End of Year 12 AS Pure & Applied - Homework 4 (2 hr) MARK SCHEME

Section A: Pure Mathematics

Question 1

a (i) $LHS = (2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2$

$= 4\cos^2 x + \cancel{4\sin x\cos x} + \sin^2 x + \cos^2 x - \cancel{4\cos x\sin x} + 4\sin^2 x$

$= 5\cos^2 x + 5\sin^2 x$

$= 5(\cos^2 x + \sin^2 x)$ $\quad [\cos^2 x + \sin^2 x = 1]$

$= 5(1)$

$= 5$

$= RHS$

(ii) $LHS = \cos x + \sin x \tan x$

$= \cos x + \sin x\left(\frac{\sin x}{\cos x}\right)$

$= \cos x + \frac{\sin^2 x}{\cos x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos x}$ $\quad [\cos^2 x + \sin^2 x = 1]$

$= \frac{1}{\cos x}$

$= RHS$

(b)	$\tan(2\theta + 30) = \frac{3}{7}$	B1ft
	$\tan^{-1}\text{"}\frac{3}{7}\text{"}\ (\alpha)$	M1
	One of $\theta =$ awrt 87 or awrt 177 or awrt 267 or awrt 357	A1
	Follow through any of their final θs for $\theta \pm 90n$ within range	A1ft
	All of $\theta =$ 86.6, 176.6, 266.6, 356.6	A1
		[5]

Question 2

Question Number	Scheme	Marks
(i)	$\log_a x + \log_a 3 = \log_a 27 - 1$ so $\log_a \frac{3x}{27} = -1$ Or $\log_a x + \log_a 3 = \log_a 27 - \log_a a$ so $\log_a 3x = \log_a \frac{27}{a}$ Or $\log_a x + 1 = \log_a 27 - \log_a 3 = \log_a 9$ so $\log_a ax = \log_a 9$	M1 A1
	$\frac{3x}{27} = a^{-1}$	M1
	$x = 9a^{-1}$ or $\frac{9}{a}$	A1
		[4]
(ii)	$x^2 - 7x + 12 = 0$ and attempt to solve to give $x = \ldots$ or $\log_5 y = \ldots$ (implied by correct answers)	M1
	x (or $\log_5 y$) = 3 and 4	A1
	$y = 5^3$ or 5^4	dM1
	$y = 125$ and 625	A1
		[4]
		8 marks

Question 3

Question Number	Scheme	Marks
(a)	$320\ (°C)$	B1 [1]
(b)	$T = 180 \Rightarrow 300e^{-0.04t} = 160, \Rightarrow e^{-0.04t} = \frac{160}{300}$ (awrt 0.53)	M1, A1
	$t = \frac{1}{-0.04}\ln\left(\frac{160}{300}\right)$ *or* $\frac{1}{0.04}\ln\left(\frac{300}{160}\right)$	dM1
	15.7 (minutes) cao	A1cso [4]
(c)	$\frac{dT}{dt} = (-0.04) \times 300e^{-0.04t} = (-0.04) \times (T - 20)$	M1 A1
	$= \frac{20 - T}{25}$ *	A1* [3]
		(8 marks)
Alt (b)	Puts $T = 180$ so $180 = 300e^{-0.04t} + 20$ and $300e^{-0.04t} = 160$	M1
	$\ln 300 - 0.04t = \ln 160 \Rightarrow t = \ldots,\ \frac{\ln 300 - \ln 160}{0.04}$	dM1, A1
	15.7 (minutes) cao	A1cso [4]

Question 4

(a)	$\log_{10} P = mt + c$	M1
	$\log_{10} P = \frac{1}{200}t + 5$	A1 (2)
(b)	As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	M1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	M1
	so a = 100 000 **or** b = 1.0116	A1
	both a = 100 000 **and** b = 1.0116 (awrt 1.01)	A1 (4)
(c)	(i) The initial population	B1
	(ii) The proportional increase of population each year	B1 (2)
(d)	(i) 300 000, to nearest hundred thousand	B1
	(ii) Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give t =	M1
	60.2 years to 3 sf	A1 ft (3)
(e)	Any two valid reasons, e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth	B2 (2) **(13 marks)**

Question 5

(a) Recall that $\cos(x)$ oscillates between 1 and -1. So the maximum value occurs when $\cos(15t) = -1$ to give $T_{\max} = 12 - 3(-1) = 15$. Similarly the minimum value is $T_{\min} = 12 - 3(1) = 9$.

Answer: Maximum = 15°C. Minimum = 9°C. **A1 A1**

(b) We want to solve $12 - 3\cos(15t) = 10$ for $0 \leq t < 24$. Here is one way to solve:

Let $X = 15t$, then the equation becomes $12 - 3\cos X = 10$ for $0 \leq X < 360$. **M1**

This gives $\cos X = \frac{2}{3} \Rightarrow X_{\text{principal}} = 48.18....$ **A1**

The other value of X in range is $360 - 48.18... = 311.81...$ **A1**

Now we reverse our substitution to get: $15t = 48.18...$ or $311.81... \Rightarrow t = 3.21$ or $t = 20.8$ to 3 sf.

Answer: $t = 3.21$ or $t = 20.8$ to 3 sf. **A1A1**

(c) We want to add 273 to all of our values of T from the original model. So a refined model that does this is $T = 12 - 3\cos(15t) + 273 = 285 - 3\cos(15t)$.

Answer: $T = 285 - 3\cos(15t)$ **A1**

Question 6

Since the rate of flow of liquid out of the tank is proportional to the amount of liquid in the tank, our model for V must be exponential, i.e. it must have the form

$$V = Ae^{kt}$$ **B1**

We know the initial amount of water in the tank is 300 cm^3, so $A = 300$. Then to find k, we use that at $t = 2$, $V = 80$, so

$$80 = 300e^{2k}$$ **M1**

$$\Rightarrow e^{2k} = \frac{8}{30}$$

$$\Rightarrow \ln(e^{2k}) = \ln\frac{8}{30}$$ **M1A1**

$$\Rightarrow 2k = \ln\frac{8}{30}$$

$$\Rightarrow k = -0.6608...$$ **A1**

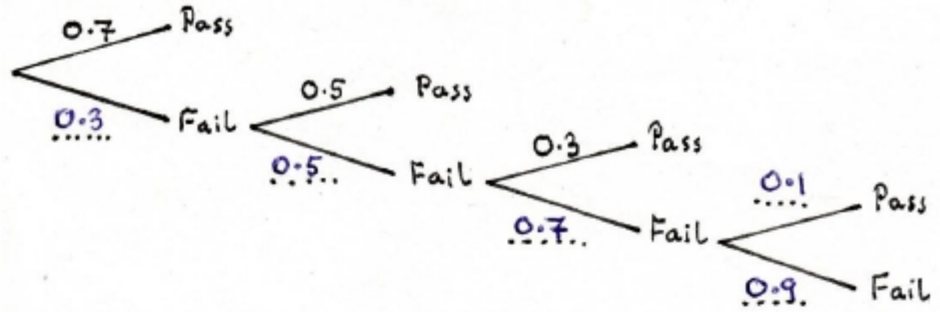
so our model is $V = 300e^{-0.661t}$.

Answer: $V = 300e^{-0.661t}$. **A1**

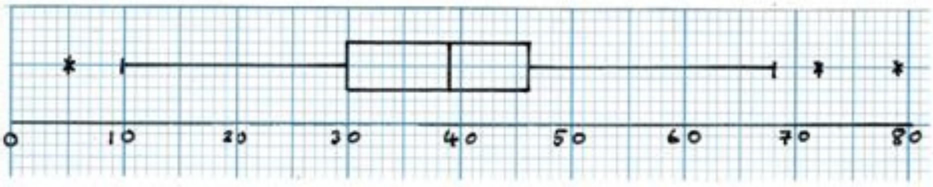
Always give values to three significant figures in Pure unless stated otherwise

Section B: Statistics

Question 7

Question Number	Scheme	Marks
(a)		B1 B1 (2)
(b)	$1-0.3\times0.5\times0.7\times0.9$ <u>or</u> $0.7+(0.3\times0.5)+(0.3\times0.5\times0.3)+(0.3\times0.5\times0.7\times0.1)$ $=\underline{0.9055}$	M1 A1 (2)
(c)	$p+(1-p)(p-0.2)$ <u>or</u> $1-(1-p)(1.2-p)$ (o.e.) e.g. $p+p-p^2+0.2p-0.2=0.95 \rightarrow p^2-2.2p+1.15=0$ (*)	M1 dM1A1cso (3)
(d)	$p=\frac{2.2\pm\sqrt{2.2^2-4\times1.15}}{2}$ <u>or</u> Complete the sq: $(p-1.1)^2-1.1^2+1.15=0$ $=\frac{2.2\pm0.4898...}{2}$ <u>or</u> $\frac{2.2\pm\sqrt{0.24}}{2}$ <u>or</u> $1.1\pm\sqrt{0.06}$ <u>or</u> $(1.34\ldots)$, $0.855\ldots$ $p=0.85505102\ldots$ $p=\underline{0.855}$	M1 A1 A1 (3)

Question 8

Question Number	Scheme	Marks
(a)	[pass for] $\underline{30}$ (labelled or 1st answer)	B1 (1)
(b)	[merit for] $\underline{46}$ (labelled or 2nd answer)	B1 (1)
(c)	$[1.5(Q_3 - Q_1) = 1.5\times 16 = 24]$ so $c = \underline{70}$ and $d = \underline{6}$	B1, B1 (2)
(d)	$\underline{68, 72, 79}$	B2/1/0 (2)
(e)	$5 < d$ therefore 5 is an outlier 	M1 A1 A1 (3)
(f)	$\frac{1}{2}\times\left(\frac{1}{4}\right)^2\times 3$ $= \underline{\frac{3}{32}}$	M1M1 A1 (3) [12 marks]

Question 9

(a) **Answer:** It corresponds to a total daily sunshine reading that is greater than 24 hours, which is impossible.

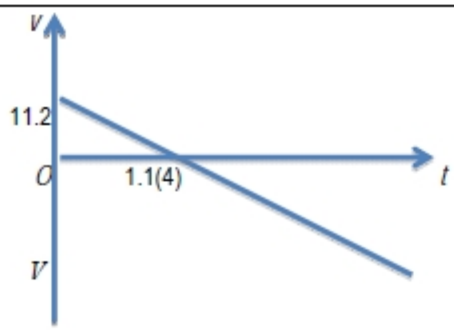
(b) **Answer:** For every 1°C increase in temperature, the total daily sunshine increases by 0.297 hours.

(c) **Answer:** This is the regression line for s on t which should only be used to find values of s given t / should use the regression line for t on s instead.

(d) **Answer:** Pryia's claim must be wrong. It is not possible for the data to be associated with Perth, since there is no data on total daily sunshine for Perth (or any of the overseas locations).

Section C: Mechanics

Question 10

Question Number	Scheme	Marks
(a)	$0^2 = 11.2^2 - 2gd$ $d = 6.4$ max ht. $= 3.6 + 6.4 = 10$ m	M1 A1 A1 A1 (4)
ALT	$11.2^2 = u^2 - 2g \text{ x } 3.6$ $u = 14$ $0^2 = 14^2 - 2gh$ $h = 10$ m	M1 A1 A1 A1 (4)
(b)	$10 = \frac{1}{2}gt^2$ $t = \frac{10}{7}$ $\text{Total} = 2\text{x}\frac{10}{7} = 2.9 \text{ or } 2.86$	M1 A1 A1 dM1 A1 (5)
(c)		B1 single line dB1 $V <$ -11.2 B1 11.2 B1 1.1(4) (4) **13**

Question 11

Question Number	Scheme	Marks
(a)	For truck: $D - 600 - 400 = 2400 \text{ x } 0.5$ $D = 2200\text{N}$	M1 A1 A1 (3)
(b)	For both: $D - 600 = (M + 2400) \text{ x } 0.5$ (or trailer: 600 - 200 = M x0.5) $M = 800$ $M = 800$	M1 A1 A1 (3)
(c)	Truck and trailer have same acceleration.	B1 (1) 7
	Notes **Can mark (a) and (b) 'together' if it helps the candidate, provided no wrong working seen.**	
1(a)	M1 for NL2 for truck only (or for a complete method if they find M first), with correct no. of terms, in D only. (M0 if 600 or 400 is replaced by 200) First A1 for a correct equation . Second A1 for 2200 (N). .	
1(b)	M1 for NL2 for whole system or trailer only, with correct no. of terms. First A1 for a correct equation. (Allow 'D' or their D) Second A1 for 800. **N.B.** In both parts of this question use the mass which is being used in their equation to guide you as to which part of the system is being considered.	
1(c)	B0 if extras included. E.g if 'tension is same' is included. B1 Must include 'truck and trailer' or 'both particles' or 'accln is same throughout the system' B0 for 'accln is same'	