi}{3}$ to $60°$ and work in degrees for all marks