End of Year 12 AS Pure & Applied - Homework 1 (2 hr) MARK SCHEME

Section A: Pure Mathematics

Question 1

Question Number	Scheme	Marks
(a)	Area of triangle = $'\frac{1}{2}ab\sin C' = \frac{1}{2}\times 2x \times 2x \times \sin 60 = \sqrt{3}x^2$	M1
	$S = 2\times\sqrt{3}x^2 + 3\times 2xl = 2x^2\sqrt{3} + 6xl$	dM1A1*
		(3)
(b)	$960 = 2x^2\sqrt{3} + 6xl \Rightarrow l = \frac{960 - 2x^2\sqrt{3}}{6x}$	M1A1
	$V = x^2\sqrt{3}\, l$	B1
	Substitute $l = \frac{960 - 2x^2\sqrt{3}}{6x}$ into $V = x^2\sqrt{3}\, l$	
	$\Rightarrow V = x^2\sqrt{3}\times\left(\frac{960 - 2x^2\sqrt{3}}{6x}\right) = 160x\sqrt{3} - x^3$	dM1A1*
		(5)
(c)	$\frac{dV}{dx} = 160\sqrt{3} - 3x^2 = 0$	M1A1
	$\Rightarrow x =$ awrt 9.6	A1
	$\Rightarrow V = 160\times 9.611\times\sqrt{3} - 9.611^3 = 1776$	dM1 A1
		(5)
(d)	$\frac{d^2V}{dx^2} = -6x < 0 \Rightarrow$ Maximum	M1A1
		(2)
		(15 marks)

Question 2

Question Number	Scheme	Marks
(a)	Sub $x = 3$ in $y = x^2 - \frac{1}{3}x^3 = 9 - 9 = 0$	B1*
		(1)
(b)	$y = x^2 - \frac{1}{3}x^3 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2x - x^2$	M1A1
	Subs $x = 3$ in $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - x^2 = 6 - 9 = (-3)$	dM1
	Equation of tangent is $-3 = \frac{y-0}{x-3} \Rightarrow y = -3x + 9$	ddM1A1*
		(5)
(c)	Sets $x^2 - \frac{1}{3}x^3 = -3x + 9 \Rightarrow x^3 - 3x^2 - 9x + 27 = 0$ oe	M1, A1
	Solves $x^3 - 3x^2 - 9x + 27 = 0 \Rightarrow (x-3)^2(x+3) = 0 \Rightarrow x = -3$	dM1, A1
		(4)
(d)	Area under curve $= \int x^2 - \frac{1}{3}x^3 \mathrm{d}x = \left[\frac{1}{3}x^3 - \frac{1}{12}x^4\right]$	M1A1
	Area of triangle $= \frac{1}{2} \times (3 - x_B) \times y_B = (54)$	M1
	Shaded area = Triangle – area under curve = $'54' - \left(\left(\frac{1}{3}\times 3^3 - \frac{1}{12}\times 3^4\right) - \left(\frac{1}{3}\times(-3)^3 - \frac{1}{12}\times(-3)^4\right)\right) = 36$	dM1A1
		(5) (15 marks)
	Alternative to (d) by integration Area $= \int_{x=-3}^{x=3} (-3x+9) - \left(x^2 - \frac{1}{3}x^3\right) \mathrm{d}x$ Either way around $\left[-3\frac{x^2}{2} + 9x - \frac{1}{3}x^3 + \frac{1}{3}\frac{x^4}{4}\right]_{x=-3}^{x=3}$ $= \left(-3\times\frac{3^2}{2} + 9\times 3 - \frac{1}{3}\times 3^3 + \frac{1}{3}\times\frac{3^4}{4}\right) - \left(-3\times\frac{(-3)^2}{2} + 9\times(-3) - \frac{1}{3}\times(-3)^3 + \frac{1}{3}\times\frac{(-3)^4}{4}\right)$ $=36$	

Question 3

Question Number	Scheme	Marks
(a)	Uses $1-\sin^2 x = \cos^2 x$	M1
	$\dfrac{\cos^2 x - \sin^2 x}{1-\sin^2 x} = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x} = 1 - \dfrac{\sin^2 x}{\cos^2 x} = 1 - \tan^2 x$	A1*
		(2)
(b)	$\dfrac{\cos^2 x - \sin^2 x}{1-\sin^2 x} + 2 = 0 \Rightarrow 1 - \tan^2 x + 2 = 0$	M1
	$\tan^2 x = 3$	A1
	$\tan x = (\pm)\sqrt{3} \Rightarrow x = ..$	dM1
	$x = 60^0, 120^0, 240^{\circ}, 300^0$	A1,A1
		(5)

Question 4

Question Number	Scheme	Marks
(a)	$h = 3.7 + 2.5\cos(30t-40)^{\circ}, \quad 0 \le t < 24$	
	Max = 3.7+2.5=6.2m	B1
	Occurs when $30t - 40 = 0 \Rightarrow t = \dfrac{40}{30}, = 1:20am\,(01:20)$	M1A1,A1
		(4)
(b)	$3.7 + 2.5\cos(30t-40)^{\circ} = 3 \Rightarrow \cos(30t-40)^{\circ} = -\dfrac{0.7}{2.5} = (-0.28)$	M1
	$30t - 40 = 106.3, (253.7)$	A1
	$t =$ awrt 4.9 or 9.8	A1
	2nd Value $30t - 40 = 253.7 \Rightarrow t = ...$	M1
	$t =$ awrt 4.9 and 9.8	A1
	Boat cannot enter the harbour between 04:53 and 09:47	A1
		(6)
		(10 marks)

Question 5

$$2\log_2 x + \log_2(x-1) - \log_2(5x+4) = 1$$

$$\log_2 x^2 + \log_2(x-1) - \log_2(5x+4) = \log_2 2$$ Converting 1 into $\log_2 2$

$$\log_2 x^2(x-1) - \log_2(5x+4) = \log_2 2 \quad [\log_a a = 1]$$

$$\Rightarrow \log_2\left[\frac{x^2(x-1)}{5x+4}\right] = \log_2 2$$

$$\Rightarrow \frac{x^3-x^2}{5x+4} = 2$$

$$\Rightarrow x^3 - x^2 = 10x + 8$$

$\Rightarrow$ By inspection $(x+1)$ is a factor

$\Rightarrow$ $x=-1$ is a solution

$$\Rightarrow x^2(x+1) - 2x(x+1) - 8(x+1) = 0$$

$$\Rightarrow \cancel{(x+1)}(x^2-2x-8) = 0$$

$(x+1)$ can't be a solution of the log equation

$$\therefore (x+2)(x-4) = 0$$

$$x = \cancel{-2} \text{ or } 4$$

Section B: Statistics

Question 6

Question	Scheme	Marks
(a)	Width = $\frac{5}{3} \times 1.5 = \underline{2.5 \text{ (cm)}}$	B1
	Area = $6 \times 1.5 = 9$ cm^2 has frequency = 12 so 1.5 cm^2 = 2 people (o.e.)	M1
	Frequency of 10 corresponds to area of 7.5 so height = $\underline{3 \text{ (cm)}}$	A1 (3)
(b)	$Q_2 = [2.5+]\frac{(25/25.5-16)}{12} \times 3 = 4.75$ (or 4.875 if use $n+1$) awrt $\underline{4.75}$	M1 A1
		(2)
(c)(i)	$[\bar{x} =]\ \frac{394}{50} = 7.88$ (*)	B1cso
(ii)	$[\sigma_x =]\ \sqrt{\frac{6500}{50} - \bar{x}^2} = \sqrt{67.9056}$	M1A1
	= $\underline{\text{awrt } 8.24}$ (Accept s = awrt 8.32)	A1 (4)
(d) (i)	There is no effect on the mean	B1
(ii)	The median will increase	B1
(iii)	The standard deviation will decrease	B1 (3)

Question 7

Question	Scheme	Marks
(a)	$P(S) = 0.31 + p$, $P(D) = 0.35$, $P(S \cap D) = 0.14$	M1
	$(0.31 + p)(0.35) = 0.14$ oe	M1
	$P(S) = 0.4$ or $0.31 + p = 0.4$ or $0.35p = 0.0315$	A1
	$p = \underline{0.09}$	A1
		(4)
(b)	$P(S \cup M \cup D) = 1$ so $q = 1 - (0.17+0.10+0.15+0.06+0.04) - p$ or $0.48 - p$	M1
	$q = \underline{0.39}$	A1ft
		(2)

Question 8

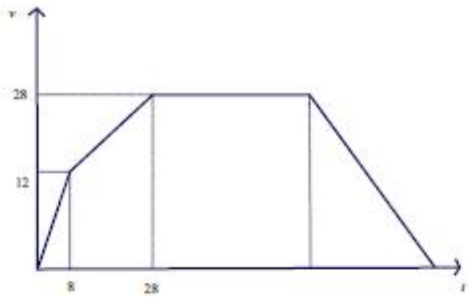
Question Number	Scheme	Marks
(a)(i)	$H_0 : p = 0.35 \quad H_1 : p \neq 0.35$	B1
(ii)	B(15,0.35)	M1
	CR $\quad X \leq 1 \cup X \geq 10$ (Allow any letter)	A1A1
		(4)
(b)	8 is not in CR	M1
	There is evidence that the Company's claim is true	A1ft
		(2)
(c)	$0.0142 + 0.0124 = 0.0266$	B1
		(1)
		[7]

Section C: Mechanics

Question 9

Question Number	Scheme	Marks	Notes
(a)	$h = -20\times5+\frac{1}{2}\times9.8\times25$	M1	Use of $s = ut + \frac{1}{2}at^2$ to find h. Must quote the correct formula and be using 20 & 5, but condone slips in substitution. Accept complete alternative solutions working via the maximum height. (max ht 20.4..., time to top 2.04...) Accept complete alternative methods using other *suvat* equations.
		A1	Correctly substituted equation(s) Condone use of a premature approximation.
	$h = 22.5$	A1 (3)	Final answer. Accept 22.5 or 23. Maximum 3sf. -22.5 is A0.
	NB Do not ignore subsequent working if they reach 22.5 and then move on to do further work.		
(b)	$V^2 = 20^2 + 2\times9.8\times22.5$ OR $V = -20+(5\times9.8)$	M1	First ball - use of *suvat* to find V or V^2 Follow their h.
	$(V^2 = 841)$ $= 29$	A1	Correct only (condone -29)
	$\left(\frac{3}{4}V\right)^2 = w^2 + 2\times9.8\times22.5$	M1	Second ball - *suvat* equation in V (or their V) to find w. Must be using the $\frac{3}{4}$.
	$w^2 = \frac{9}{16}\times841 - 2\times9.8\times22.5$	A1ft	Correctly substituted equation with their V and their h.
	$w = 5.66$	A1	or 5.7. Answer correct to 2 s.f. or to 3 s.f.
		(5)	
		[8]	

Question 10

Question Number	Scheme	Marks	Notes
(a)	$v_1 = 8 \times 1.5\ (=12)$	M1	Use of $v = u + at$ or equivalent for $t = 8$
	$v_2 = 12 + 0.8 \times 20$	M1	Follow their 12
	$v_2 = 28\ \text{m s}^{-1}$	A1 (3)	
(b)		B1 B1ft	shape nos: 8,28; 12,28 indicated. Follow their 12, 28
		(2)	
(c)	first 8 s: dist $= \frac{1}{2} \times 8 \times 12\ (=48)$	M1	Correct method for distance for the triangle (0-8) or the trapezium (8-28)
		A1ft	Follow their 12
	next 20 s: dist $= \frac{1}{2} \times (12+28) \times 20\ (=400)$	A1ft	Follow their 12, 28
	Total dist = 448 m	A1 (4)	Correct answer only (cao)
(d)	$0 = 28^2 - 2 \times 2.8s$	M1	Find area of right hand triangle or an expression in T for the trapezium (rectangle + triangle).
	$s = \frac{28^2}{2 \times 2.8}\ (=140)$	A1ft	Follow their 28
	$448 + 140 + 28T = 2000$	DM1	Form an equation in T for their 16, 448 and 140
	$T = \frac{2000 - 448 - 140}{28} = 50.4$	A1 (4)	Or better (50.42857...) Accept 50.
		[13]	