Name ....

Marks............................../99

End of Year 12 AS Pure & Applied - Homework 1 (2 hr)

Section A: Pure Mathematics

Question 1

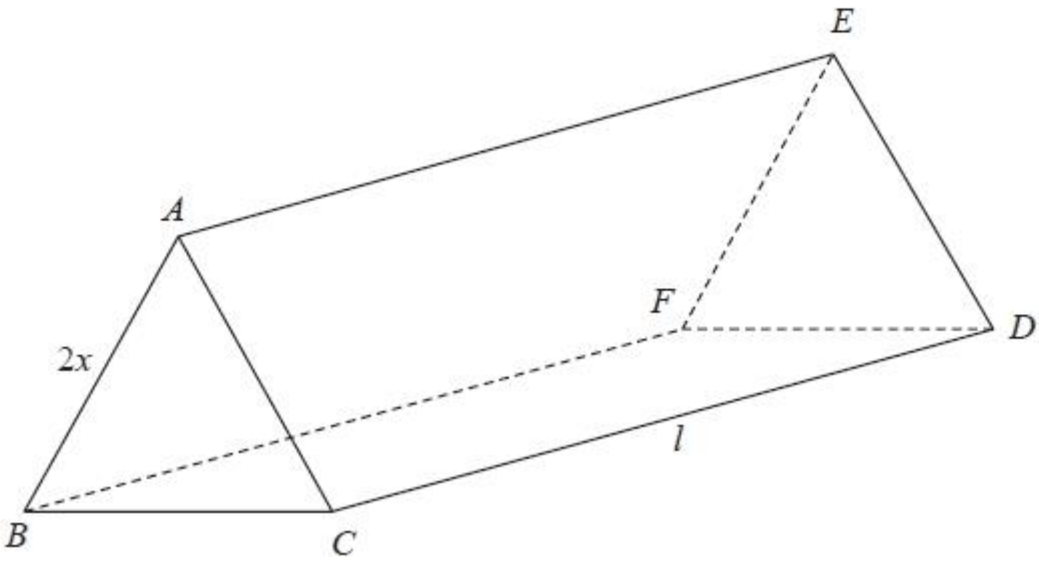


Figure 6

Figure 6 shows a solid triangular prism *ABCDEF* in which *AB* = 2*x* cm and *CD* = *l* cm.

The cross section *ABC* is an equilateral triangle.

The rectangle *BCDF* is horizontal and the triangles *ABC* and *DEF* are vertical.

The total surface area of the prism is *S* cm^2 and the volume of the prism is *V* cm^3.

(a) Show that $S = 2x^2\sqrt{3} + 6xl$ (3)

Given that *S* = 960,

(b) show that $V = 160x\sqrt{3} - x^3$ (5)

(c) Use calculus to find the maximum value of *V*, giving your answer to the nearest integer. (5)

(d) Justify that the value of *V* found in part (c) is a maximum. (2)

(Total for question = 15 marks)

Question 2

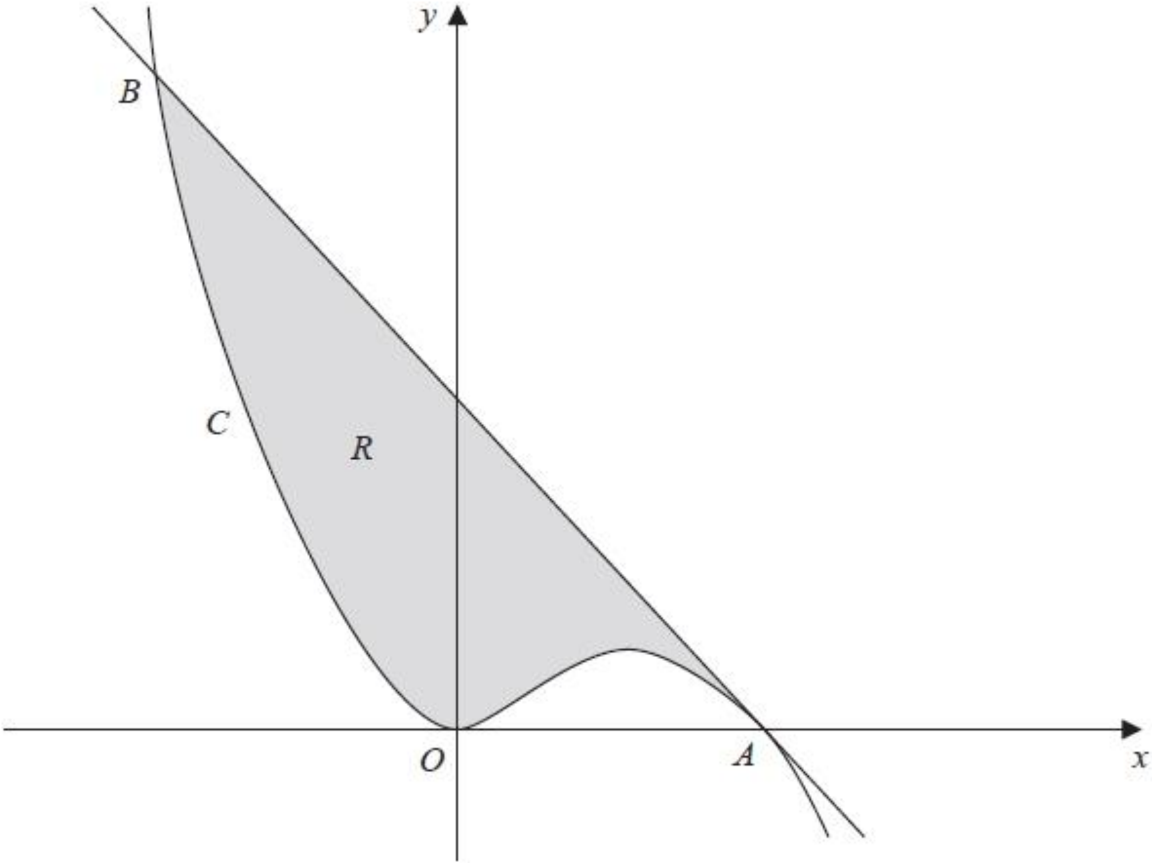


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = x^2 - \frac{1}{3}x^3$

C touches the x-axis at the origin and cuts the x-axis at the point A.

(a) Show that the coordinates of A are (3, 0). **(1)**

(b) Show that the equation of the tangent to C at the point A is $y = -3x + 9$ **(5)**

The tangent to C at A meets C again at the point B, as shown in Figure 5.

(c) Use algebra to find the x coordinate of B. **(4)**

The region R, shown shaded in Figure 5, is bounded by the curve C and the tangent to C at A.

(d) Find, by using calculus, the area of region R.
(*Solutions based entirely on graphical or numerical methods are not acceptable*.) **(5)**

(Total for question = 15 marks)

Question 3

(a) Show that

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} \equiv 1 - \tan^2 x, \qquad x \neq (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$$

(2)

(b) Hence solve, for $0 \leq x < 360$,

$$\frac{\cos^2 x - \sin^2 x}{1 - \sin^2 x} + 2 = 0$$

(5)

(Total for question = 7 marks)

Question 4

The height of sea water, h metres, on a harbour wall at time t hours after midnight is given by

$$h = 3.7 + 2.5\cos(30t - 40)°, \qquad 0 \leq t < 24$$

(a) Calculate the maximum value of h and the exact time of day when this maximum first occurs. (4)

Fishing boats cannot enter the harbour if h is less than 3

(b) Find the times during the morning between which fishing boats cannot enter the harbour.

Give these times to the nearest minute.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.) (6)

(Total for question = 10 marks)

Question 5 (Grade A*)

$$2\log_2 x + \log_2(x-1) - \log_2(5x+4) = 1$$

Find the only real root of the above logarithmic equation (6)

(Total for question = 6 marks)

Section B: Statistics

Question 6.

The table below shows the distances (to the nearest km) travelled to work by the 50 employees in an office.

Distance (km)	Frequency (f)	Distance midpoint (x)
0 – 2	16	1.25
3 – 5	12	4
6 – 10	10	8
11 – 20	8	15.5
21 – 40	4	30.5

[You may use $\sum fx = 394$, $\sum fx^2 = 6500$]

A histogram has been drawn to represent these data.

The bar representing the distance of 3 – 5 has a width of 1.5 cm and a height of 6 cm.

(a) Calculate the width and height of the bar representing the distance of 6 – 10 **(3)**

(b) Use linear interpolation to estimate the median distance travelled to work. **(2)**

(c) (i) Show that an estimate of the mean distance travelled to work is 7.88 km.

(ii) Estimate the standard deviation of the distances travelled to work. **(4)**

Peng starts to work in this office as the 51st employee.

She travels a distance of 7.88 km to work.

(d) Without carrying out any further calculations, state, giving a reason, what effect Peng's addition to the workforce would have on your estimates of the

(i) mean,

(ii) median,

(iii) standard deviation

of the distances travelled to work. **(3)**

(Total for question = 12 marks)

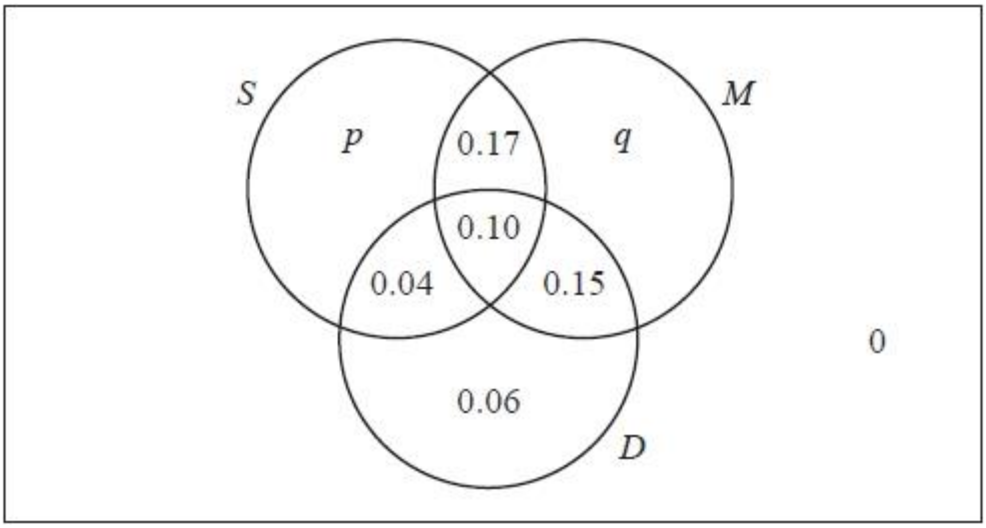
Question 7.

The Venn diagram below shows the probabilities of customers having various combinations of a starter, main course or dessert at Polly's restaurant.

S = the event a customer has a starter.

M = the event a customer has a main course.

D = the event a customer has a dessert.



Given that the events S and D are statistically independent

(a) find the value of p. (4)

(b) Hence find the value of q. (2)

(Total for question = 6 marks)

Question 8

A company claims that 35% of its peas germinate. In order to test this claim Ann decides to plant 15 of these peas and record the number which germinate.

(a) (i) State suitable hypotheses for a two-tailed test of this claim.
(ii) Using a 5% level of significance, find an appropriate critical region for this test. The probability in each of the tails should be as close to 2.5% as possible. (4)

(b) Ann found that 8 of the 15 peas germinated. State whether or not the company's claim is supported. Give a reason for your answer. (2)

(c) State the actual significance level of this test. (1)

(Total for question = 7 marks)

Section C: Mechanics

Question 9

A ball is thrown vertically upwards with speed 20 m s^{-1} from a point *A*, which is *h* metres above the ground. The ball moves freely under gravity until it hits the ground 5 s later.

(a) Find the value of *h*. (3)

A second ball is thrown vertically downwards with speed *w* m s^{-1} from *A* and moves freely under gravity until it hits the ground.

The first ball hits the ground with speed *V* m s^{-1} and the second ball hits the ground with speed $\frac{3}{4}V$ m s^{-1}

(b) Find the value of *w*. (5)

(Total for question = 8 marks)

Question 10

A car starts from rest at a point *A* and moves along a straight horizontal road. The car moves with constant acceleration 1.5 m s^{-2} for the first 8 s. The car then moves with constant acceleration 0.8 m s^{-2} for the next 20 s. It then moves with constant speed for *T* seconds before slowing down with constant deceleration 2.8 m s^{-2} until it stops at a point *B*.

(a) Find the speed of the car 28 s after leaving *A*. (3)

(b) Sketch, in the space provided, a speed–time graph to illustrate the motion of the car as it travels from *A* to *B*. (2)

(c) Find the distance travelled by the car during the first 28 s of its journey from *A*. (4)

The distance from *A* to *B* is 2 km.

(d) Find the value of *T*. (4)

(Total for question = 13 marks)