Trigonometry 2 (Addition, Double Angle & R Formulae) - Edexcel Past Exam Questions MARK SCHEME

Question 1

Question No	Scheme	Marks
	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$	M1A1
	$= \frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$ $(\div cosAcosB)$	M1
	$= \frac{tanA + tanB}{1 - tanAtanB}$	A1 *
		(4)
	(b) $\tan\left(\theta+\frac{\pi}{6}\right) = \frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$	M1
	$= \frac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}$	M1
	$= \frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$	A1 *
		(3)
	(c) $\tan\left(\theta+\frac{\pi}{6}\right) = \tan(\pi-\theta)$.	M1
	$\left(\theta+\frac{\pi}{6}\right) = (\pi-\theta)$	dM1
	$\theta = \frac{5}{12}\pi$	ddM1 A1
	$\tan\left(\theta+\frac{\pi}{6}\right) = \tan(2\pi-\theta)$	dddM1
	$\theta = \frac{11}{12}\pi$	A1
		(6)
		(13 MARKS)

Question 2

Question Number	Scheme	Marks
	(a) $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$	B1 B1
		(2)
	(b) $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta} = \dfrac{\cancel{4}}{\cancel{4}\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1
	Using $1-\cos^2\theta = \sin^2\theta$ $\quad = \dfrac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}\cos^2\theta}$	M1
	$= \dfrac{1}{\cos^2\theta} = \sec^2\theta$	M1A1*
		(4)
	(c) $\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\dfrac{1}{2}$	M1
	$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$	A1,A1
		(3)
		(9 marks)

Note (a) and (b) can be scored together

(a) B1 One term correct. Eg. writes $4\operatorname{cosec}^2 2\theta$ as $\dfrac{4}{(2\sin\theta\cos\theta)^2}$ **or** $\operatorname{cosec}^2\theta$ as $\dfrac{1}{\sin^2\theta}$. Accept terms like $\operatorname{cosec}^2\theta = 1+\cot^2\theta = 1+\dfrac{\cos^2\theta}{\sin^2\theta}$. The question merely asks for an expression in $\sin\theta$ and $\cos\theta$

B1 A fully correct expression in $\sin\theta$ and $\cos\theta$. Eg. $\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$ **Accept equivalents**

Allow a different variable say x's instead of θ's but do not allow mixed units.

b) M1 Attempts to combine their expression in $\sin\theta$ and $\cos\theta$ using a common denominator. The terms can be separate but the denominator must be correct and one of the numerators must have been adapted

M1 Attempts to form a 'single' term on the numerator by using the identity $1-\cos^2\theta = \sin^2\theta$

M1 Cancels correctly by $\sin^2\theta$ terms and replaces $\dfrac{1}{\cos^2\theta}$ with $\sec^2\theta$

A1* Cso. This is a given answer. All aspects must be correct

IF IN ANY DOUBT SEND TO REVIEW OR CONSULT YOUR TEAM LEADER

c) M1 For $\sec^2\theta = 4$ leading to a solution of $\cos\theta$ by taking the root and inverting in either order.
Similarly accept $\tan^2\theta = 3, \sin^2\theta = \dfrac{3}{4}$ leading to solutions of $\tan\theta, \sin\theta$. Also accept $\cos 2\theta = -\dfrac{1}{2}$

A1 Obtains one correct answer usually $\theta = \dfrac{\pi}{3}$ Do not accept decimal answers or degrees

A1 Obtains both correct answers. $\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ Do not award if there are extra solutions inside the range.
Ignore solutions outside the range.

Question 3

Question Number	Scheme	Marks
	(a) $R = 25$	B1
	$\tan\alpha = \frac{24}{7} \Rightarrow \alpha = \text{(awrt)}73.7^0$	M1A1
		(3)
	(b) $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$	M1
	$2x + their\,'\alpha' = 60^0$	A1
	$2x + their\,'\alpha' = \text{their } 300^0 \text{ or their } 420^0 \Rightarrow x = ..$	M1
	$x = awrt\ 113.1^0, 173.1^0$	A1A1
		(5)
	(c) Attempts to use $\cos 2x = 2\cos^2 x - 1$ **AND** $\sin 2x = 2\sin x\cos x$ in the expression	M1
	$14\cos^2 x - 48\sin x\cos x = 7(\cos 2x + 1) - 24\sin 2x$	
	$= 7\cos 2x - 24\sin 2x + 7$	A1
		(2)
	(d) $14\cos^2 x - 48\sin x\cos x = R\cos(2x+\alpha) + 7$	
	Maximum value $= 'R' + 'c'$	M1
	$= 32$ cao	A1
		(2)
		(12 marks)

(a) B1 Accept 25, awrt 25.0, $\sqrt{625}$. Condone ± 25

M1 For $\tan\alpha = \pm\frac{24}{7}$ $\tan\alpha = \pm\frac{7}{24}$ $\sin\alpha = \pm\frac{24}{their\ R}$, $\cos\alpha = \pm\frac{7}{their\ R}$

A1 $\alpha = \text{(awrt)}73.7^0$. The answer 1.287 (radians) is A0

(b) M1 For using part (a) and dividing by their R to reach $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$

A1 Achieving $2x + their\ \alpha = 60^{(0)}$. This can be implied by $113.1^{(0)}/113.2^{(0)}$ or $173.1^{(0)}/173.2^{(0)}$ or $-6.8^{(0)}/-6.85^{(0)}/-6.9^{(0)}$

M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark

Look for $\frac{360 \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$

A1 $x = awrt\ 113.1^0/113.2^0$ OR $173.1^0/173.2^0$.

A1 $x = awrt\ 113.1^0$ AND 173.1^0. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

(c) M1 Attempts to use $\cos 2x = 2\cos^2 x - 1$ **and** $\sin 2x = 2\sin x\cos x$ in expression.

Allow slips in sign on the $\cos 2x$ term. So accept $2\cos^2 x = \pm\cos 2x \pm 1$

A1 Cao $= 7\cos 2x - 24\sin 2x + 7$. The order of terms is not important. Also accept a=7, b=-24, c=7

(d) M1 This mark is scored for adding their R to their c

A1 cao 32

Radian solutions- they will lose the first time it occurs (usually in a with 1.287 radians) Part b will then be marked as follows

(b) M1 For using part (a) and dividing by their R to reach $\cos(2x + their\,\alpha) = \frac{12.5}{their\ R}$

A1 The correct principal value of $\frac{\pi}{3}$ or awrt 1.05 radians. Accept $60^{(0)}$

This can be implied by awrt -0.12 radians or awrt or 1.97 radians or awrt 3.02 radians

M1 Finding a secondary value of x from their principal value. A correct answer will imply this mark

Look for $\frac{2\pi \pm \text{'their' principal value} \pm \text{'their' } \alpha}{2}$ Do not allow mixed units.

A1 $x = awrt\ 1.97$ OR 3.02.

A1 $x = awrt\ 1.97$ AND 3.02. Ignore solutions outside of range. Penalise this mark for extra solutions inside the range

Question 4

Question Number	Scheme	Marks
	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $\text{p}(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
	$\text{p}(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - \text{'}their\ \alpha\text{'} = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)

(a) M1 Using Pythagoras' Theorem with 6 and 8 to find R. Accept $R^2 = 6^2 + 8^2$
If α has been found first accept $R = \pm\frac{8}{\sin\text{'}\alpha\text{'}}$ or $R = \pm\frac{6}{\cos\text{'}\alpha\text{'}}$

A1 $R = 10$. Many candidates will just write this down which is fine for the 2 marks.
Accept ±10 but not -10

M1 For $\tan\alpha = \pm\frac{8}{6}$ or $\tan\alpha = \pm\frac{6}{8}$

If R is used then only accept $\sin\alpha = \pm\frac{8}{R}$ or $\cos\alpha = \pm\frac{6}{R}$

A1 $\alpha = \text{awrt } 0.927$. Note that 53.1° is A0

(b) Note that (b)(i) and (b)(ii) can be marked together

(i) M1 Award for $\text{p}(x) = \frac{4}{12 - \text{'}R\text{'}}$.

A1 Cao $\text{p}(x)_{\max} = 2$.
The answer is acceptable for both marks as long as no incorrect working is seen

(ii) M1 For setting $\theta - \text{'}their\ \alpha\text{'} = \pi$ and proceeding to θ=..
If working exclusively in degrees accept $\theta - \text{'}their\ \alpha\text{'} = 180$
Do not accept mixed units

A1 $\theta = \text{awrt } 4.07$. If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1

Question 5

Question Number	Scheme	Marks
	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=\sin 2x \Rightarrow 2\sin 22.5\cos 22.5=\sin 45$	M1
	$(\sin 22.5+\cos 22.5)^2=1+\sin 45$	A1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ cso	A1
		(5)
	(ii) (a) $\cos 2\theta+\sin\theta=1 \Rightarrow 1-2\sin^2\theta+\sin\theta=1$	M1
	$\sin\theta-2\sin^2\theta=0$	
	$2\sin^2\theta-\sin\theta=0$ or $k=2$	A1*
		(2)
	(b) $\sin\theta(2\sin\theta-1)=0$	M1
	$\sin\theta=0,\quad \sin\theta=\frac{1}{2}$	A1
	Any two of 0,30,150,180	B1
	All four answers 0,30,150,180	A1
		(4)
		(11 marks)

(i) M1 Attempts to expand $(\sin 22.5+\cos 22.5)^2$. Award if you see $\sin^2 22.5+\cos^2 22.5+......$
There must be > two terms. Condone missing brackets ie $\sin 22.5^2+\cos 22.5^2+......$

B1 Stating or using $\sin^2 22.5+\cos^2 22.5=1$. Accept $\sin 22.5^2+\cos 22.5^2=1$ as the intention is clear.
Note that this may also come from using the double angle formula
$\sin^2 22.5+\cos^2 22.5=(\frac{1-\cos 45}{2})+(\frac{1+\cos 45}{2})=1$

M1 Uses $2\sin x\cos x=\sin 2x$ to write $2\sin 22.5\cos 22.5$ as sin 45 or $\sin(2\times 22.5)$

A1 Reaching the intermediate answer $1+\sin 45$

A1 Cso $1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$. Be aware that both 1.707 and $\frac{2+\sqrt{2}}{2}$ can be found by using a calculator for 1+sin45. Neither can be accepted on their own without firstly seeing one of the two answers given above. **Each stage should be shown as required by the mark scheme.**
Note that if the candidates use $(\sin\theta+\cos\theta)^2$ they can pick up the first M and B marks, but no others until they use $\theta=22.5$. All other marks then become available.

(iia) M1 Substitutes $\cos 2\theta=1-2\sin^2\theta$ in $\cos 2\theta+\sin\theta=1$ to produce an equation in $\sin\theta$ only.
It is acceptable to use $\cos 2\theta=2\cos^2\theta-1$ *or* $\cos^2\theta-\sin^2\theta$ as long as the $\cos^2\theta$ is subsequently replaced by $1-\sin^2\theta$

A1* Obtains the correct simplified equation in $\sin\theta$.
$\sin\theta-2\sin^2\theta=0$ or $\sin\theta=2\sin^2\theta$ must be written in the form $2\sin^2\theta-\sin\theta=0$ as required by the question. Also accept $k=2$ as long as no incorrect working is seen.

(iib) M1 Factorises or divides by $\sin\theta$. For this mark $1='k'\sin\theta$ is acceptable. If they have a 3 TQ in $\sin\theta$ this can be scored for correct factorisation

A1 **Both** $\sin\theta=0$, and $\sin\theta=\frac{1}{2}$

B1 Any two answers from 0, 30, 150, 180.

A1 All four answers 0, 30, 150, 180 with no extra solutions inside the range. Ignore solutions outside the range.

Question Number	Scheme	Marks
alt 1	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=2\sqrt{\dfrac{1-\cos 2x}{2}}\sqrt{\dfrac{\cos 2x+1}{2}}\Rightarrow\sqrt{1-\cos 45}\sqrt{1+\cos 45}$	M1
	$=\sqrt{1-\cos^2 45}$	A1
	Hence $(\sin 22.5+\cos 22.5)^2=1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
alt 2	(i) Uses Factor Formula $(\sin 22.5+\sin 67.5)^2=\left(2\sin 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
alt 3	(i) Uses Factor Formula $(\cos 67.5+\cos 22.5)^2=\left(2\cos 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\dfrac{\sqrt{2}}{2}$ or $1+\dfrac{1}{\sqrt{2}}$	A1
		(5)

Question 6

Question Number	Scheme	Marks
(a)	$7\cos x + \sin x = R\cos(x-\alpha)$	
	$R = \sqrt{(7^2+1^2)} = \sqrt{50} = (5\sqrt{2})$	B1
	$\alpha = \arctan\left(\frac{1}{7}\right) = 8.13... = \text{awrt } 8.1^0$	M1A1
		(3)
(b)	$\sqrt{50}\cos(x-8.1) = 5 \Rightarrow \cos(x-8.1) = \frac{5}{\sqrt{50}}$	M1
	$x - 8.1 = 45 \Rightarrow x = 53.1^0$	M1,A1
	AND $x - 8.1 = 315 \Rightarrow x = 323.1^0$	M1A1
		(5)
(c)	One solution if $\frac{k}{\sqrt{50}} = \pm 1, \Rightarrow k = \pm\sqrt{50}$ ft on R	M1A1ft
		(2)
		(10 marks)

Notes for Question

(a)

B1 $R=\sqrt{50}$. Accept $5\sqrt{2}$ Accept $R=\pm\sqrt{50}$

Do not accept $R=\sqrt{(7^2+1^2)}$ or the decimal equivalent 7.07…unless you see $\sqrt{50}$ or $5\sqrt{2}$ as well

M1 For $\tan\alpha=\pm\frac{1}{7}$ or $\tan\alpha=\pm\frac{7}{1}$. Condone if this comes from $\cos\alpha=7, \sin\alpha=1$

If R is used then only accept $\sin\alpha=\pm\frac{1}{R}$ or $\cos\alpha=\pm\frac{7}{R}$

A1 $\alpha=$ awrt 8.1.

Be aware that $\tan\alpha=7\Rightarrow\alpha=81.9$ can easily be mistaken for the correct answer

Note that the radian answer awrt 0.1418… is A0

(b)

M1 For using their answers to part (a) and moving from $R\cos(x\pm\alpha)=5\Rightarrow\cos(x\pm\alpha)=\frac{5}{R}$ using their numerical values of R and α

This may be implied for sight of 53.1 if R and α were correct

M1 For achieving $x\pm\alpha=$ awrt 45^0 or 315, leading to one value of x **in the range**

Note that for this to be scored R has to be correct (to 2sf) as awrt 45, 315 must be achieved

This may be implied for achieving an answer of either $45+\textit{their }\alpha$ or $315+\textit{their }\alpha$

A1 One correct answer, either awrt 53.1^0 or 323.1^0

M1 For an attempt at finding a secondary value of x **in the range**.

Usually this is an attempt at solving $x-\textit{their }8.1^0=360^0-\textit{their }45^0\Rightarrow x=..$

A1 Both values correct awrt 53.1^0 and 323.1^0.

Withhold this mark if there are extra values in the range.

Ignore extra values outside the range

(c)

M1 For stating that $\frac{k}{\textit{their }R}=1$ OR $\frac{k}{\textit{their }R}=-1$

This may be implied by seeing $k=(\pm)\textit{their }R$

A1ft Both values $k=\pm\sqrt{50}$ oe . Follow through on their numerical R

Answers all in radians. Lose the first time that it appears but demand an accuracy of 2dp.

Part (a) $R=\sqrt{50}$ $\alpha=awrt\ 0.14$

Part (b) $x=awrt\ 0.927,\ 5.64$. Accuracy must be to 3 sf.

With correct working this would score (a) B1M1A0 (b) M1A1A1M1A1

Mixed degrees and radians refer to the main scheme

Question 7

Question Number	Scheme	Marks
(i)	$\text{cosec}2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x \cos x}$	M1
	$= \dfrac{1}{2}\text{cosec}x \sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$	A1
		(3)
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1
		(6)
		(9 marks)
ALT (ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\times\dfrac{1}{\cos^2\theta} + 3\times\dfrac{1}{\cos\theta} = 2\times\dfrac{\sin^2\theta}{\cos^2\theta}$	
	$3 + 3\cos\theta = 2\sin^2\theta$	
	$3 + 3\cos\theta = 2(1 - \cos^2\theta)$	M1
	$2\cos^2\theta + 3\cos\theta + 1 = 0$	
	$(2\cos\theta + 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = -0.5, -1$	M1A1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	M1,A1,A1
		(6)
		(9 marks)

Notes for Question

(i)

M1 Uses the identity $\text{cosec}2x = \frac{1}{\sin 2x}$

M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ in their expression.
Accept $\sin 2x = \sin x\cos x + \cos x\sin x$

A1 $\lambda = \frac{1}{2}$ following correct working

(ii)

M1 Replaces $\tan^2\theta$ by $\pm\sec^2\theta \pm 1$ to produce an equation in just $\sec\theta$

M1 Award for a forming a 3TQ=0 in $\sec\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\sec\theta$
If they replace $\sec\theta = \frac{1}{\cos\theta}$ it is for forming a 3TQ in $\cos\theta$ and applying a correct method for finding two answers to $\cos\theta$

A1 Correct answers to $\sec\theta = -2, -1$ or $\cos\theta = -\frac{1}{2}, -1$

M1 Award for using the identity $\sec\theta = \frac{1}{\cos\theta}$ and proceeding to find at least one value for θ.
If the 3TQ was in cosine then it is for finding at least one value of θ.

A1 Two correct values of θ. All method marks must have been scored.
Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.
Accept $0.\dot{6}\pi, 1.\dot{3}\pi, \pi$
Withhold this mark if further values in the range are given. All method marks must have been scored.
Ignore any answers outside the range.

Alt (ii)

M1 Award for replacing $\sec^2\theta$ with $\frac{1}{\cos^2\theta}$, $\sec\theta$ with $\frac{1}{\cos\theta}$, $\tan^2\theta$ with $\frac{\sin^2\theta}{\cos^2\theta}$ multiplying through by $\cos^2\theta$ (seen in at least 2 terms) and replacing $\sin^2\theta$ with $\pm 1 \pm \cos^2\theta$ to produce an equation in just $\cos\theta$

M1 Award for a forming a 3TQ=0 in $\cos\theta$ and applying a correct method for factorising, or using the formula, or completing the square to find two answers to $\cos\theta$

A1 $\cos\theta = -\frac{1}{2}, -1$

M1 Proceeding to finding at least one value of θ from an equation in $\cos\theta$.

A1 Two correct values of θ. All method marks must have been scored
Accept two of $120°, 180°, 240°$ or two of $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$ or two of awrt 2dp 2.09, 3.14, 4.19

A1 All three answers correct. They must be given in terms of π as stated in the question.

Question 8

Question Number	Scheme	Marks
(a)	$\text{cosec}2x + \cot 2x = \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$	M1
	$= \dfrac{1+\cos 2x}{\sin 2x}$	M1
	$= \dfrac{1+2\cos^2 x - 1}{2\sin x \cos x}$	
	$= \dfrac{2\cos^2 x}{2\sin x \cos x}$	M1 A1
	$= \dfrac{\cos x}{\sin x} = \cot x$	A1*
		(5)
(b)	$\text{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$	
	$\cot(2\theta \pm ..°) = \sqrt{3}$	M1
	$2\theta \pm ... = 30° \Rightarrow \theta = 12.5°$	dM1, A1
	$2\theta \pm ... = 180 + PV° \Rightarrow \theta = ..°$	dM1
	$\theta = 102.5°$	A1
		(5)
		(10 marks)

(a)

M1 Writing $\text{cosec}2x = \frac{1}{\sin 2x}$ **and** $\cot 2x = \frac{\cos 2x}{\sin 2x}$ *or* $\frac{1}{\tan 2x}$

M1 Writing the lhs as a single fraction $\frac{a+b}{c}$. The denominator must be correct for their terms.

M1 Uses the appropriate double angle formulae/trig identities to produce a fraction in a form containing no addition or subtraction signs. A form $\frac{p \times q}{s \times t}$ or similar

A1 A correct intermediate line. Accept $\frac{2\cos^2 x}{2\sin x \cos x}$ or $\frac{2\sin x \cos x}{2\sin x \cos x \tan x}$ or similar

This cannot be scored if errors have been made

A1* Completes the proof by cancelling and using either $\frac{\cos x}{\sin x} = \cot x$ or $\frac{1}{\tan x} = \cot x$

The cancelling could be implied by seeing $\frac{2}{2}\frac{\cos x}{\sin x}\frac{\cos x}{\cos x} = \cot x$

The proof cannot rely on expressions like $\cot = \frac{\cos}{\sin}$ (with missing x's) for the final A1

(b)

M1 Attempt to use the solution to part (a) with $2x = 4\theta + 10 \Rightarrow$ to write or imply $\cot(2\theta \pm ...°) = \sqrt{3}$

Watch for attempts which start $\cot \alpha = \sqrt{3}$. The method mark here is not scored until the α has been replaced by $2\theta \pm ...°$

Accept a solution from $\cot(2x \pm ...°) = \sqrt{3}$ where θ has been replaced by another variable.

dM1 Proceeds from the previous method and uses $\tan .. = \frac{1}{\cot ..}$ and

$\arctan\left(\frac{1}{\sqrt{3}}\right) = 30°$ to solve $2\theta \pm ...° = 30° \Rightarrow \theta = ..$

A1 $\theta = 12.5°$ or exact equivalent. Condone answers such as $x = 12.5°$

dM1 This mark is for the correct method to find a second solution to θ. It is dependent upon the first M only.
Accept $2\theta \pm ... = 180 + PV° \Rightarrow \theta = ..°$

A1 $\theta = 102.5°$ or exact equivalent. Condone answers such as $x = 102.5°$
Ignore any solutions outside the range. This mark is withheld for any extra solutions within the range.
If radians appear they could just lose the answer marks. So for example

$2\theta \pm ... = \frac{\pi}{6}(0.524) \Rightarrow \theta = ..$is M1dM1A0 followed by

$2\theta \pm ... = \pi + '\frac{\pi}{6}' \Rightarrow \theta = ..$dM1A0

Special case 1: For candidates in (b) who solve $\cot(4\theta \pm ...°) = \sqrt{3}$ the mark scheme is severe, so we are awarding a special case solution, scoring 00011.

$\cot(4\theta + \beta°) = \sqrt{3} \Rightarrow 4\theta + \beta = 30° \Rightarrow \theta = ..$ is M0M0A0 where $\beta = 5°$ or $10°$

$\Rightarrow 4\theta + \beta = 210° \Rightarrow \theta = ..$ can score M1A1 Special case.

If $\beta = 5°,\ \theta = 51.25$ If $\beta = 10°,\ \theta = 50$

Special case 2: Just answers in (b) **with no working** scores 1 1 0 0 0 for 12.5 and 102.5
BUT $\cot(2\theta \pm 5°) = \sqrt{3} \Rightarrow \theta = 12.5°, 102.5°$ scores all available marks.

Question Number	Scheme	Marks
(a)Alt 1	$\operatorname{cosec}2x+\cot 2x=\dfrac{1}{\sin 2x}+\dfrac{1}{\tan 2x}$	1^{ST} M1
	$=\dfrac{1}{2\sin x\cos x}+\dfrac{1-\tan^2 x}{2\tan x}$	
	$=\dfrac{\tan x+(1-\tan^2 x)\sin x\cos x}{2\sin x\cos x\tan x}$ or $=\dfrac{2\tan x+2(1-\tan^2 x)\sin x\cos x}{4\sin x\cos x\tan x}$	2^{nd} M1
	$=\dfrac{\tan x+\sin x\cos x-\tan^2 x\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\dfrac{\tan x+\sin x\cos x-\tan x\sin^2 x}{2\sin x\cos x\tan x}$	
	$=\dfrac{\tan x(1-\sin^2 x)+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\dfrac{\tan x\cos^2 x+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\dfrac{\sin x\cos x+\sin x\cos x}{2\sin x\cos x\tan x}$	
	$=\dfrac{2\sin x\cos x}{2\sin x\cos x\tan x}$ oe	3^{rd} M1A1
	$=\dfrac{1}{\tan x}=\cot x$	A1* (5)
(a)Alt 2	**Example of how main scheme could work in a roundabout route**	
	$\operatorname{cosec}2x+\cot 2x=\cot x\Leftrightarrow\dfrac{1}{\sin 2x}+\dfrac{1}{\tan 2x}=\dfrac{1}{\tan x}$	1^{st} M1
	$\Leftrightarrow \tan 2x\tan x+\sin 2x\tan x=\sin 2x\tan 2x$	2^{nd} M1
	$\Leftrightarrow\dfrac{2\tan x}{1-\tan^2 x}\times\tan x+2\sin x\cancel{\cos x}\times\dfrac{\sin x}{\cancel{\cos x}}=2\sin x\cos x\times\dfrac{2\tan x}{1-\tan^2 x}$	
	$\Leftrightarrow\dfrac{2\tan^2 x}{1-\tan^2 x}+2\sin^2 x=\dfrac{4\sin^2 x}{1-\tan^2 x}$	
	$\times(1-\tan^2 x)\Leftrightarrow 2\tan^2 x+2\sin^2 x(1-\tan^2 x)=4\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x-2\sin^2 x\tan^2 x=2\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x(1-\sin^2 x)=2\sin^2 x$	3^{rd} M1
	$\div 2\tan^2 x\Leftrightarrow 1-\sin^2 x=\cos^2 x$	A1
	As this is true, initial statement is true	A1*
		(5)

Question 9

Question Number	Scheme	Marks
(a)	$R = \sqrt{20}$	B1
	$\tan\alpha = \frac{4}{2} \Rightarrow \alpha = \text{awrt } 1.107$	M1A1
		(3)
(b)(i)	$'4 + 5R^2' = 104$	B1ft
(ii)	$3\theta - '1.107' = \frac{\pi}{2} \Rightarrow \theta = \text{awrt } 0.89$	M1A1
		(3)
(c)(i)	4	B1
(ii)	$3\theta - '1.107' = 2\pi \Rightarrow \theta = \text{awrt } 2.46$	M1A1
		(3)
		(9 marks)

(a)

B1 Accept $R=\sqrt{20}$ or $2\sqrt{5}$ or awrt 4.47

Do not accept $R=\pm\sqrt{20}$

This could be scored in parts (b) or (c) as long as you are certain it is R

M1 for sight of $\tan\alpha=\pm\frac{4}{2}, \tan\alpha=\pm\frac{2}{4}$. Condone $\sin\alpha=4, \cos\alpha=2 \Rightarrow \tan\alpha=\frac{4}{2}$

If R is found first only accept $\sin\alpha=\pm\frac{4}{R}, \cos\alpha=\pm\frac{2}{R}$

A1 $\alpha=$ awrt 1.107. The degrees equivalent $63.4°$ is A0.

If a candidate does all the question in degrees they will lose just this mark.

(b)(i)

B1ft Either 104 or if R was incorrect allow for the numerical value of their $'4+5R^2'$.

Allow a tolerance of 1 dp on decimal R's.

(b)(ii)

M1 Using $3\theta \pm$ their $'1.107' = \frac{\pi}{2} \Rightarrow \theta = ..$

Accept $3\theta \pm$ their $'1.107' = (2n+1)\frac{\pi}{2} \Rightarrow \theta = ..$ where n is an integer

Allow slips on the lhs with an extra bracket such as

$3(\theta \pm \text{their}'1.107') = \frac{\pi}{2} \Rightarrow \theta = ..$

The degree equivalent is acceptable $3\theta -$ their $'63.4°' = 90° \Rightarrow \theta =$

Do not allow mixed units in this question

A1 awrt 0.89 radians or $51.1°$. Do not allow multiple solutions for this mark.

(c)(i)

B1 4

(c)(ii)

M1 Using $3\theta \pm$ their $'1.107' = 2\pi \Rightarrow \theta = ...$

Accept $3\theta \pm$ their $'1.107' = n\pi \Rightarrow \theta = ..$ where n is an integer, including 0

Allow slips on the lhs with an extra bracket such as

$3(\theta \pm \text{their}'1.107') = 2\pi \Rightarrow \theta = ..$

The degree equivalent is acceptable $3\theta -$ their $'63.4°' = 360° \Rightarrow \theta =$ but

Do not allow mixed units in this question

A1 $\theta =$ awrt 2.46 radians or $141.1°$ Do not allow multiple solutions for this mark.

Question 10

Question Number	Scheme	Marks
(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57) = +77.1.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 51.8°$	A1
	$2\theta - 26.57 = '-77.1...' \Rightarrow \theta = -\text{awrt } 25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)

You can marks parts (a) and (b) together as one.

(a)

B1 For $R = \sqrt{20} = 2\sqrt{5}$. Condone $R = \pm\sqrt{20}$

M1 For $\alpha = \arctan\left(\pm\frac{1}{2}\right)$ or $\alpha = \arctan(\pm 2)$ leading to a solution of α

Condone any solutions coming from $\cos\alpha = 4, \sin\alpha = 2$

Condone for this mark $2\alpha = \arctan\left(\pm\frac{1}{2}\right) \Rightarrow \alpha = ..$

If R has been used to find α award for only $\alpha = \arccos\left(\pm\frac{4}{'R'}\right)$ $\alpha = \arcsin\left(\pm\frac{2}{'R'}\right)$

A1 $\alpha = \text{awrt } 26.57°$

(b)

M1 Using part (a) and proceeding as far as $\cos(2\theta \pm \text{their } 26.57) = \dfrac{1}{\text{their } R}$.

This may be implied by $(2\theta \pm \text{their } 26.57) = \arccos\left(\dfrac{1}{theirR}\right)$

Allow this mark for $\cos(\theta \pm \text{their } 26.57) = \dfrac{1}{\text{their } R}$

dM1 Dependent upon the first M1- it is for a correct method to find θ from their principal value
Look for the correct order of operations, that is dealing with the "26.57" before the "2".
Condone subtracting 26.57 instead of adding.

$\cos(2\theta \pm \text{their } 26.57) = \ldots \Rightarrow 2\theta \pm \text{their } 26.57 = \beta \Rightarrow \theta = \dfrac{\beta \pm \text{their } 26.57}{2}$

A1 awrt $\theta = 51.8°$

ddM1 For a correct method to find a secondary value of θ in the range
Either $2\theta \pm 26.57 = '-\beta' \Rightarrow \theta =$ OR $2\theta \pm 26.57 = 360 - '\beta' \Rightarrow \theta =$ THEN MINUS 180

A1 awrt $\theta = -25.3°$
Withhold this mark if there are extra solutions in the range.

Radian solution: Only lose the first time it occurs.
FYI. In radians desired accuracy is awrt 2 dp (a) $\alpha = 0.46$ and (b) $\theta_1 = 0.90, \theta_2 = -0.44$
Mixing degrees and radians only scores the first M

(c)

B1ft Follow through on their R. Accept decimals here including $\sqrt{20} \approx$ awrt 4.5 .

Score for one of the ends $k > \sqrt{20}$, $k < -\sqrt{20}$

Condone versions such as $g(\theta) > \sqrt{20}$, $y > \sqrt{20}$

or both ends including the boundaries $k \geqslant \sqrt{20}$, $k \leqslant -\sqrt{20}$

B1 ft For both intervals in terms of k.

Accept $k > \sqrt{20}$ or $k < -\sqrt{20}$. Accept $|k| > \sqrt{20}$ Accept $k \in (\sqrt{20}, \infty)(-\infty, -\sqrt{20})$

Condone $k > \sqrt{20}, k < -\sqrt{20}$ $k > \sqrt{20}$ and $k < -\sqrt{20}$ for both marks

but $-\sqrt{20} > k > \sqrt{20}$ is B1 B0

Question 11

Question Number	Scheme	Marks
(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A+\sin A)(\cos A+\sin A)}{(\cos A+\sin A)(\cos A-\sin A)}$	M1
	$= \dfrac{\cos A+\sin A}{\cos A-\sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta = awrt\ 2.820, 5.961$	dM1A1
		(4)
		(9 marks)

(a)

B1 A correct identity for $\sec 2A = \dfrac{1}{\cos 2A}$ **OR** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

M1 For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\dfrac{1+\cos 2A\tan 2A}{\cos 2A}$ or $\dfrac{1+\sin 2A}{\cos 2A}$

M1 For getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities

$\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1 - 2\sin^2 A$.

Alternatively for getting an expression in just $\sin A$ and $\cos A$ by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ with $\tan A = \dfrac{\sin A}{\cos A}$.

For example $= \dfrac{1}{\cos^2 A - \sin^2 A} + \dfrac{2\sin A/\cos A}{1-\sin^2 A/\cos^2 A}$ is B1M0M1 so far

M1 In the main scheme it is for replacing 1 by $\cos^2 A + \sin^2 A$ **and** factorising both numerator and denominator

A1* Cancelling to produce given answer with no errors.
Allow a consistent use of another variable such as θ, but mixing up variables will lose the A1*.

(b)
M1 For using part (a), cross multiplying, dividing by $\cos\theta$ to reach $\tan\theta = k$
Condone $\tan 2\theta = k$ for this mark only

A1 $\tan\theta = -\frac{1}{3}$

dM1 Scored for $\tan\theta = k$ leading to at least one value (with 1 dp accuracy) for θ between 0 and 2π. You may have to use a calculator to check. Allow answers in degrees for this mark.

A1 $\theta =$ awrt $2.820, 5.961$ with no extra solutions within the range. Condone 2.82 for 2.820.

You may condone different/ mixed variables in part (b)

There are some long winded methods. Eg. M1, dM1 applied as in main scheme

$$\Rightarrow (2\cos\theta + 2\sin\theta)^2 = (\cos\theta - \sin\theta)^2 \Rightarrow 4 + 4\sin 2\theta = 1 - \sin 2\theta$$

$$\Rightarrow \sin 2\theta = -\frac{3}{5} \text{ is M1 (for} \sin 2\theta = k\text{) A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ for dM1 (for } \theta = \frac{\arcsin k}{2}\text{) A1}$$

$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\cos(\theta - 1.25) = 0 \text{ M1 for..} \cos(\theta - \alpha) = 0, \alpha = \arctan\left(\pm\frac{3}{1} \text{ or } \pm\frac{1}{3}\right) \text{) A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta + 3\sin\theta = 0 \Rightarrow (\sqrt{10})\sin(\theta + 0.32) = 0 \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{10} \Rightarrow \sin\theta = (\pm)\sqrt{\frac{1}{10}} \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

$$\cos\theta = -3\sin\theta \Rightarrow \cos^2\theta = 9\sin^2\theta \Rightarrow \cos^2\theta = \frac{9}{10} \Rightarrow \cos\theta = (\pm)\sqrt{\frac{9}{10}} \text{ M1 A1}$$

$$\Rightarrow \theta = 2.820, 5.961 \text{ dM1 A1}$$

Question Number	Scheme	Marks
Alt I **From RHS**	$\frac{\cos A+\sin A}{\cos A-\sin A} = \frac{\cos A+\sin A}{\cos A-\sin A}\times\frac{\cos A+\sin A}{\cos A+\sin A}$ $=\frac{\cos^2 A+\sin^2 A+2\sin A\cos A}{\cos^2 A-\sin^2 A}$ $=\frac{1+\sin 2A}{\cos 2A}$ $=\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}$ $=\sec 2A+\tan 2A$	(Pythagoras) **M1** (Double Angle) **M1** (Single Fraction) **M1** **B1**(Identity), **A1***
Alt II **Both sides**	Assume true $\sec 2A+\tan 2A=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1}{\cos 2A}+\frac{\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+\sin 2A}{\cos 2A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\frac{1+2\sin A\cos A}{\cos^2 A-\sin^2 A}=\frac{\cos A+\sin A}{\cos A-\sin A}$ $\times(\cos A-\sin A)\Rightarrow\frac{1+2\sin A\cos A}{\cos A+\sin A}=\cos A+\sin A$ $1+2\sin A\cos A=\cos^2 A+2\sin A\cos A+\sin^2 A=1+2\sin A\cos A$ **True**	B1 (identity) M1 (single fraction) M1(double angles) M1(Pythagoras)A1*
Alt 111 **Very difficult**	$\sec 2A+\tan 2A=\frac{1}{\cos 2A}+\tan 2A$ $=\frac{1}{\cos 2A}+\frac{2\tan A}{1-\tan^2 A}$ $=\frac{1-\tan^2 A+2\tan A\cos 2A}{\cos 2A(1-\tan^2 A)}$ $=\frac{1-\tan^2 A+2\tan A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(1-\tan^2 A)}$ $=\frac{1-\frac{\sin^2 A}{\cos^2 A}+2\frac{\sin A}{\cos A}(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)\left(1-\frac{\sin^2 A}{\cos^2 A}\right)}$ $\times\cos^2 A=\frac{\cos^2 A-\sin^2 A+2\sin A\cos A(\cos^2 A-\sin^2 A)}{(\cos^2 A-\sin^2 A)(\cos^2 A-\sin^2 A)}$ $=\frac{\cancel{(\cos^2 A-\sin^2 A)}(1+2\sin A\cos A)}{\cancel{(\cos^2 A-\sin^2 A)}(\cos^2 A-\sin^2 A)}$ Final two marks as in main scheme	(Identity) **B1** (Single fraction) **M1** (Double Angle and in just sin and cos) **M1** **M1A1***

Question 12

Question	Scheme	Marks
(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(awrt\ 0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$	A1
	$\theta+26.6°=360°-\text{their}\ '59.5°'$	dM1
	$\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$	A1
		(5)
(c)	$\theta-\text{their}\ 26.57°=\text{their}\ 59.54°\Rightarrow\theta=...$	M1
	$\theta=\text{awrt}\ 86.1°$	A1
		(2)
		(10 marks)

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\frac{1}{2}$, $\tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$

If their value of R is used to find the value of α only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=\text{awrt}\ 26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their}\ 26.6°)=K$, $|K|\leq 1$

A1 $\cos(\theta\pm\text{their}\ 26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt}\ 0.507)$. Can be implied by $(\theta\pm\text{their}\ 26.6°)=\text{awrt}\ 59.5°/59.6°$

A1 One solution correct, $\theta=awrt\ 33.0°$ or $\theta=awrt\ 273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm\text{their}\ 26.6°=360°-\text{their}\ 59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=awrt\ 33.0°$ and $awrt\ 273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\leq\theta<360°$

(c)

M1 $\theta-\text{their}\ 26.57°=\text{their}\ 59.54°\Rightarrow\theta=...$
Alternatively $-\theta+\text{their}\ 26.6°=-\text{their}\ 59.5°\Rightarrow\theta=...$
If the candidate has an incorrect sign for α, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+\text{their}\ 26.57°=\text{their}\ 59.54°\Rightarrow\theta=...$

A1 awrt $86.1°$ ONLY. Allow both marks following a correct (a) and (b)
They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b)

Answers in radians: Withhold only one A mark, the first time a solution in radians appears
FYI (a) $\alpha=0.46$ (b) $\theta_1=\text{awrt}\ 0.58$ and $\theta_2=\text{awrt}\ 4.78$ (c) $\theta_3=\text{awrt}\ 1.50$. Require 2 dp accuracy

Question 13

Question	Scheme	Marks
(a)	$2\cot 2x + \tan x \equiv \dfrac{2}{\tan 2x} + \tan x$	B1
	$\equiv \dfrac{(1-\tan^2 x)}{\tan x} + \dfrac{\tan^2 x}{\tan x}$	M1
	$\equiv \dfrac{1}{\tan x}$	M1
	$\equiv \cot x$	A1*
		(4)
(b)	$6\cot 2x + 3\tan x = \mathrm{cosec}^2 x - 2 \Rightarrow 3\cot x = \mathrm{cosec}^2 x - 2$	
	$\Rightarrow 3\cot x = 1 + \cot^2 x - 2$	M1
	$\Rightarrow 0 = \cot^2 x - 3\cot x - 1$	A1
	$\Rightarrow \cot x = \dfrac{3 \pm \sqrt{13}}{2}$	M1
	$\Rightarrow \tan x = \dfrac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)
		(10 marks)
(a)alt 1	$2\cot 2x + \tan x \equiv \dfrac{2\cos 2x}{\sin 2x} + \tan x$	B1
	$\equiv 2\dfrac{\cos^2 x - \sin^2 x}{2\sin x \cos x} + \dfrac{\sin x}{\cos x}$	M1
	$\equiv \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} + \dfrac{\sin^2 x}{\sin x \cos x} \equiv \dfrac{\cos^2 x}{\sin x \cos x}$	M1
	$\equiv \dfrac{\cos x}{\sin x}$	
	$\equiv \cot x$	A1*
(a)alt 2	$2\cot 2x + \tan x \equiv 2\dfrac{(1-\tan^2 x)}{2\tan x} + \tan x$	B1M1
	$\equiv \dfrac{2}{2\tan x} - \dfrac{2\tan^2 x}{2\tan x} + \tan x$ or $\dfrac{(1-\tan^2 x) + \tan^2 x}{\tan x}$	
	$\equiv \dfrac{2}{2\tan x} = \cot x$	M1A1*

Alt (b)	$6\cot 2x + 3\tan x = \mathrm{cosec}^2 x - 2 \Rightarrow \dfrac{3\cos x}{\sin x} = \dfrac{1}{\sin^2 x} - 2$	
	$(\times \sin^2 x) \Rightarrow 3\sin x \cos x = 1 - 2\sin^2 x$	M1
	$\Rightarrow \dfrac{3}{2}\sin 2x = \cos 2x$	M1A1
	$\Rightarrow \tan 2x = \dfrac{2}{3} \Rightarrow x = ..$	M1
	$\Rightarrow x = 0.294, -2.848, -1.277, 1.865$	A2,1,0
		(6)

(a)

B1 States or uses the identity $2\cot 2x = \frac{2}{\tan 2x}$ or alternatively $2\cot 2x = \frac{2\cos 2x}{\sin 2x}$

This may be implied by $2\cot 2x = \frac{1-\tan^2 x}{\tan x}$. Note $2\cot 2x = \frac{1}{2\tan 2x}$ is B0

M1 Uses the correct double angle identity $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

Alternatively uses $\sin 2x = 2\sin x\cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ oe and $\tan x = \frac{\sin x}{\cos x}$

M1 Writes their two terms with a single common denominator and simplifies to a form $\frac{ab}{cd}$.

For this to be scored the expression must be in either $\sin x$ and $\cos x$ or just $\tan x$.

In alternative 2 it is for splitting the complex fraction into parts and simplifying to a form $\frac{ab}{cd}$.

You are awarding this for a correct method to proceed to terms like $\frac{\cos^2 x}{\sin x\cos x}$, $\frac{2\cos^3 x}{2\sin x\cos^2 x}$, $\frac{2}{2\tan x}$

A1* cso. For proceeding to the correct answer. This is a given answer and all aspects must be correct including the consistent use of variables. If the candidate approaches from both sides there must be a conclusion for this mark to be awarded. Occasionally you may see a candidate attempting to prove $\cot x - \tan x \equiv 2\cot 2x$. This is fine but again there needs to be a conclusion for the A1*
If you are unsure of how some items should be marked then please use review

(b)

M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

WITH an attempt at using $\operatorname{cosec}^2 x = \pm 1 \pm \cot^2 x$ to produce a quadratic equation in just $\cot x / \tan x$

A1 $\cot^2 x - 3\cot x - 1 = 0$ The $= 0$ may be implied by subsequent working

Alternatively accept $\tan^2 x + 3\tan x - 1 = 0$

M1 Solves a 3TQ=0 in $\cot x$ (or tan) using the formula or any suitable method for their quadratic to find at least one solution. Accept answers written down from a calculator. You may have to check these from an incorrect quadratic. FYI answers are $\cot x =$ awrt 3.30, -0.30

Be aware that $\cot x = \frac{3 \pm \sqrt{13}}{2} \Rightarrow \tan x = \frac{-3 \pm \sqrt{13}}{2}$

M1 For $\tan x = \frac{1}{\cot x}$ and using arctan producing at least one answer for x in degrees or radians.

You may have to check these with your calculator.

A1 Two of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3dp) in radians or degrees.

In degrees the answers you would accept are (awrt 2dp) $x = 16.8°, 106.8°, -73.2°, -163.2°$

A1 All four of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3 dp) with no extra solutions in the range $-\pi$ „ xx π

See main scheme for Alt to (b) using Double Angle formulae still entered M A M M A A in epen

1st M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

then using $\cot x = \frac{\cos x}{\sin x}$, $\operatorname{cosec}^2 x = \frac{1}{\sin^2 x}$ and $\times \sin^2 x$ to form an equation sin and cos

1st A1 For $\frac{3}{2}\sin 2x = \cos 2x$ or equivalent. **Attached to the next M**

2nd M1 For using both correct double angle formula

3rd M1 For moving from $\tan 2x = C$ to x=..using the correct order of operations.

Question 14

Question Number	Scheme	Marks
(a)	$R=\sqrt{29}$	B1
	$\tan\alpha=\frac{2}{5}\Rightarrow\alpha=\text{awrt } 0.381$	M1A1
		(3)
(b)	$5\cot 2x-3\operatorname{cosec}2x=2\Rightarrow 5\frac{\cos 2x}{\sin 2x}-\frac{3}{\sin 2x}=2$	M1
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1
		(2)
(c)	$5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\frac{3}{\sqrt{29}}$	M1
	$2x+0.381=\arccos\left(\frac{3}{\sqrt{29}}\right)\Rightarrow x=\ldots$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)
		(9 marks)
Alt I (c)	$5\cos 2x-2\sin 2x=3\Rightarrow 10\cos^2 x-5-4\sin x\cos x=3$	
	$\Rightarrow 4\tan^2 x+2\tan x-1=0$	M1
	$\Rightarrow\tan x=\frac{-1\pm\sqrt{5}}{4}\Rightarrow x=..$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)
Alt II (c)	$5\cos 2x-2\sin 2x=3\Rightarrow(5\cos 2x)^2=(3+2\sin 2x)^2$ & $\cos^2 2x=1-\sin^2 2x$	
	$\Rightarrow 29\sin^2 2x+12\sin 2x-16=0$	M1
	$\Rightarrow\sin 2x=\frac{-12\pm\sqrt{2000}}{58}\Rightarrow 2x=..\Rightarrow x=..$	dM1
	$x=\text{awrt } 0.30,\ 2.46$	A1A1
		(4)

(a)

B1 $R=\sqrt{29}$

Condone $R=\pm\sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$)

M1 $\tan\alpha=\pm\frac{2}{5},\ \tan\alpha=\pm\frac{5}{2}\Rightarrow\alpha=\ldots$

If R is used to find α accept $\sin\alpha=\pm\frac{2}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=\ldots$

A1 $\alpha=$ awrt 0.381 Note that the degree equivalent $\alpha=$ awrt $21.8°$ is A0

(b)

M1 Replaces $\cot 2x$ by $\dfrac{\cos 2x}{\sin 2x}$ **and** $\operatorname{cosec} 2x$ by $\dfrac{1}{\sin 2x}$ in the lhs

Do not be concerned by the coefficients 5 and -3.

Replacing $\cot 2x$ by $\dfrac{1}{\tan 2x}$ does not score marks until the $\tan 2x$ has been replaced by $\dfrac{\sin 2x}{\cos 2x}$

They may state $\times\sin 2x \Rightarrow 5\cos 2x-3=2\sin 2x$ which implies this mark

A1 cso $5\cos 2x-2\sin 2x=3$ There is no need to state the value of 'c'

The notation must be correct. They cannot mix variables within their equation

Do not accept for the final A1 $\tan 2x=\dfrac{\sin}{\cos}2x$ within their equations

(c)

M1 Attempts to use part (a) and (b). They must be using their R and α from part (a) and their c from part (b)

Accept $\cos(2x\pm'\alpha')=\dfrac{'c'}{'R'}$ Condone $\cos(\theta\pm'\alpha')=\dfrac{'c'}{'R'}$ or even $\cos(x\pm'\alpha')=\dfrac{'c'}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x=..$

Don't be concerned if they change the variable in the question and solve for $\theta=$ (as long as all operations have been undone). You may not see any working. It is implied by one correct answer.

You may need to check with a calculator.

Eg for an incorrect α $\cos(2x+1.19)=\dfrac{3}{\sqrt{29}}\Rightarrow x=-0.105$ would score M1 dM1 A0 A0

A1 One solution correct, usually $x=0.3/0.30$ or $x=2.46$ or in degrees $17.2°$ **or** $141.(0)°$

A1 Both solutions correct awrt $x=$ awrt $0.30, 2.46$ and no extra values in the range.

Condone candidates who write 0.3 and 2.46 without any (more accurate) answers

In degrees accept awrt 1 dp $17.2°, 141.(0)°$ and no extra values in the range.

Special case: For candidates who are misreading the question and using their part (a) with 2 on the rhs.

They will be allowed to score a maximum of SC M1 dM1 A0 A0

M1 Attempts to use part (a) with 2. They must be using their R and α from part (a)

Accept $\cos(2x\pm'\alpha')=\dfrac{2}{'R'}$ Condone $\cos(\theta\pm'\alpha')=\dfrac{2}{'R'}$ or even $\cos(x\pm'\alpha')=\dfrac{2}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x=..$

You may not see any working. It is implied by one correct answer. You may need to check with a calculator.

Eg for an correct α and R $\cos(2x+0.381)=\dfrac{2}{\sqrt{29}}\Rightarrow x=0.405$

Alt to part (c)

M1 Attempts both double angle formulae condoning sign slips on $\cos 2x$, divides by $\cos^2 x$ and forms a quadratic in tan by using the identity $\pm1\pm\tan^2 x=\sec^2 x$

dM1 Attempts to solve their quadratic in $\tan x$ leading to a solution for x.

A1 A1 As above

Question 15

Question Number	Scheme	Marks
(a)	$\sin 2x - \tan x = 2\sin x\cos x - \tan x$	M1
	$= \dfrac{2\sin x\cos^2 x}{\cos x} - \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\sin x}{\cos x} \times (2\cos^2 x - 1)$	
	$= \tan x\cos 2x$	dM1 A1*
		(4)
(b)	$\tan x\cos 2x = 3\tan x\sin x \Rightarrow \tan x(\cos 2x - 3\sin x) = 0$	
	$\cos 2x - 3\sin x = 0$	M1
	$\Rightarrow 1 - 2\sin^2 x - 3\sin x = 0$	M1
	$\Rightarrow 2\sin^2 x + 3\sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-3 \pm \sqrt{17}}{4} \Rightarrow x = ...$	M1
	Two of $x = 16.3°, 163.7°, 0, 180°$	A1
	All four of $x = 16.3°, 163.7°, 0, 180°$	A1
		(5)
		(9 marks)

(a)

M1 Uses a correct double angle identity involving $\sin 2x$ Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$

M1 Uses $\tan x = \dfrac{\sin x}{\cos x}$ with $\sin 2x = 2\sin x\cos x$ and attempts to combine the two terms using a common denominator. This can be awarded on two separate terms with a common denominator.
Alternatively uses $\sin x = \tan x\cos x$ and attempts to combine two terms using factorisation of $\tan x$

dM1 Both M's must have been scored. Uses a correct double angle identity involving $\cos 2x$.

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent
Withhold this mark if for instance they write $\tan x = \dfrac{\sin}{\cos}$

If the candidate $\times\cos x$ on line 1 and/or $\div\sin x$ they cannot score any more than one mark unless they are working with both sides of the equation or it is fully explained.

(b)

M1 The $\tan x$ must be cancelled or factorised out to produce $\cos 2x - 3\sin x = 0$ or $\dfrac{\cos 2x}{\sin x} = 3$ oe Condone slips

M1 Uses $\cos 2x = 1 - 2\sin^2 x$ to form a 3TQ=0 in $\sin x$ The = 0 may be implied by later work

M1 Uses the formula/completion of square or GC with invsin to produce at least one value for x
It may be implied by one correct value.
This mark **can** be scored from factorisation of their 3TQ in $\sin x$ **but only if** their quadratic factorises.

A1 Two of $x = 0, 180°$, awrt $16.3°$, awrt $163.7°$ or in radians two of awrt 0.28, 2.86, 0 and π or 3.14
This mark can be awarded as a SC for those students who just produce $0, 180°$ (or 0 and π) from $\tan x = 0$ or $\sin x = 0$.

A1 All four values in degrees $x = 0, 180°$, awrt $16.3°$, awrt $163.7°$ and no extra's inside the range $0 ,, x < 360°$.
Condone $0 = 0.0$ and $180° = 180.0°$ Ignore any roots outside range.

Alternatives to parts (a) and (b)

(a) Alt 1	$\tan x\cos 2x = \tan x\left(2\cos^2 x-1\right)$	M1
	$= 2\tan x\cos^2 x - \tan x$	
	$= 2\dfrac{\sin x}{\cos x}\cos^2 x - \tan x$	M1
	$= 2\sin x\cos x - \tan x$	
	$= \sin 2x - \tan x$	dM1 A1
		(4)

a) Alt 1 Starting from the rhs

M1 Uses a correct double angle identity for $\cos 2x$. Accept any correct version including $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 Uses $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and attempts to multiply out the bracket

dM1 Both M's must have been scored. It is for using $2\sin x\cos x = \sin 2x$

A1* A fully correct solution with no errors or omissions. All notation must be correct and variables must be consistent.

See Main scheme for how to deal with candidates who $\div\tan x$

(a) Alt 2	$\sin 2x - \tan x \equiv \tan x\cos 2x$	
	$2\sin x\cos x - \tan x \equiv \tan x(2\cos^2 x - 1)$	M1
	$2\sin x\cos x - \cancel{\tan x} \equiv 2\tan x\cos^2 x - \cancel{\tan x}$	
	$2\sin x\cos x \equiv 2\dfrac{\sin x}{\cos x}\cos^2 x$	M1
	$2\sin x\cos x \equiv 2\sin x\cos x$	dM1
	+statement that it must be true	A1*

a) Alt 2 Candidates who use both sides

M1 Uses a correct double angle identity involving $\sin 2x$ **or** $\cos 2x$. Can be scored from either side

Accept $\sin(x+x) = \sin x\cos x + \cos x\sin x$ or $\cos(x+x) = \cos x\cos x - \sin x\sin x$

M1 Uses $\tan x = \dfrac{\sin x}{\cos x}$ with $\cos 2x = 2\cos^2 x - 1$ and cancels the $\tan x$ term from both sides

dM1 Uses a correct double angle identity involving $\sin 2x$ Both previous M's must have been scored

A1* A fully correct solution with no errors or omissions AND statement "hence true", "a tick", "QED". W[5]

All notation must be correct and variables must be consistent

---

It is possible to solve part (b) without using the given identity. There are various ways of doing this, one of which is shown below.

$\sin 2x - \tan x = 3\tan x\sin x \Rightarrow \quad 2\sin x\cos x - \dfrac{\sin x}{\cos x} = 3\dfrac{\sin x}{\cos x}\sin x$

$2\sin x\cos^2 x - \sin x = 3\sin^2 x$ M1 Equation in $\sin x$ and $\cos x$

$2\sin x\left(1-\sin^2 x\right) - \sin x = 3\sin^2 x$ M1 Equation in $\sin x$ only

$\left(2\sin^2 x + 3\sin x - 1\right)\sin x = 0$

$x = ..$ M1 Solving equation to find at least one x

Two of $x = 16.3°, 163.7°, 0, 180°$ A1

All four of $x = 16.3°, 163.7°, 0, 180°$ and no extras A1
