End of Year 12 AS Pure & Applied - Homework 3 (2 hr) MARK SCHEME

Section A: Pure Mathematics

Question 1

Question Number	Scheme	Marks
(a)	$\pi R^2 H + \frac{2}{3}\pi R^3 = 800\pi$ so $H = \frac{800}{R^2} - \frac{2}{3}R$ *	M1 A1*
(b)	$A = \pi R^2 + 2\pi RH + 2\pi R^2$	B1
	$A = 3\pi R^2 + 2\pi R\left(\frac{800}{R^2} - \frac{2}{3}R\right)$ so $A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$ *	M1 A1 *
		[5]
(c)	Find $\frac{dA}{dR} = \frac{10}{3}\pi R - \frac{1600\pi}{R^2}$	M1 A1
	Put derivative equal to zero and obtain $R^3 = 480$	dM1 A1
	So $R = 7.83$	A1
		[5]
(d)	Consider $\frac{d^2A}{dR^2} = \frac{10\pi}{3} + 3200\pi R^{-3} > 0$ so minimum	M1A1
		[2]
(e)	$H =$ awrt 7.83	B1
		[1]
		13 marks

Question 2

Question Number	Scheme	Marks
(ii)	Use or state $\log_2 16 = 4$ or $2^4 = 16$	B1
	Use $\log_2(x+3) - \log_2(2x+4) = \log_2 \frac{(x+3)}{(2x+4)}$	M1
	Then $\frac{x+3}{(2x+4)} = 16$, and so $x = -\frac{61}{31}$	A1 , A1
		[4]
		6 marks
Alt (ii)	$\log_2(x+3) - \log_2(2x+4) = \log_2 16$	B1
	$\log_2(x+3) = \log_2 16 + \log_2(2x+4)$	
	$\log_2(x+3) = \log_2 16(2x+4)$	M1
	$(x+3) = 16(2x+4)$	A1
	$x = -\frac{61}{31}$	A1

Question 2 (ii)

$$e^{2y} = x + 1 \quad \text{——— (1)}$$

$$\ln(x-2) = 2y - 1 \Rightarrow x - 2 = e^{2y-1}$$

$$x - 2 = e^{2y} . e^{-1}$$

$$x - 2 = \frac{e^{2y}}{e} \quad \text{——— (2)}$$

Sub (1) in (2)

$$x - 2 = \frac{x+1}{e}$$ [Replacing e^{2y} by $e^{2y} = x + 1$]

$$e(x-2) = x + 1$$

$$ex - 2e = x + 1$$

$$ex - x = 1 + 2e$$

$$x(e-1) = 1 + 2e$$

$$x = \frac{1+2e}{e-1}$$

$$x = 3.75 \quad (3 \text{ s.f})$$

Sub $x = \frac{1+2e}{e-1}$ in $\ln(x-2) = 2y - 1$

$$\Rightarrow \ln\left[\frac{1+2e}{e-1} - 2\right] = 2y - 1$$

$$0.5572\ldots = 2y - 1$$

$$y = 0.779 \quad (3 \text{ s.f})$$

Question 3

Question Number	Scheme	Marks
(a)	$25e$ or equivalent decimal - Accept awrt 68	B1
		(1)
(b)	$50 = 25e^{1-10k}$	M1
	Way 1 $e^{1-10k} = 2$ $\Rightarrow 1-10k = \ln 2$ $\Rightarrow k = \frac{\ln e - \ln 2}{10}$ | Way 2 $e^{-10k} = 2/e$ $-10k = \ln(2/e)$ $k = \frac{-\ln\left(\frac{2}{e}\right)}{10}$ | Way 3 $e^{10k} = e/2$ $10k = \ln(e/2)$ No intermediate step needed	A1 M1
	$\Rightarrow k = \frac{\ln\left(\frac{1}{2}e\right)}{10}$ *	A1*
		(4)
(c)	Uses $m = 20$ and their numerical k so $20 = 25e^{1-'k't} \Rightarrow e^{1-'k't} = 0.8$ o.e.	M1
	$\Rightarrow t = \frac{1-\ln 0.8}{'k'}$	dM1
	$\Rightarrow t =$ awrt 40 (years)	A1
		(3)
		(8 marks)

Question 4

Question Number	Scheme	Marks
(i)	$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin x + \cos x - \sin^2 x \cos x - \cos^2 x \sin x$	1st M1
	$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin x + \cos x - (1-\cos^2 x)\cos x - (1-\sin^2 x)\sin x$	2nd M1
	$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$	A1 *
		[3]
Alt I (i)	Use LHS $= (\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)$	2nd M1
	$\equiv \sin^3 x + \sin x \cos^2 x - \sin^2 x \cos x + \sin^2 x \cos x - \cos^2 x \sin x + \cos^3 x$	1st M1
	$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x$	A1 *
		[3]
Alt II (i)	Use RHS $\equiv \sin^3 x + \cos^3 x = (\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)$	M1
	$= (\sin x + \cos x)(1 - \sin x \cos x)$	M1 A1
		[3]

Question 4 (ii)

$$\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + (1+\cos\alpha)^2}{\sin\alpha(1+\cos\alpha)}$$ (Cross multiply

$$= \frac{\sin^2\alpha + (1+2\cos\alpha+\cos^2\alpha)}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{\underbrace{\sin^2\alpha + \cos^2\alpha}_{\sin^2\alpha+\cos^2\alpha=1} + 1 + 2\cos\alpha}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{1+1+2\cos\alpha}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{2+2\cos\alpha}{\sin\alpha(1+\cos\alpha)}$$

$$= \frac{2\cancel{(1+\cos\alpha)}}{\sin\alpha\cancel{(1+\cos\alpha)}}$$

$$= \frac{2}{\sin\alpha}$$

Question 5

5.	Finds $\frac{dy}{dx} = 8x - 6$	M1
	Gradient of curve at *P* is –2	M1
	Normal gradient is $\frac{-1}{m} = \frac{1}{2}$	M1
	Thus equation of normal is $(y - 2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x + 7$	A1
	Eliminate *y* between $= \frac{1}{2}x + \ln(2x)$ and normal equation to give an equation in *x*	M1
	Solve $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$	M1
	Subsitute to find a value for *y*	M1
	Point *Q* is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}}, \frac{7}{4}\right)$	A1 (8)
		(8 marks)

Section B: Statistics

Question 6

Question	Scheme	Marks
(a)	[P(both blue) $=\frac{1}{20}\times\frac{1}{20}=$] $\frac{1}{400}$ oe	B1
		(1)
(b)	P(exactly 1 red) $=2\times\frac{1}{20}\times\frac{19}{20}, = \frac{19}{200}$ oe	M1, A1
		(2)
(c)	P(2 yellow and 1 green)$=3\times\frac{4}{9}\times\frac{5}{8}\times\frac{4}{7}=\frac{10}{21}$ oe	B1 M1 A1
		(3)
(d)	P(All beads are yellow) $=\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}$	M1
	P(At least 1 bead is green) = 1 – P(All beads are yellow)	
	$1-\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}=\frac{121}{126}$	M1A1
		(3)
		Total 9

	Notes	
(a)	B1 $\frac{1}{400}$ or 0.0025	
(b)	M1 for a correct equivalent expression $\frac{1}{20}\times\frac{19}{20}+\frac{19}{20}\times\frac{1}{20}$	
	A1 $\frac{19}{200}$ or 0.095	
(c)	B1 for 3 × … <u>or</u> for the sum of exactly 3 identical products attempted	
	M1 for any one product correct	
	A1 $\frac{10}{21}$ (allow awrt 0.476 from correct working)	
(d)	1st M1 $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}$	
	2nd M1 Use of $1-p$ (where p is a product of 4 probabilities)	
	A1 $\frac{121}{126}$ (condone awrt 0.960 must be at least 3sf from correct working)	
	OR	
	1st M1 List **all** 15 favourable outcomes <u>and</u> at least one correct product	
	(YYYG)×4 [(YYGY), (YGYY), (GYYY)]	
	(YYGG)×6 [(YGYG), (YGGY), (GYYG), (GYGY), (GGYY)]	
	(GGYG)×4 [(GGGY), (YGGG), (GYGG)]	
	(GGGG)	
	2nd M1 Sum all 15 correct probabilities	
	A1 $\frac{121}{126}$ (condone awrt 0.960 must be at least 3sf from correct working)	

Question 7

Question	Scheme	Marks
(a)	(Time is) continuous	B1 (1)
(b)	40 people = 8 large squares/200 small squares 200 people = 40 large squares/1000 small squares 40/(21 – 11) or correct scale on f.d. axis	B1
	$\frac{x}{40}=\frac{180}{200}$ or $\frac{x}{40}=\frac{7.2}{8}$ or $(21-18)\times 4+(25-21)\times 6$	M1
	36 people (spent between 18 and 25 minutes shopping in the supermarket)	A1 (3)
(c)	$\text{Median}=26+\frac{[30]}{36}\times 5=\text{awrt } \underline{\mathbf{30.2}}$	M1A1 (2)
(d)	$\sum fx \; =16\times 40+23.5\times 30+28.5\times 36+33.5\times 40+38.5\times 14+46\times 20+61\times 20$	M1
	$=6390$ **	A1cso (2)
(e)	i $\bar{x}=\frac{6390}{200}=31.95$	B1
	ii $\sigma=\sqrt{\frac{238430}{200}-31.95^2}=\sqrt{171.3475}=13.09\,(\text{or } s=13.122)$ awrt $\underline{\mathbf{13.1}}$	M1A1 (3)

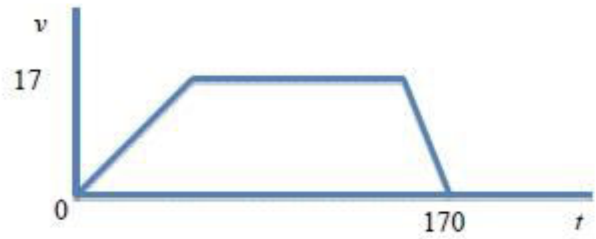
	Notes	
(a)	Allow not discrete. Condone misspellings if intention of 'continuous' is clear.	
(b)	B1 for establishing a ratio (usually 5 or 1/5) between people and area <u>or</u> calculating f.d. (may be implied by M1)	
	M1 for a correct ratio <u>or</u> expression using areas for the people from 18 to 25	
	A1 36 cao (Answer of 36 scores 3 out of 3).	
(c)	M1 for an attempt at the medians (should have 26, 36 and 5). If working down $31-\frac{[6]}{36}\times 5$	
	A1 awrt 30.2 (can come from using (n+1))	
(d)	M1 for a correct expression for $\sum fx$ condone one incorrect product	
	A1cso for 6390 and all correct	
(e)(i)	B1 31.95 or equivalent fraction	
(ii)	M1 for correct expression for standard deviation including root	
	A1 awrt 13.1 (answer of awrt 13.1 scores 2 out of 2) [NB (s = 13.122)]	

Section C: Mechanics

Question 8

Question Number	Scheme	Marks	Notes
		M1	Use $s = ut + \frac{1}{2}at^2$ or a complete *suvat* route to find h in terms of t
	$h = \frac{1}{2}gt^2$	A1	Or $h = \frac{1}{2}g(t+1)^2$. The expression for time used in the first equation defines the expression expected in the second equation.
	$h = 19.6(t-1) + \frac{1}{2}g(t-1)^2$	A1	Or $h = 19.6(t) + \frac{1}{2}g(t)^2$ or $h = 4.9 + \left(9.8t + \frac{1}{2}gt^2\right)$
	$\frac{1}{2}gt^2 = 19.6(t-1) + \frac{1}{2}g(t-1)^2$	M1	Equate the two expressions for h.
		DM1	Solve for t. Dependent on the previous M1.
	$t = 1.5$	A1	Using the "Or" approach gives $t = 0.5$
	$h = 11$ m or 11.0 m	A1	Accept 2 or 3 s.f. only
		7	

Question 9

Question Number	Scheme	Marks	Notes
(a)		B1	shape
		B1	rel grad - RHS steeper than LHS
		B1 (3)	17 and 170 shown
(b)	$T; 2T$	B1	Correct ratios of times for acceleration and deceleration seen or implied.
	$\frac{170+(170-3T)}{2}17=2125$ Or $\frac{1}{2}\times T_1\times 17+17(170-(T_1+T_2))+\frac{1}{2}\times 17\times T_2=2125$ Or $2125=\frac{17}{2}(170+T')$	M1	Form an equation for total distance with their times
		A2	-1 each error
	$T=30$ Or $T_1+T_2=90$	A1	Use their equation and the correct ratio to find the value for time decelerating or the total of time accelerating and decelerating
		M1	Use of $v=u+at$ or equivalent
	decel $=\frac{17}{30}$ oe	A1	$(0.5\dot{6})$ 3sf or better. Must be positive.
		(7)	
		10	

Question 10

Question Number	Scheme	Marks	Notes
(a)			
		M1	One equation of motion. Requires all terms but condone sign errors
	$4mg - T = \pm 4ma$	A1	
		M1	A second equation of motion of P. Requires all terms but condone sign errors
	$T - F = \pm ma$	A1	Signs of a must be consistent
			Condone use of $4mg - F = 5ma$ in place of either of the above equations.
	$4mg - 0.5mg = 5ma$ $a = 0.7g$ or $4mg - T = 4T - 2mg$	DDM1	Solve for T Dependent on the two preceding M marks
	$T = 1.2mg$	A1	
		(6)	
(b)	$v^2 = 2 \text{ x } 0.7gh$	M1	Complete method to an equation in v or v^2
	$v = \sqrt{1.4gh}$ *	A1	Obtain **given answer** or exact equivalent from exact working with no errors seen.
		(2)	
(c)	$-0.5mg = ma'$	M1	Complete method to find the deceleration of P
	$\Rightarrow a' = -0.5g$	A1	
		M1	Complete method to find additional distance on terms of h $(a \neq 0.7g, a \neq g)$
	$0^2 = 1.4gh - 2 \text{ x } 0.5g \text{ x } d$	A1	Correctly substituted equation. Follow their $a \neq 0.7g$, $a \neq g$.
	$d = 1.4h$	A1	
	Hence, length of string is greater than $1.4h + h = 2.4h$	A1	Obtain **given answer** with no errors seen. Their statement needs to reflect the inequality.