

**End of Year 12 AS Pure & Applied - Homework 3 (2 hr)**

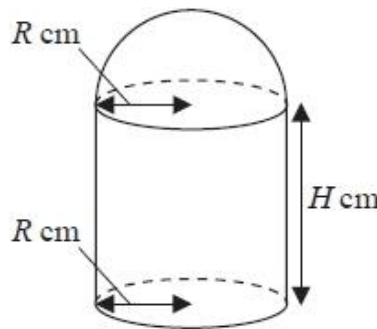
**Section A: Pure Mathematics**

**Question 1**

[In this question you may assume the formula for the area of a circle and the following formulae:

a **sphere** of radius  $r$  has volume  $V = \frac{4}{3}\pi r^3$  and surface area  $S = 4\pi r^2$

a **cylinder** of radius  $r$  and height  $h$  has volume  $V = \pi r^2 h$  and curved surface area  $S = 2\pi rh$



**Figure 5**

Figure 5 shows the model for a building. The model is made up of three parts. The roof is modelled by the curved surface of a hemisphere of radius  $R$  cm. The walls are modelled by the curved surface of a circular cylinder of radius  $R$  cm and height  $H$  cm. The floor is modelled by a circular disc of radius  $R$  cm. The model is made of material of negligible thickness, and the walls are perpendicular to the base.

It is given that the volume of the model is  $800\pi$  cm<sup>3</sup> and that  $0 < R < 10.6$

(a) Show that

$$H = \frac{800}{R^2} - \frac{2}{3}R \tag{2}$$

(b) Show that the surface area,  $A$  cm<sup>2</sup>, of the model is given by

$$A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R} \tag{3}$$

(c) Use calculus to find the value of  $R$ , to 3 significant figures, for which  $A$  is a minimum. (5)

(d) Prove that this value of  $R$  gives a minimum value for  $A$ . (2)

(e) Find, to 3 significant figures, the value of  $H$  which corresponds to this value for  $R$ . (1)

**(Total for question = 13 marks)**

---

## Question 2

- (i) Solve, giving an exact answer, the equation

$$\log_2(x + 3) - \log_2(2x + 4) = 4$$

(You should show each step in your working.) (4)

- (ii) Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} = x + 1$$

$$\ln(x - 2) = 2y - 1$$

(7)

**(Total for question = 11 marks)**

---

## Question 3

The mass,  $m$  grams, of a radioactive substance  $t$  years after first being observed, is modelled by the equation

$$m = 25e^{1-kt}$$

where  $k$  is a positive constant.

- (a) State the value of  $m$  when the radioactive substance was first observed. (1)

Given that the mass is 50 grams, 10 years after first being observed,

- (b) show that  $k = \frac{1}{10} \ln\left(\frac{1}{2}e\right)$  (4)

- (c) Find the value of  $t$  when  $m = 20$ , giving your answer to the nearest year. (3)

**(Total for question = 8 marks)**

---

## Question 4

- (i) Showing each step in your reasoning, prove that

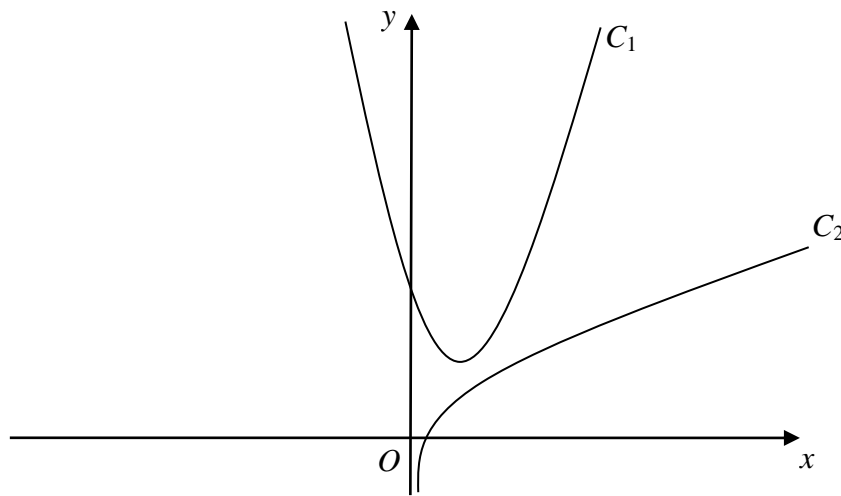
$$(\sin x + \cos x)(1 - \sin x \cos x) \equiv \sin^3 x + \cos^3 x \quad (3)$$

- (ii)  $\frac{\sin Q}{1 + \cos Q} + \frac{1 + \cos Q}{\sin Q} \equiv \frac{2}{\sin Q}$  (4)

**(Total for question = 7 marks)**

---

---

**Question 5**Diagram not  
drawn to scale**Figure 3**

The curve  $C_1$ , shown in Figure 3, has equation  $y = 4x^2 - 6x + 4$ .

The point  $P\left(\frac{1}{2}, 2\right)$  lies on  $C_1$ .

The curve  $C_2$ , also shown in Figure 3, has equation  $y = \frac{1}{2}x + \ln(2x)$ .

The normal to  $C_1$  at the point  $P$  meets  $C_2$  at the point  $Q$ .

Find the exact coordinates of  $Q$ .

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

**(8)****(Total for Question 15 is 8 marks)**

---



---

## Section B: Statistics

### Question 6

A bag contains 19 red beads and 1 blue bead only.

Linda selects a bead at random from the bag. She notes its colour and replaces the bead in the bag. She then selects a second bead at random from the bag and notes its colour

Find the probability that

- (a) both beads selected are blue, (1)
- (b) exactly one bead selected is red. (2)

In another bag there are 9 beads, 4 of which are green and the rest are yellow.

Linda selects 3 beads from this bag at random without replacement.

- (c) Find the probability that 2 of these beads are yellow and 1 is green. (3)

Linda replaces the 3 beads and then selects another 4 at random without replacement.

- (d) Find the probability that at least 1 of the beads is green. (3)

**(Total for question = 9 marks)**

---

### Question 7

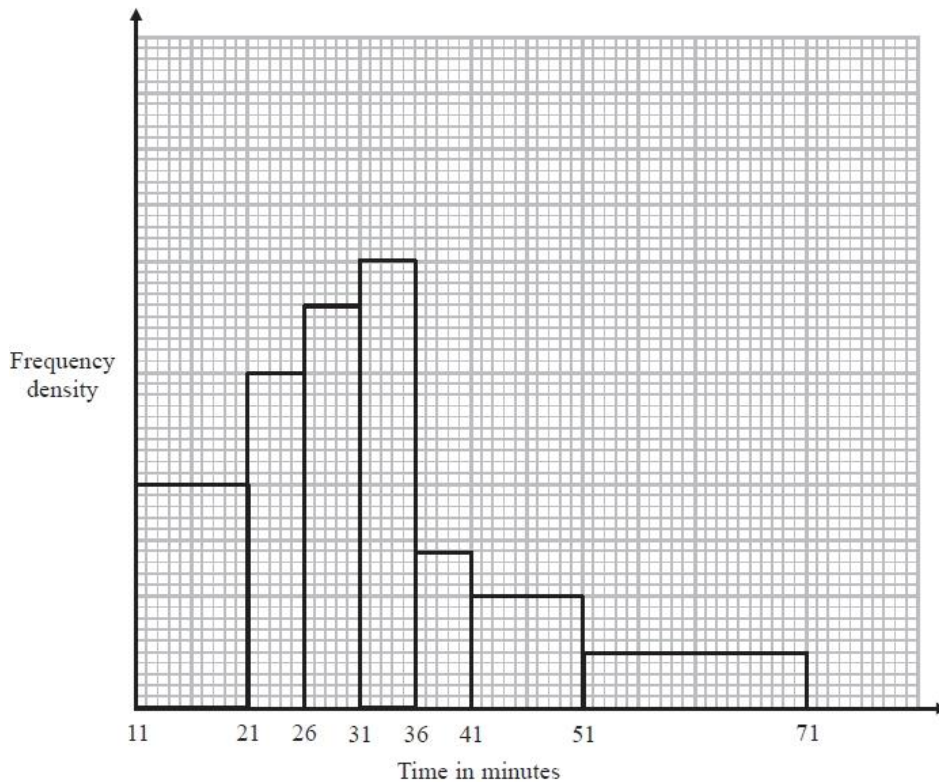


Figure 1

The histogram in Figure 1 summarises the times, in minutes, that 200 people spent shopping in a supermarket.

(a) Give a reason to justify the use of a histogram to represent these data. (1)

Given that 40 people spent between 11 and 21 minutes shopping in the supermarket, estimate

(b) the number of people that spent between 18 and 25 minutes shopping in the supermarket, (3)

(c) the median time spent shopping in the supermarket by these 200 people. (2)

The mid-point of each bar is represented by  $x$  and the corresponding frequency by  $f$ .

(d) Show that  $\sum f x = 6390$  (2)

Given that  $\sum f x^2 = 238\,430$

(e) for the data shown in the histogram, calculate estimates of

(i) the mean,

(ii) the standard deviation. (3)

**(Total for question = 11 marks)**

**Section C: Mechanics**

**Question 8**

A small stone is released from rest from a point  $A$  which is at height  $h$  metres above horizontal ground. Exactly one second later another small stone is projected with speed  $19.6 \text{ m s}^{-1}$  vertically downwards from a point  $B$ , which is also at height  $h$  metres above the horizontal ground. The motion of each stone is modelled as that of a particle moving freely under gravity. The two stones hit the ground at the same time.

Find the value of  $h$ .

(7)

**(Total for question = 7 marks)**

---

**Question 9**

A car travelling along a straight horizontal road takes 170s to travel between two sets of traffic lights at  $A$  and  $B$  which are 2125m apart. The car starts from rest at  $A$  and moves with constant acceleration until it reaches a speed of  $17\text{ms}^{-1}$ . The car then maintains this speed before moving with constant deceleration, coming to rest at  $B$ . The magnitude of the deceleration is twice the magnitude of the acceleration.

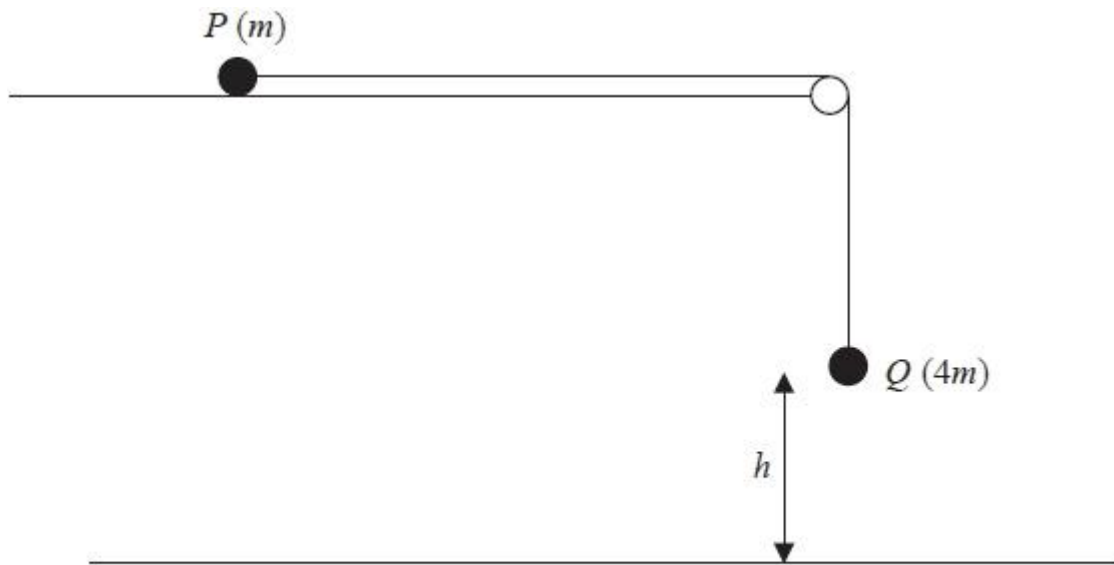
(a) Sketch a speed-time graph for the motion of the car between  $A$  and  $B$ . (3)

(b) Find the deceleration of the car. (7)

**(Total for question = 10 marks)**

---

**Question 10**



**Figure 3**

Two particles  $P$  and  $Q$  have masses  $m$  and  $4m$  respectively. The particles are attached to the ends of a light inextensible string. Particle  $P$  is held at rest on a rough horizontal table.

The string lies along the table and passes over a small smooth light pulley which is fixed at the edge of the table. Particle  $Q$  hangs at rest vertically below the pulley, at a height  $h$  above a horizontal plane, as shown in Figure 3.  $P$  experiences a frictional force of  $0.5mg$ . Particle  $P$  is released from rest with the string taut and slides along the table.

- (a) Find, in terms of  $mg$ , the tension in the string while both particles are moving. (6)

The particle  $P$  does not reach the pulley before  $Q$  hits the plane.

- (b) Show that the speed of  $Q$  immediately before it hits the plane is  $\sqrt{1.4gh}$  (2)

When  $Q$  hits the plane,  $Q$  does not rebound and  $P$  continues to slide along the table. Given that  $P$  comes to rest before it reaches the pulley,

- (c) show that the total length of the string must be greater than  $2.4h$  (6)

**(Total for question = 14 marks)**

**TOTAL FOR PAPER IS 98 MARKS**