End of Year 12 AS Pure & Applied - Homework 2 (2 hr) MARK SCHEME

Section A: Pure Mathematics

Question 1

Question Number	Scheme	Marks
(a)	$y = x^3 + 10x^{\frac{3}{2}} + kx$ $\frac{dy}{dx} = 3x^2 + 10\times\frac{3}{2}x^{\frac{1}{2}} + k$	M1 A1 [2]
(b)	Substitutes $x = 4$ and $\frac{dy}{dx} = 0$ to give $3(4)^2 + 15(4)^{\frac{1}{2}} + k = 0 \quad \Rightarrow k = -78$ *	M1 A1* [2]
(c)	When $x = 4$, $y = -168$ (see this stated – or see rectangle has height 168) $\int x^3 + 10x^{\frac{3}{2}} - 78x\ (+168)dx = \frac{1}{4}x^4 + \frac{10}{\frac{5}{2}}x^{\frac{5}{2}} - \frac{78}{2}x^2 (+168x + c)$ Use limits 0 and 4 to give ± 432 or if $168x$ included to give ± 240 Rectangle area is 4 × "168" (= 672) or see $168x$ in integrated answer with limits So R has area "672 – 432" or see +168 in original integrand = 240	B1 M1 A1 dB1 M1 M1 A1 [7]
		11 marks
	Notes	
(a)	**M1:** Fractional power dealt with correctly so becomes $\frac{3}{2}x^{\frac{1}{2}}$ (may be implied by simplification to 15) **A1:** All terms correct, may not be simplified	
(b)	**M1: Substitutes** $x = 4$ and $\frac{dy}{dx} = 0$ Must see $3(4)^2 + 15(4)^{\frac{1}{2}} + k = 0$ or $48 + 30 + k = 0$ ***A1:** This is a printed answer so all must be correct in the working and conclusion k = -78 is needed.	
(c)	**B1:** Substitute into y = to find y (This may appear anywhere in the answer) **M1:** Attempt to integrate so at least one power increases **A1:** Accept unsimplified correct answer and allow with or without their $+168x$, or even with their $-168x$ **dB1**: Use limit 4 to give 432 but may be implied by later answer 240- needs to follow M1A1 for integration **M1:** Calculates rectangle area (may be by integration). Must be rectangle and not triangle area **M1:** Subtracts (either way round) numerical areas – should be (+) – (+) or (-) - (-) (subtraction may be in their original integral but penalize wrong sign here eg $-168x$ instead of $+168x$) (Again use of triangle is M0) **A1:** 240 only (Can recover from -240 to 240) Common error: If $168x$ (instead of 168) is integrated this may only gain a maximum of B1 M1 A1 dB1 (for seeing 432 calculated if integrals are separated) M0 M0 A0 4/7	

Question 2

Question Number	Scheme	Marks
	(a) Use or state $2\log_4(2x+3) = \log_4(2x+3)^2$	M1
	Use or state $\log_4 4 = 1$ or $4^1 = 4$	M1
	Use or state $\log_4 x + \log_4(2x-1) = \log_4 x(2x-1)$ or $\log_4(2x+3)^2 - \log_4 x = \log_4 \frac{(2x+3)^2}{x}$ etc	M1
	$(2x+3)^2 = 4x(2x-1)$ or equivalent including correct rational equations	A1
	Then $4x^2+12x+9 = 8x^2-4x$ and so $4x^2-16x-9=0$ *	A1*
		[5]
	(b) $(2x+1)(2x-9) = 0$ so $x =$ (or use other method e.g formula or completion of square)	M1
	$x = (-\frac{1}{2}$ or $)\ \frac{9}{2}$	A1
		[2]
		7 marks
	Notes	

(a) M1: Uses power law for logs

M1: Connects 1 with 4 correctly

M1: Uses addition (or subtraction) law correctly

e.g. $\log_4 x + \log_4(2x-1) = \log_4 x(2x-1)$ or $\log_4(2x+3)^2 - \log_4 x = \log_4 \frac{(2x+3)^2}{x}$ or

$\log_4(2x+3)^2 - \log_4 x - \log_4(2x-1) = \log_4 \frac{(2x+3)^2}{x(2x-1)}$ or even $\log_4 x + \log_4 4 = \log_4 4x$ or

$\log_4(2x-1) + \log_4 4 = \log_4 4(2x-1)$ or $\log_4(2x-1) + \log_4 4 + \log_4 x = \log_4 4x(2x-1)$ etc...

A1: Correct equation (unsimplified) after correct work. e.g. $\frac{(2x+3)^2}{x(2x-1)} = 4$

A1: Obtains printed answer correctly (This is a given answer so needs previous A mark to have been awarded and needs correct expansion)

Special case :

$\log_4(2x+3)^2 = 1 + \log_4 x(2x-1)$ *so* $\frac{\log_4(2x+3)^2}{\log_4 x(2x-1)} = 1$ *so* $\frac{4x^2+12x+9}{2x^2-x} = 4$

This can have M1, M1, M1, A0, A0 so 3/5 losing accuracy because of the error in the second step.

(b) Some candidates who did not achieve marks in part (a) begin the log work again and make more progress here. Mark the better work. So credit for (a) may be given in (b). Credit for (b) should not be given in (a)

M1: Uses solution of their quadratic or of printed quadratic(see notes). This must be in part (b)

A1: $x = 4.5$ and discards $x = -0.5$ (any equivalent form) Giving $x = -\frac{1}{2}, \frac{9}{2}$ is A0 This must be in part (b)

Question 3

Question Number	Scheme	Marks
(a)	$1000 < V \le 23000$	B1,B1
		(2)
(b)	$\frac{dV}{dt} = 18000 \times -0.2e^{-0.2t} + 4000 \times -0.1e^{-0.1t}$	M1
	$\left.\frac{dV}{dt}\right\|_{t=10} = 18000 \times -0.2e^{-2} + 4000 \times -0.1e^{-1} = \text{awrt}(-)634$	M1A1
		(3)
(c)	$15000 = 18000e^{-0.2t} + 4000e^{-0.1t} + 1000$	
	$0 = 9e^{-0.2t} + 2e^{-0.1t} - 7$	
	$0 = (9e^{-0.1t} - 7)(e^{-0.1t} + 1)$	M1A1
	$9e^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\frac{9}{7}\right)$ oe	dM1A1
		(4)
		(9 marks)

(a)

B1 Accept either boundary: $V < 23000$ or $V \le 23000$ or $V_{\max}\,23000$ for the upper boundary and $V > 1000$ or $V \ge 1000$ or $V_{\min}\,1000$ for the lower boundary. Answers like $V \ge 23000$ are B0

B1 Completely correct solution.
Accept $1000 < V \le 23000$, $1000 < \text{Range}$ or $y \le 23000$, $(1000,23000]$, $V > 1000$ *and* $V \le 23000$

(b)

M1 Score for a $\frac{dV}{dt} = Ae^{-0.2t} + Be^{-0.1t}$, where $A \ne 18000, B \ne 4000$

M1 Sub $t = 10$ into a $\frac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t}$ where $A \ne 18000, B \ne 4000$

Condone substitution of $t = 10$ into a $\frac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t} + 1000$ $A \ne 18000, B \ne 4000$

A1 Correct solution and answer only. Accept ± 634 following correct $\frac{dV}{dt} = -3600e^{-0.2t} - 400e^{-0.1t}$

Watch for students who sub $t = 10$ into their V first and then differentiate. This is 0,0,0.

Watch for students who achieve +634 following $\frac{dV}{dt} = 3600e^{-0.2t} + 400e^{-0.1t}$. This is 1,1,0

A correct answer with no working can score all marks.

(c)

M1 Setting up 3TQ in $e^{\pm 0.1t}$ AND correct attempt to factorise or solve by the formula.
For this to be scored the $e^{\pm 0.2t}$ term must be the x^2 term.

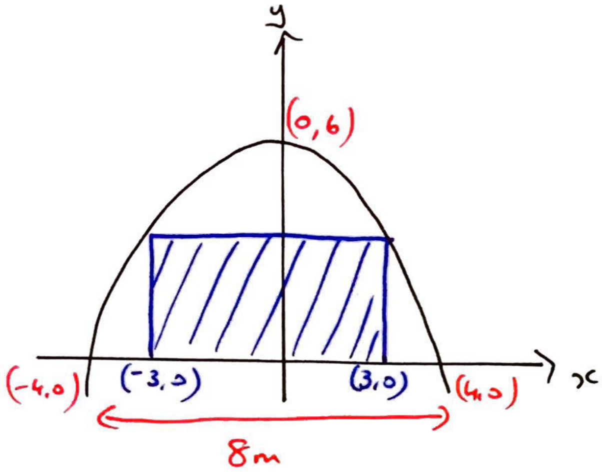
A1 Correct factors $(9e^{-0.1t} - 7)(e^{-0.1t} + 1)$ or $(7e^{0.1t} - 9)(e^{0.1t} + 1)$ or a root $e^{-0.1t} = \frac{7}{9}$

dM1 Dependent upon the previous M1.
This is scored for setting the $ae^{\pm 0.1t} - b = 0$ and proceeding using correct ln work to $t = \ldots$

A1 $t = 10\ln\left(\frac{9}{7}\right)$. Accept alternatives such as $t = \frac{1}{0.1}\ln\left(\frac{9}{7}\right), \frac{1}{-0.1}\ln\left(\frac{7}{9}\right), -10\ln\left(\frac{7}{9}\right)$

If any extra solutions are given withhold this mark.

Question 4



Setting up quadratic equation for the arch

$\Rightarrow$ Negative parabola $\therefore -x^2$

Start with $\quad y = A - Bx^2$ B1

$y = 6 - Bx^2$ [A = 6 highest point]

Using (4, 0)

$\Rightarrow \quad 0 = 6 - B(4)^2$ M1

$0 = 6 - 16B$

$B = \frac{6}{16}$

$= \frac{3}{8}$

$\therefore \quad \boxed{y = 6 - \frac{3}{8}x^2}$ A1

Using (3, 0) $\Rightarrow y = 6 - \frac{3}{8}(3^2)$ M1

$= 6 - \frac{27}{8}$

$= \frac{21}{8}$

$= 2\frac{5}{8}$ A1

Since $\quad 2\frac{5}{8} > 2$

$\therefore$ Lorry can pass through the tunnel with a clearance of $\frac{5}{8} = 0.625$ m B1

Question 5

a) Using proof by exhaustion

n can be either even or odd number

Suppose n is even, then $n = 2k$, where $k \in \mathbb{Z}$

$$\therefore 2n^2 + n = 2(2k)^2 + 2k \quad \text{M1}$$
$$= 2(4k^2) + 2k$$
$$= 8k^2 + 2k$$
$$= 4(2k^2) + 2k \quad \text{A1}$$

which is not a multiple of 4

Suppose n is odd, then $n = 2k+1$, where $k \in \mathbb{Z}$

$$\therefore 2n^2 + n = 2(2k+1)^2 + 2k \quad \text{M1}$$
$$= 2(4k^2 + 4k + 1) + 2k$$
$$= 8k^2 + 8k + 2 + 2k$$
$$= 8k^2 + 10k + 2$$
$$= 4(2k^2 + 2k) + 2k + 2 \quad \text{A1}$$

which is not a multiple of 4

$\therefore 2n^2 + n$ is not divisible by 4 for $n \in \mathbb{Z}$ A1

b) Using proof by exhaustion

Since $x \in \mathbb{R}$, x can be either negative ($x < 0$) or positive ($x > 0$) or 0

$x < 0$, $x = -1 \Rightarrow e^{4(-1)} < e^{3(-1)}$ M1

$e^{-4} < e^{-3}$ $\therefore$ statement that $e^{4x} > e^{3x}$ is not true when $x < 0$

$x > 0,\ x = 1 \Rightarrow e^{4(1)} > e^{3(1)}$ M1

$e^4 > e^3 \quad \therefore\ e^{4x} > e^{3x}$ is true when $x > 0$

If $x = 0 \Rightarrow e^{4(0)} = e^{3(0)}$

$e^0 = e^0$

$1 = 1 \quad \therefore\ e^{4x} > e^{3x}$ is not true when $x = 0$

B1

$\therefore$ Alroy's claim is sometimes true

Question 6

$$4\tan x \sin x \cos x + 4\tan x \cos x + 1 = 0$$

$$\Rightarrow 4\tan x \cos x(\sin x + 1) + 1 = 0$$

$$\Rightarrow 4\frac{\sin x}{\cancel{\cos x}}\cancel{\cos x}(\sin x + 1) + 1 = 0$$

$$\Rightarrow 4\sin x(\sin x + 1) + 1 = 0$$

$$\Rightarrow 4\sin^2 x + 4\sin x + 1 = 0$$

$$\Rightarrow (2\sin x + 1)^2 = 0$$

$$\Rightarrow \sin x = \frac{-1}{2}$$

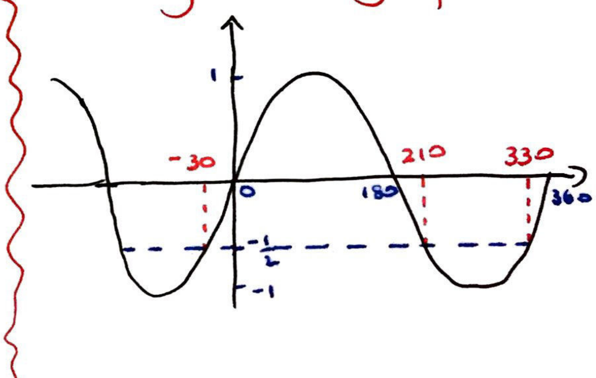
$$x = \arcsin\left(\frac{-1}{2}\right)$$

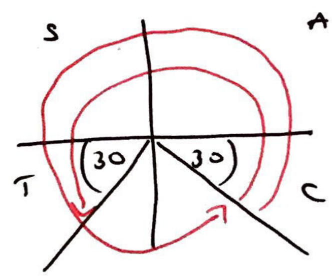
$$x = -30^\circ$$

$$\therefore x = 180 + 30^\circ,\ 360 - 30^\circ$$

$$x = 210^\circ,\ 330^\circ$$

Alternative Method: Using sine graph





Section B: Statistics

Question 7

Q	Marking Instructions	AO	Marks	Typical Solution
	Sets up enumerated population using valid numbering stating range used (PI)	AO2.4	E1	Give each student a number from (000)1 to 3200 or equivalent
	Explains how to obtain sample with respect to a specified range of random numbers. Accept random number generator / calculator set to give numbers from 1 to 3200 Do not accept 'drawn from a hat' (impractical for 3200 population)	AO2.4	E1	Generate random four digit integers using calculator
	Explains how to deal with repeats and random numbers outside range (PI by **both** 'different' numbers and 'numbers from 1 to 3200' seen).	AO2.4	E1	Ignore repeats and any (random) numbers outside the range Continue until 60 different numbers have been identified and select the students given those numbers
	Explains how to select the 60 (expresses idea of matching numbers to students or selecting them)	AO2.4	E1	
	Total		**4**	

Question 8

Question	Scheme	Marks
(a)(i)	$x + 0.1$ [P($x + 0.1$) is B0]	B1
(b)	$x+y+0.1$ (o.e.) [P($x+y+0.1$) is B0]	B1
		(1)
(c)	$x+y+0.1+0.32=1$ <u>or</u> $x+y+0.1=0.68$ <u>or</u> "(b)" $+ 0.32 = 1$ o.e.	M1
	$x+0.1=2(y+0.1)$	M1
	Eliminating x gives $3y = 0.48$	M1
	$x = \underline{0.42}$ $y = \underline{0.16}$	A1 A1
		(5)
	Notes	

Question 9

Question Number	Scheme	Marks
(a)	$P(X=5) = {}^{20}C_5(0.3)^5(0.7)^{15}$ or $0.4164 - 0.2375$	M1
	$= 0.17886...$ awrt 0.179	A1
		(2)

(b)	$H_0: p = 0.3$ $H_1: p > 0.3$	B1
	$X \sim B(20, 0.3)$	M1
	$P(X \geq 8) = 0.2277$ or $P(X \geq 10) = 0.0480$, so CR $X \geq 10$	A1
	Insufficient evidence to reject H_0 or Not Significant or 8 does not lie in the critical region.	dM1
	There is no evidence to support the Director (of Studies') belief/There is no evidence that the proportion of parents that do not support the new curriculum is greater than 30%	A1cso
		(5)
(c)	$X \sim B(2n, 0.25)$	
	$X \sim B(8, 0.25)$ $P(X \geq 4) = 0.1138$	M1
	$X \sim B(10, 0.25)$ $P(X \geq 5) = 0.0781$	
	$2n = 10$	A1
	$n = 5$	A1
		(3)

	Notes	
(a)	M1 ${}^{20}C_5(p)^5(1-p)^{15}$ or using $P(X \leq 5) - P(X \leq 4)$	
(b)	M1 use of $20 \times 0.7 \times 0.3$ (with or without the square root)	
(c)	B1 both hypotheses correct (p or π)	
	M1 using $X \sim B(20, 0.3)$ (may be implied by 0.7723, 0.2277, 0.8867 or 0.1133)	
	A1 awrt 0.228 or CR $X \geq 10$	
	dM1 a correct comment (dependent on previous M1)	
	A1 cso requires correct contextual conclusion with underlined words and all previous marks in (c) to be scored.	

Section C: Mechanics

Question 10

	$h = \frac{1}{2}gt^2$	B1
	$h = 7.35(t - \frac{1}{2}) + \frac{1}{2}g(t - \frac{1}{2})^2$	M1 A1
	$\frac{1}{2}gt^2 = 7.35(t - \frac{1}{2}) + \frac{1}{2}g(t - \frac{1}{2})^2$	DM1
	$t = 1$	M1 A1
	$h = 4.9$	A1 7
	NOTES **Question** B1 for $h = \frac{1}{2}gt^2$ or $h = \frac{1}{2}g(t + \frac{1}{2})^2$ First M1 for $h = 7.35(t - \frac{1}{2}) + \frac{1}{2}g(t - \frac{1}{2})^2$ or $h = 7.35t + \frac{1}{2}gt^2$ M0 if different t used in the two terms and M0 if two terms have opposite signs. First A1 for appropriate t value used Second M1, dependent, for equating their two expressions for h, but must have different t's in the two expressions Third M1, independent, for solving for their t (must have used two expressions etc.) Second A1 for $t = 1$ (or $t = ½$) Third A1 for $h = 4.9$ N.B. See alternative below where t is eliminated: $h = \frac{1}{2}gt^2$ B1 $h = 7.35(t - \frac{1}{2}) + \frac{1}{2}g(t - \frac{1}{2})^2$ M1A1 $h = 7.35(\sqrt{\frac{2h}{g}} - \frac{1}{2}) + \frac{1}{2}g(\sqrt{\frac{2h}{g}} - \frac{1}{2})^2$ DM1 $h = 7.35\sqrt{\frac{2h}{g}} - 3.675 + 4.9(\frac{2h}{g} - \sqrt{\frac{2h}{g}} + 0.25)$ A1 $h = 4.9$ M1 A1	

Question 11

(a)	For B: $1=\frac{1}{2}a.2^2 \Rightarrow a=\frac{1}{2}\text{ m s}^{-2}$	M1 A1 (2)
(b)	$T-F=3m\times 0.5$ M1A1 $2mg-T=2m\times 0.5$ M1A1 $F = 2mg - 2.5m$ $F = 19.6m - 2.5m$ M1 $F = 17.1m$ A1	
(c)	$v=\frac{1}{2}\times 2=1$ $-17.1m = 3ma$ $a = -5.7$ $0^2 = 1^2 + 2(-5.7)s$ $s=0.0877\ldots(0.09 \text{ or better})$ $s<0.3$ correct conclusion	B1 ft M1 M1 A1 DM1A1 cso (6) 16