Name ....

Marks............................./100

End of Year 12 AS Pure & Applied - Homework 4 (2 hr)

Section A: Pure Mathematics

Question 1

a) Prove the following trigonometric equations:

(i) $(2\cos x + \sin x)^2 + (\cos x - 2\sin x)^2 \equiv 5$ (3)

(ii) $\cos x + \sin x \tan x \equiv \frac{1}{\cos x}$ (3)

(b) Solve for $0 \leq \theta \leq 360°$

$$7\sin(2\theta + 30°) = 3\cos(2\theta + 30°)$$

giving your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.) (5)

(Total for question = 11 marks)

Question 2

(i) Given that

$$\log_a x + \log_a 3 = \log_a 27 - 1, \text{ where } a \text{ is a positive constant}$$

find, in its simplest form, an expression for x in terms of a. (4)

(ii) Solve the equation

$$(\log_5 y)^2 - 7(\log_5 y) + 12 = 0$$

showing each step of your working. (4)

(Total for question = 8 marks)

Question 3

A hot piece of metal is dropped into a cool liquid. As the metal cools, its temperature T degrees Celsius, t minutes after it enters the liquid, is modelled by

$$T = 300e^{-0.04t} + 20, \qquad t \geq 0$$

(a) Find the temperature of the piece of metal as it enters the liquid. **(1)**

(b) Find the value of t for which $T = 180$, giving your answer to 3 significant figures.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(4)

(c) Show, by differentiation, that the rate, in degrees Celsius per minute, at which the temperature of the metal is changing, is given by the expression.

$$\frac{20 - T}{25}$$

(3)

(Total for question = 8 marks)

Question 4

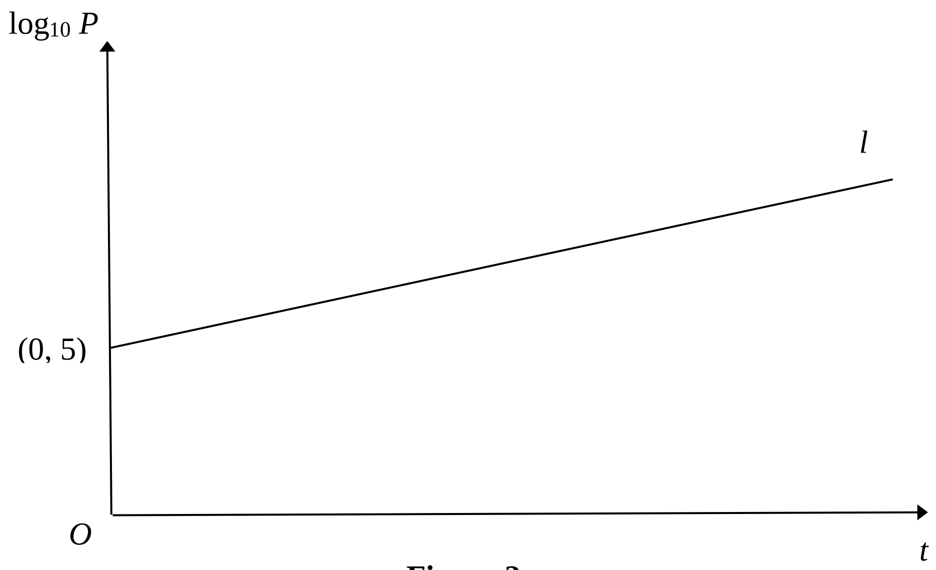


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded. The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l. **(2)**

(b) Find the value of a and the value of b. **(4)**

(c) With reference to the model, interpret

(i) the value of the constant a,

(ii) the value of the constant b. **(2)**

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model. **(3)**

(e) State two reasons why this may not be a realistic population model. **(2)**

(Total for Question = 13 marks)

Question 5

The temperature of a room, T °C, is modelled using the equation

$$T = 12 - 3\cos(15t)°$$

where t is the time in hours since midnight and $0 \leqslant t < 24$. **(2)**

(a) Write down the maximum and minimum temperature of the room as given by the model.

(b) Calculate the values of t for which the temperature of the room is 10 °C. **(5)**

One other unit for temperature is the Kelvin.

To convert a temperature from degrees Celsius (°C) to Kelvin (K), 273 is added to the value. For example, 0 °C = 273 K.

(c) Refine the model for T so that the units for T are in Kelvin. **(1)**

(Total for question 5 = 8 marks)

Question 6

A tank is initially completely filled with liquid. An outlet is opened at the bottom of the tank and the liquid begins to drain from the tank.

At time t minutes after the outlet is opened, the amount of liquid in the tank is V cm^3.

Kyle creates a model for the liquid flow out of the tank. The model includes the following assumptions:

- the initial volume of liquid in the tank is 300 cm^3,
- it takes two minutes for the volume of liquid in the tank to reach 80 cm^3,
- the rate of flow of liquid out of the tank is proportional to the amount of liquid in the tank.

Using Kyle's model, find an expression for V in terms of t. **(6)**

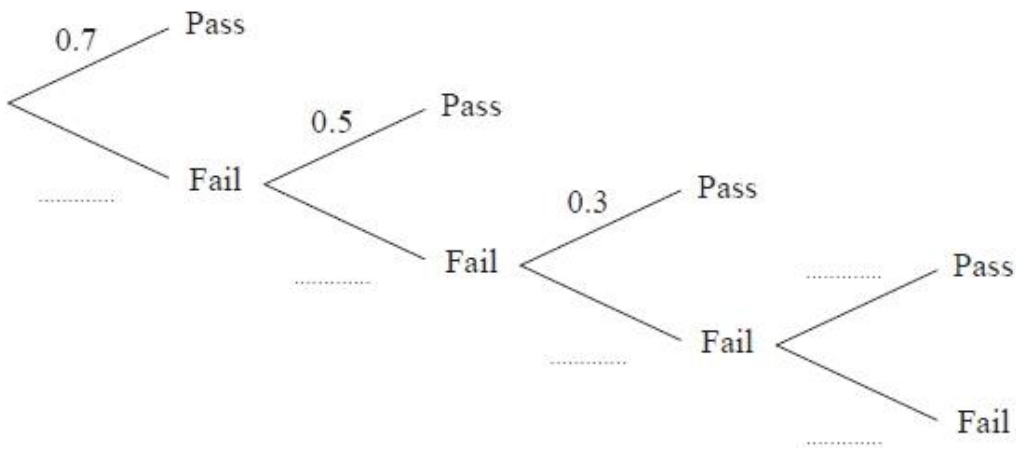
(Total for question 6 = 6 marks)

Section B: Statistics

Question 7

A training agency awards a certificate to each student who passes a test while completing a course.
Students failing the test will attempt the test again up to 3 more times, and, if they pass the test, will be awarded a certificate.
The probability of passing the test at the first attempt is 0.7, but the probability of passing reduces by 0.2 at each attempt.

(a) Complete the tree diagram below to show this information.



(2)

A student who completed the course is selected at random.

(b) Find the probability that the student was awarded a certificate. (2)

The training agency decides to alter the test taken by the students while completing the course, but will not allow more than 2 attempts. The agency requires the probability of passing the test at the first attempt to be p, and the probability of passing the test at the second attempt to be $(p - 0.2)$. The percentage of students who complete the course and are awarded a certificate is to be 95%

(c) Show that p satisfies the equation

$$p^2 - 2.2p + 1.15 = 0$$ (3)

(d) Hence find the value of p, giving your answer to 3 decimal places. (3)

(Total for question = 10 marks)

Question 8

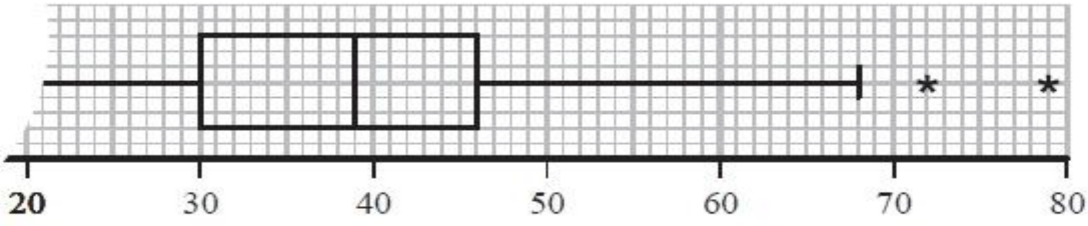


Figure 1

Figure 1 shows part of a box and whisker plot for the marks in an examination with a large number of candidates. Part of the lower whisker has been torn off.

(a) Given that 75% of the candidates passed the examination, state the lowest mark for the award of a pass.

(1)

(b) Given that the top 25% of the candidates achieved a merit grade, state the lowest mark for the award of a merit grade. (1)

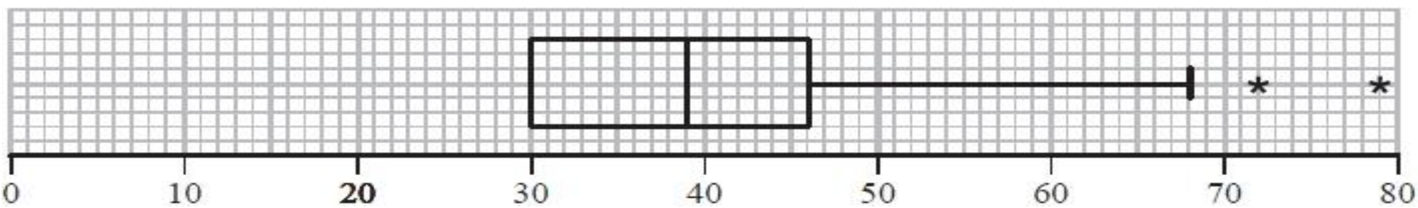
An outlier is defined as any value greater than c or any value less than d where

$$c = Q_3 + 1.5(Q_3 - Q_1)$$
$$d = Q_1 - 1.5(Q_3 - Q_1)$$

(c) Find the value of c and the value of d. (2)

(d) Write down the 3 highest marks scored in the examination. (2)

The 3 lowest marks in the examination were 5, 10 and 15



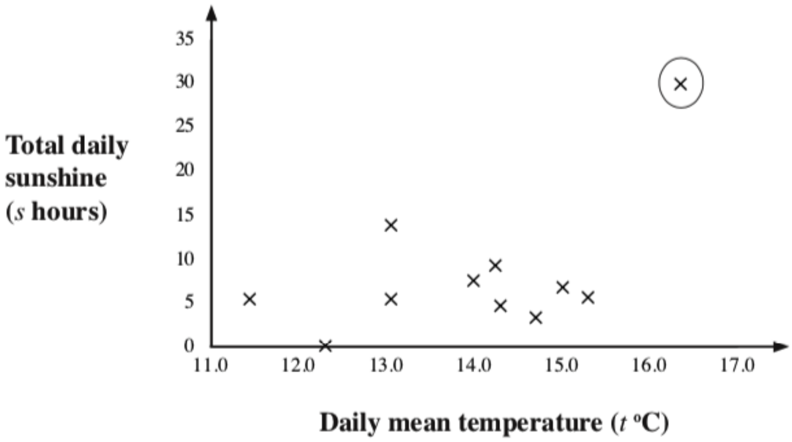
(e) On the diagram, complete the box and whisker plot. (3)

Three candidates are selected at random from those who took this examination.

(f) Find the probability that all 3 of these candidates passed the examination but only 2 achieved a merit grade. (3)

(Total for question = 12 marks)

Question 9



Priya is investigating the relationship between total daily sunshine, s hours, and daily mean temperature, t °C.

Priya's teacher gives her data about s and t for one location in the large data set.

The scatter graphs above show her data.

The point circled on the graph is an outlier.

(a) Using your knowledge of the large data set, explain why this outlier must be an error. **(1)**

Excluding the outlier, the regression line for s on t is given by $s = 0.297t + 2.160$.

(b) Interpret the figure 0.297 in the regression line. **(1)**

(c) Explain why the regression line above should **not** be used to estimate the daily mean temperature on a day with 10 hours of sunshine. **(1)**

Priya suggests the location chosen by her teacher is Perth.

(d) Comment on Priya's suggestion using your knowledge of the large data set. **(1)**

(Total for question = 4 marks)

Section C: Mechanics

Question 10

A small stone is projected vertically upwards from the point *O* and moves freely under gravity. The point *A* is 3.6 m vertically above *O*. When the stone first reaches *A*, the stone is moving upwards with speed 11.2 m s^{-1}. The stone is modelled as a particle.

(a) Find the maximum height above *O* reached by the stone. **(4)**

(b) Find the total time between the instant when the stone was projected from *O* and the instant when it returns to *O*. **(5)**

(c) Sketch a velocity-time graph to represent the motion of the stone from the instant when it passes through *A* moving upwards to the instant when it returns to *O*. Show, on the axes, the coordinates of the points where your graph meets the axes. **(4)**

(Total for question = 13 marks)

Question 11

A truck of mass 2400 kg is pulling a trailer of mass *M* kg along a straight horizontal road. The tow bar, connecting the truck to the trailer, is horizontal and parallel to the direction of motion. The tow bar is modelled as being light and inextensible. The resistance forces acting on the truck and the trailer are constant and of magnitude 400 N and 200 N respectively. The acceleration of the truck is 0.5 m s^{-2} and the tension in the tow bar is 600 N.

(a) Find the magnitude of the driving force of the truck. **(3)**

(b) Find the value of *M*. **(3)**

(c) Explain how you have used the fact that the tow bar is inextensible in your calculations. **(1)**

(Total for question = 7 marks)

TOTAL FOR PAPER IS 100 MARKS