Name .....

Marks...../90

End of Year 12 AS Pure & Applied - Homework 2 (2 hr)

Section A: Pure Mathematics

Question 1

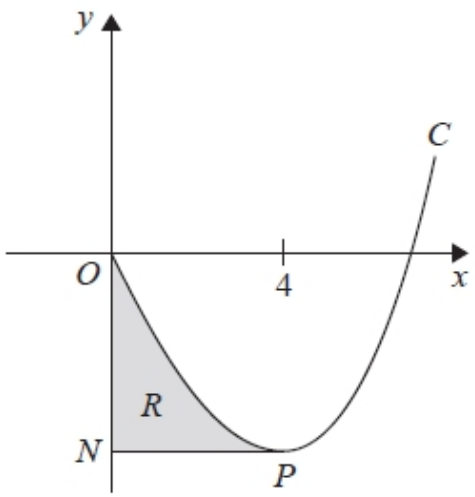


Figure 5

Figure 5 shows a sketch of part of the curve C with equation

$$y = x^3 + 10x^{\frac{3}{2}} + kx, \qquad x \geqslant 0$$

where k is a constant.

(a) Find $\dfrac{dy}{dx}$ **(2)**

The point P on the curve C is a minimum turning point.
Given that the x coordinate of P is 4

(b) show that $k = -78$ **(2)**

The line through P parallel to the x-axis cuts the y-axis at the point N.

The finite region R, shown shaded in Figure 5, is bounded by C, the y-axis and PN.

(c) Use integration to find the area of R. **(7)**

(Total for question = 11 marks)

Question 2

Given that

$$2\log_4 (2x + 3) = 1 + \log_4 x + \log_4 (2x - 1),\ x > \tfrac{1}{2}$$

(a) show that

$$4x^2 - 16x - 9 = 0 \qquad \textbf{(5)}$$

(b) Hence solve the equation

$$2\log_4 (2x + 3) = 1 + \log_4 x + \log_4 (2x - 1),\ x > \tfrac{1}{2} \qquad \textbf{(2)}$$

(Total for question = 7 marks)

Question 3

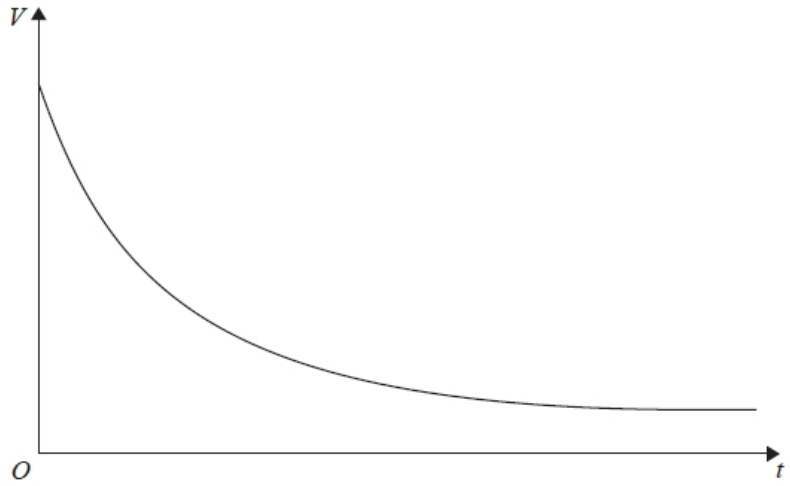


Figure 1

The value of Lin's car is modelled by the formula

$$V = 18\,000e^{-0.2t} + 4000e^{-0.1t} + 1000, \quad t \geqslant 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(a) State the range of V. **(2)**

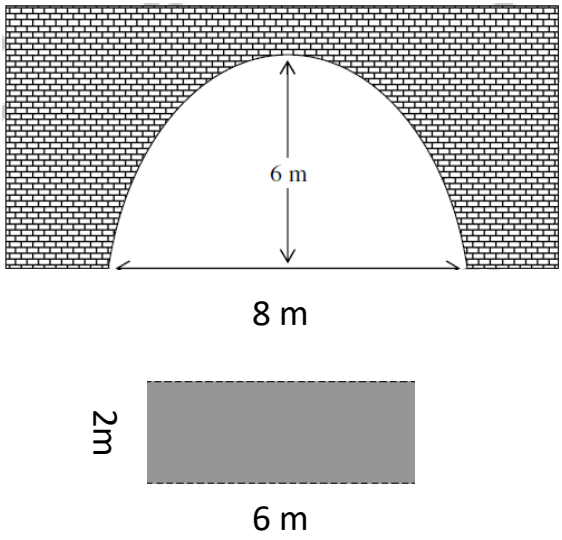
According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$
Give your answer in pounds per year. **(3)**

(c) Calculate the exact value of t when $V = 15000$. **(4)**

(Total for question = 9 marks)

Question 4



The figure above shows the entrance of a tunnel. The shape of the entrance in the form of a parabolic arch.

The width of the arc at its lowest level is 8 metres and the highest point of the arch is 6 metres from the ground.

By using an algebraic method, determine whether a lorry of width 6 metres and height 2 metres can pass through the tunnel

(Total for question = 6 marks)

Question 5

(a) Prove that $2n^2 + n$ is not divisible by 4 for $n \in \mathbb{Z}$ **(5)**

(b) Alroy claims that $e^{4x} > e^{3x}$ for $x \in \mathbb{R}$

Determine whether Alroy's claim is always true, sometimes true or never true, justifying your answer **(3)**

(Total for question = 8 marks)

Question 6

Solve for $0 \leq x < 360^0$

$$4 \tan x \sin x \cos x + 4 \tan x \cos x + 1 = 0$$

(Total for question = 8 marks)

Section B: Statistics

Question 7

Kevin is the Principal of a college.

He wishes to investigate types of transport used by students to travel to college.

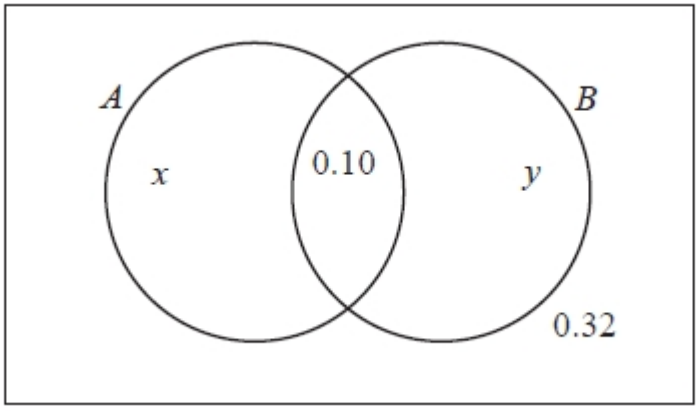
There are 3200 students in the college and Kevin decides to survey 60 of them.

Describe how he could obtain a simple random sample of size 60 from the 3200 students.

[4 marks]

Question 8

Events A and B are shown in the Venn diagram below where x, y, 0.10 and 0.32 are probabilities.



(a) Find an expression in terms of x for

P(A) **(1)**

(b) Find an expression in terms of x and y for P(A ∪ B) **(1)**

Given that P(A) = 2P(B)

(c) find the value of x and the value of y **(5)**

(Total for question = 7 marks)

Question 9

The Headteacher of a school claims that 30% of parents do not support a new curriculum.
In a survey of 20 randomly selected parents, the number, X, who do not support the new curriculum is recorded.

Assuming that the Headteacher's claim is correct, find

(a) the probability that $X = 5$ **(2)**

The Director of Studies believes that the proportion of parents who do not support the new curriculum is greater than 30%. Given that in the survey of 20 parents 8 do not support the new curriculum,

(b) test, at the 5% level of significance, the Director of Studies' belief. State your hypotheses clearly. **(5)**

The teachers believe that the sample in the original survey was biased and claim that only 25% of the parents are in support of the new curriculum. A second random sample, of size $2n$, is taken and exactly half of this sample supports the new curriculum.

A test is carried out at a 10% level of significance of the teachers' belief using this sample of size $2n$

Using the hypotheses H_0: $p = 0.25$ and H_1: $p > 0.25$

(c) find the minimum value of n for which the outcome of the test is that the teachers' belief is rejected. **(3)**

(Total for question = 10 marks)

Section C: Mechanics

Question 10

The points P and Q are at the same height h metres above horizontal ground. A small stone is dropped from rest from P. Half a second later a second small stone is thrown vertically downwards from Q with speed 7.35 m s^{-1}. Given that the stones hit the ground at the same time, find the value of h. **(7)**

(Total for question = 7 marks)

Question 11

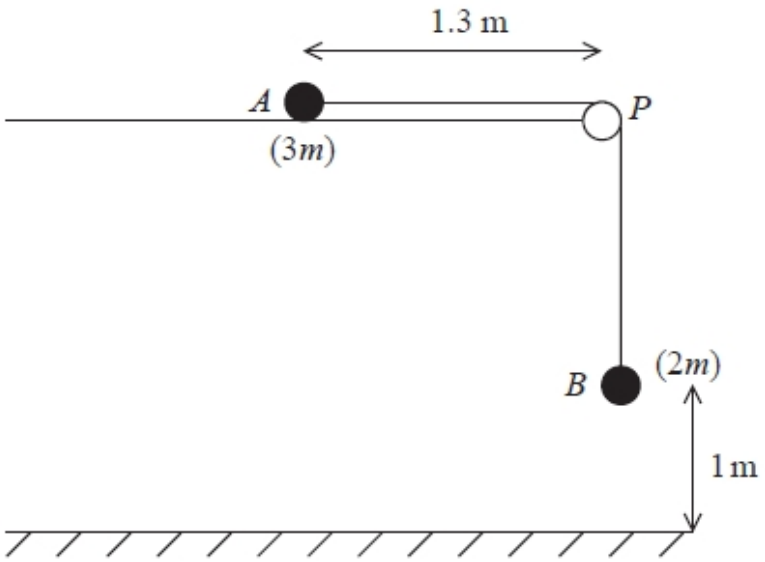


Figure 3

A particle A of mass $3m$ is held at rest on a rough horizontal table. The particle is attached to one end of a light inextensible string. The string passes over a small smooth pulley P which is fixed at the edge of the table. The other end of the string is attached to a particle B of mass $2m$, which hangs freely, vertically below P. The system is released from rest, with the string taut, when A is 1.3 m from P and B is 1 m above the horizontal floor, as shown in Figure 3. Particle A experiences a frictional force, F N

Given that *B* hits the floor 2 s after release and does not rebound,

(a) find the acceleration of *A* during the first two seconds, **(2)**

(b) find the magnitude of the frictional force in terms of m, **(5)**

(c) determine whether *A* reaches the pulley. **(6)**

(Total for question = 13 marks)

TOTAL FOR PAPER IS 90 MARKS