

## Algebraic Expression - Edexcel Past Exam Questions 2 MARK SCHEME

### Question 1

Question	Scheme	Marks
(a)	$\sqrt{32} = 4\sqrt{2}$ or $\sqrt{18} = 3\sqrt{2}$ $(\sqrt{32} + \sqrt{18}) = \underline{7\sqrt{2}}$	B1 B1 (2)
(b)	$\times \frac{3-\sqrt{2}}{3-\sqrt{2}}$ or $\times \frac{-3+\sqrt{2}}{-3+\sqrt{2}}$ seen $\left[ \frac{\sqrt{32} + \sqrt{18}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} \right] = \frac{a\sqrt{2}(3-\sqrt{2})}{[9-2]} \rightarrow \frac{3a\sqrt{2}-2a}{[9-2]}$ (or better) $= \underline{3\sqrt{2}, -2}$	M1 dM1 A1, A1 (4)
ALT	$(b\sqrt{2} + c)(3 + \sqrt{2}) = 7\sqrt{2}$ leading to: $3b + c = 7$ , $3c + 2b = 0$ e.g. $3(7 - 3b) + 2b = 0$ (o.e.)	M1 dM1
		<b>6 marks</b>
<b>Notes</b>		
(a)	1 <sup>st</sup> B1 for either surd simplified 2 <sup>nd</sup> B1 for $7\sqrt{2}$ or accept $a = 7$ . Answer only scores B1B1 NB Common error is $\sqrt{32} + \sqrt{18} = \sqrt{50} = 5\sqrt{2}$ this scores B0B0 but can use their "5" in (b) to get M1M1	
(b)	1 <sup>st</sup> M1 for an attempt to multiply by $\frac{3-\sqrt{2}}{3-\sqrt{2}}$ (o.e.) Allow poor use of brackets 2 <sup>nd</sup> dM1 for using $a\sqrt{2}$ to correctly obtain a numerator of the form $p + q\sqrt{2}$ where $p$ and $q$ are non-zero integers. Allow arithmetic slips e.g. $21\sqrt{2} - 28$ or $3\sqrt{2} \times \sqrt{2} = 3$ Follow through their $a = 7$ or a new value found in (b). Ignore denominator. Allow use of letter $a$ . Dependent on 1 <sup>st</sup> M1 So $3\sqrt{32} - \sqrt{64} + 3\sqrt{18} - \sqrt{36}$ is M0 until they reduce $p + q\sqrt{2}$ 1 <sup>st</sup> A1 for $3\sqrt{2}$ or accept $b = 3$ from correct working 2 <sup>nd</sup> A1 for $-2$ or accept $c = -2$ from correct working	
ALT	<b>Simultaneous Equations</b> 1 <sup>st</sup> M1 for $(b\sqrt{2} + c)(3 + \sqrt{2}) = 7\sqrt{2}$ and forming 2 simultaneous equations. Ft their $a = 7$ 2 <sup>nd</sup> dM1 for solving their simultaneous equations: reducing to a linear equation in one variable	

## Question 2

Question Number	Scheme	Marks
(a)	$\left\{ (32)^{\frac{3}{5}} \right\} = (\sqrt[5]{32})^3 \text{ or } \sqrt[5]{(32)^3} \text{ or } 2^3 \text{ or } \sqrt[5]{32768}$ $= 8$	M1 A1 [2]
(b)	$\left\{ \left( \frac{25x^4}{4} \right)^{-\frac{1}{2}} \right\} = \left( \frac{4}{25x^4} \right)^{\frac{1}{2}} \text{ or } \left( \frac{5x^2}{2} \right)^{-1} \text{ or } \frac{1}{\left( \frac{25x^4}{4} \right)^{\frac{1}{2}}}$ $= \frac{2}{5x^2} \text{ or } \frac{2}{5}x^{-2}$ <p style="text-align: right;"><i>See notes below</i></p> <p style="text-align: right;"><i>See notes for other alternatives</i></p>	M1 A1 [2] 4
<b>Notes</b>		
(a)	<b>M1:</b> for a full correct interpretation of the fractional power. <b>Note:</b> $5 \times (32)^3$ is M0. <b>A1:</b> for 8 only. <b>Note:</b> Award M1A1 for writing down 8.	
(b)	<b>M1:</b> For use of $\frac{1}{2}$ OR use of $-1$ <b>Use of <math>\frac{1}{2}</math>:</b> Candidate needs to demonstrate they have rooted all three elements in their bracket. <b>Use of <math>-1</math>:</b> Either Candidate has $\frac{1}{\text{Bracket}}$ or $\left( \frac{Ax^c}{B} \right)$ becomes $\left( \frac{B}{Ax^c} \right)$ . <b>Allow M1 for...</b> $\bullet \left( \frac{4}{25x^4} \right)^{\frac{1}{2}} \text{ or } \left( \frac{5x^2}{2} \right)^{-1} \text{ or } \frac{1}{\left( \frac{25x^4}{4} \right)^{\frac{1}{2}}} \text{ or } \sqrt{\left( \frac{4}{25x^4} \right)} \text{ or } \frac{1}{\sqrt{\left( \frac{25x^4}{4} \right)}} \text{ or } \left( \frac{1}{\frac{25x^4}{4}} \right)^{\frac{1}{2}} \text{ or } \frac{1}{\frac{5x^2}{2}} \text{ or } \frac{1}{\frac{1}{2}} \text{ or } \frac{1}{\frac{1}{2}}$ $\text{or } -\left( \frac{5x^2}{2} \right) \text{ or } \left( \frac{-5x^{-2}}{-2} \right) \text{ or } -\left( \frac{5x^{-2}}{2} \right) \text{ or } \frac{5x^{-2}}{2}$ $\bullet \left( \frac{4}{25x^4} \right)^K \text{ or } \left( \frac{5x^2}{2} \right)^C \text{ where } K, C \text{ are any powers including } 1.$ <b>A1:</b> for either $\frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$ or $0.4x^{-2}$ or $\frac{0.4}{x^2}$ . <b>Note:</b> $\left( \sqrt{\left( \frac{25x^4}{4} \right)} \right)^{-1}$ is not enough work by itself for the method mark. <b>Note:</b> A final answer of $\frac{1}{\frac{5}{2}x^2}$ or $\frac{1}{2\frac{1}{2}x^2}$ or $\frac{1}{2.5x^2}$ is A0. <b>Note:</b> Also allow $\pm \frac{2}{5x^2}$ or $\pm \frac{2}{5}x^{-2}$ or $\pm 0.4x^{-2}$ or $\pm \frac{0.4}{x^2}$ for A1.	

### Question 3

Question Number	Scheme	Marks
	$\left\{ \frac{2}{\sqrt{12} - \sqrt{8}} \right\} = \frac{2}{(\sqrt{12} - \sqrt{8})} \times \frac{(\sqrt{12} + \sqrt{8})}{(\sqrt{12} + \sqrt{8})}$ $= \frac{\{2(\sqrt{12} + \sqrt{8})\}}{12 - 8}$ $= \frac{2(2\sqrt{3} + 2\sqrt{2})}{12 - 8}$ $= \sqrt{3} + \sqrt{2}$	<p>Writing this is sufficient for M1.</p> <p>For 12 – 8. This mark can be implied.</p> <p>M1</p> <p>A1</p> <p>B1 B1</p> <p>A1 cso</p> <p>5</p>
	Notes	
	<p><b>M1:</b> for a correct method to rationalise the denominator.</p> <p><b>1<sup>st</sup> A1:</b> <math>(\sqrt{12} - \sqrt{8})(\sqrt{12} + \sqrt{8}) \rightarrow 12 - 8</math> or <math>(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) \rightarrow 3 - 2</math></p> <p><b>1<sup>st</sup> B1:</b> for <math>\sqrt{12} = 2\sqrt{3}</math> or <math>\sqrt{48} = 4\sqrt{3}</math> seen or implied in candidate's working.</p> <p><b>2<sup>nd</sup> B1:</b> for <math>\sqrt{8} = 2\sqrt{2}</math> or <math>\sqrt{32} = 4\sqrt{2}</math> seen or implied in candidate's working.</p> <p><b>2<sup>nd</sup> A1:</b> for <math>\sqrt{3} + \sqrt{2}</math>. Note: <math>\frac{\sqrt{3} + \sqrt{2}}{1}</math> as a final answer is A0.</p> <p><b>Note:</b> The first accuracy mark is dependent on the first method mark being awarded.</p> <p><b>Note:</b> <math>\frac{1}{2}\sqrt{12} + \frac{1}{2}\sqrt{8} = \sqrt{3} + \sqrt{2}</math> with no intermediate working implies the B1B1 marks.</p> <p><b>Note:</b> <math>\sqrt{12} = \sqrt{4}\sqrt{3}</math> or <math>\sqrt{8} = \sqrt{4}\sqrt{2}</math> are not sufficient for the B1 marks.</p> <p><b>Note:</b> A candidate who writes down (by misread) <math>\sqrt{18}</math> for <math>\sqrt{8}</math> can potentially obtain M1A0B1B1A0, where the 2<sup>nd</sup> B1 will be awarded for <math>\sqrt{18} = 3\sqrt{2}</math> or <math>\sqrt{72} = 6\sqrt{2}</math></p> <p><b>Note:</b> The final accuracy mark is for a correct solution only.</p> <p><u>Alternative 1 solution</u></p> $\left\{ \frac{2}{\sqrt{12} - \sqrt{8}} \right\} = \frac{2}{(2\sqrt{3} - 2\sqrt{2})}$ <p>B1 B1</p> $= \frac{1}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})}$ <p>M1</p> $= \frac{\{(\sqrt{3} + \sqrt{2})\}}{3 - 2}$ <p>A1 for 3 – 2</p> $= \sqrt{3} + \sqrt{2}$ <p>A1</p> <p><u>Alternative 2 solution</u></p> $\left\{ \frac{2}{\sqrt{12} - \sqrt{8}} \right\} = \frac{2}{(2\sqrt{3} - 2\sqrt{2})} = \frac{1}{(\sqrt{3} - \sqrt{2})} = \sqrt{3} + \sqrt{2}, \text{ or } \frac{2}{(2\sqrt{3} - 2\sqrt{2})} = \sqrt{3} + \sqrt{2}$ <p>with no incorrect working seen is awarded M1A1B1B1A1.</p>	<p>Please record the marks in the relevant places on the mark grid.</p>



#### Question 4

Question Number	Scheme	Marks
	$x(1 - 4x^2)$ <p>Accept <math>x(-4x^2 + 1)</math> or <math>-x(4x^2 - 1)</math> or <math>-x(-1 + 4x^2)</math> or even <math>4x(\frac{1}{4} - x^2)</math> or equivalent quadratic (or initial cubic) into two brackets</p> $x(1 - 2x)(1 + 2x)$ or $-x(2x - 1)(2x + 1)$ or $x(2x - 1)(-2x - 1)$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>
		3 marks
	<b>Notes</b>	
	<p><b>B1:</b> Takes out a factor of <math>x</math> or <math>-x</math> or even <math>4x</math>. This line may be implied by correct final answer, but if this stage is shown it <b>must be correct</b>. So B0 for <math>x(1 + 4x^2)</math></p> <p><b>M1:</b> Factorises the quadratic resulting from their first factorisation using usual rules (see note 1 in General Principles). e.g. <math>x(1 - 4x)(1 + 4x)</math>. Also allow attempts to factorise cubic such as <math>(x - 2x^2)(1 + 2x)</math> etc N.B. Should not be completing the square here.</p> <p><b>A1:</b> Accept either <math>x(1 - 2x)(1 + 2x)</math> or <math>-x(2x - 1)(2x + 1)</math> or <math>x(2x - 1)(-2x - 1)</math>. (No fractions for this final answer)</p>	
	<b>Specific situations</b>	
	<p><b>Note:</b> <math>x(1 - 4x^2)</math> followed by <math>x(1 - 2x)^2</math> scores B1M1A0 as factors follow quadratic factorisation criteria</p> <p><b>And</b> <math>x(1 - 4x^2)</math> followed by <math>x(1 - 4x)(1 + 4x)</math> <b>B1M0A0</b>.</p>	
	<b>Answers with no working:</b> Correct answer gets all three marks B1M1A1	
	: $x(2x - 1)(2x + 1)$ gets B0M1A0 if <b>no working</b> as $x(4x^2 - 1)$ would earn B0	
	<p><b>Poor bracketing:</b> e.g. <math>(-1 + 4x^2) - x</math> gets B0 unless subsequent work implies bracket round the <math>-x</math> in which case candidate may recover the mark by the following correct work.</p>	

#### Question 5

Question Number	Scheme	Marks
	$(8^{2x+3} = (2^3)^{2x+3}) = 2^{3(2x+3)} \text{ or } 2^{ax+b} \text{ with } a = 6 \text{ or } b = 9$ $= 2^{6x+9} \text{ or } 2^{3(2x+3)} \text{ as final answer with no errors or } (y =) 6x + 9 \text{ or } 3(2x + 3)$	<p>M1</p> <p>A1</p> <p>[2]</p>
		2 marks
	<b>Notes</b>	
	<p><b>M1:</b> Uses <math>8 = 2^3</math>, and <b>multiplies</b> powers <math>3(2x + 3)</math>. Does <b>not</b> add powers. (Just <math>8 = 2^3</math> or <math>8^{\frac{1}{3}} = 2</math> is M0)</p> <p><b>A1:</b> Either <math>2^{6x+9}</math> or <math>2^{3(2x+3)}</math> or <math>(y =) 6x + 9</math> or <math>3(2x + 3)</math></p>	
	<p><b>Note:</b> Examples: <math>2^{6x+3}</math> scores M1A0</p> <p>: <math>8^{2x+3} = (2^3)^{2x+3} = 2^{3+2x+3}</math> gets M0A0</p> <p><b>Special case:</b> : <math>= 2^{6x} 2^9</math> without seeing as single power M1A0</p> <p><b>Alternative method using logs:</b> <math>8^{2x+3} = 2^y \Rightarrow (2x + 3) \log 8 = y \log 2 \Rightarrow y = \frac{(2x + 3) \log 8}{\log 2}</math></p> <p>So <math>(y =) 6x + 9</math> or <math>3(2x + 3)</math></p>	
		<p>M1</p> <p>A1 [2]</p>





## Question 6

Question Number	Scheme	Marks
(i)	$(5 - \sqrt{8})(1 + \sqrt{2})$ $= 5 + 5\sqrt{2} - \sqrt{8} - 4$ $= 5 + 5\sqrt{2} - 2\sqrt{2} - 4$ $= 1 + 3\sqrt{2}$ $\sqrt{8} = 2\sqrt{2}, \text{ seen or implied at any point.}$ $1 + 3\sqrt{2} \text{ or } a = 1 \text{ and } b = 3.$	M1 B1 A1 [3]
(ii)	<div> <div> <b>Method 1</b>  <b>Either</b> <math>\sqrt{80} + \frac{30}{\sqrt{5}} \left( \frac{\sqrt{5}}{\sqrt{5}} \right)</math>  <math>= 4\sqrt{5} + \dots</math>  <math>= 4\sqrt{5} + 6\sqrt{5}</math> </div> <div> <b>Method 2</b>  <b>Or</b> <math>\left( \frac{\sqrt{400} + 30}{\sqrt{5}} \right) \frac{\sqrt{5}}{\sqrt{5}}</math>  <math>= \left( \frac{20 + \dots}{\dots} \right) \dots</math>  <math>= \left( \frac{50\sqrt{5}}{5} \right)</math>  <math>= 10\sqrt{5}</math> </div> <div> <b>Method 3</b>  <math>\sqrt{80} + \frac{\sqrt{900}}{\sqrt{5}} = \sqrt{80} + \sqrt{180}</math>  <math>= 4\sqrt{5} + \dots</math>  <math>= 4\sqrt{5} + 6\sqrt{5}</math> </div> </div>	M1 B1 A1 [3]
Alternative for (i)	$(5 - 2\sqrt{2})(1 + \sqrt{2})$ $= 5 + 5\sqrt{2} - 2\sqrt{2} - 2\sqrt{2}\sqrt{2}$ $= 1 + 3\sqrt{2}$ <p>This earns the B1 mark.</p> <p>Multiplies out correctly with <math>2\sqrt{2}</math>. This may be seen or implied and may be simplified e.g. <math>= 5 + 3\sqrt{2} - 2\sqrt{4}</math> o.e.</p> <p>For earlier use of <math>2\sqrt{2}</math>  <math>1 + 3\sqrt{2}</math> or <math>a = 1</math> and <math>b = 3</math>.</p>	M1 B1 A1 [3] <b>6 marks</b>
<b>Notes</b>		
(i)	<b>M1:</b> Multiplies out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) – can appear as table <b>B1:</b> $\sqrt{8} = 2\sqrt{2}$ , seen or implied at any point <b>A1:</b> Fully and correctly simplified to $1 + 3\sqrt{2}$ or $a = 1$ and $b = 3$ .	
(ii)	<b>M1:</b> Rationalises denominator i.e. Multiplies $\left( \frac{k}{\sqrt{5}} \right)$ by $\left( \frac{\sqrt{5}}{\sqrt{5}} \right)$ or $\left( \frac{-\sqrt{5}}{-\sqrt{5}} \right)$ , seen or implied or uses Method 3 or similar e.g. $\left( \frac{30}{\sqrt{5}} \right) = \frac{6 \times 5}{\sqrt{5}} = 6\sqrt{5}$ <b>B1:</b> (Independent mark) States $\sqrt{80} = 4\sqrt{5}$ Or either $\sqrt{400} = 20$ or $\sqrt{80}\sqrt{5} = 20$ at any point if they use Method 2. <b>A1:</b> $10\sqrt{5}$ or $c = 10$ .	
	N.B There are other methods e.g. $\sqrt{80} = \frac{20}{\sqrt{5}}$ (B1) then add $\frac{20}{\sqrt{5}} + \frac{30}{\sqrt{5}} = \frac{50}{\sqrt{5}}$ then M1 A1as before Those who multiply initial expression by $\sqrt{5}$ to obtain $\sqrt{400} + 30 = 20 + 30 = 50$ earn M0 B1 A0	



## Question 7

Question Number	Scheme	Notes	Marks
	$\frac{15}{\sqrt{3}} = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{3}$	M1: Attempts to multiply numerator and denominator by $\sqrt{3}$ . This may be implied by a correct answer. A1: $5\sqrt{3}$	M1A1
	$\sqrt{27} = 3\sqrt{3}$		B1
	$\frac{15}{\sqrt{3}} - \sqrt{27} = 2\sqrt{3}$		A1
	Correct answer only scores full marks		
			[4]
Way 2	$\frac{15}{\sqrt{3}} - \sqrt{27} = \frac{15 - \sqrt{81}}{\sqrt{3}} \left( = \frac{6}{\sqrt{3}} \right)$	Terms combined correctly with a common denominator (Need not be simplified)	B1
	$\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3}$	M1: Attempts to multiply numerator and denominator by $\sqrt{3}$ . This may be implied by a correct answer. A1: $\frac{6\sqrt{3}}{3}$	M1A1
	$\frac{15}{\sqrt{3}} - \sqrt{27} = 2\sqrt{3}$		A1
			[4]
	Note that $\frac{15}{\sqrt{3}} - \sqrt{27} = \frac{15\sqrt{3}}{3} - 3\sqrt{3} = 15\sqrt{3} - 9\sqrt{3} = 6\sqrt{3}$ is quite common and scores M1A0B1A0 (i.e. $5\sqrt{3}$ is never seen)		

### Question 8

Question Number	Scheme	Notes	Marks
(a)	$2^y = 8 \Rightarrow y = 3$	Cao (Can be implied i.e. by $2^3$ )	B1
	(Alternative: Takes logs base 2: $\log_2 2^y = \log_2 8 \Rightarrow y \log_2 2 = 3 \log_2 2 \Rightarrow y = 3$ )		
			(1)
(b)	$8 = 2^3$	Replaces 8 by $2^3$ (May be implied)	M1
	$4^{x+1} = (2^2)^{x+1}$ or $(2^{x+1})^2$	Replaces 4 by $2^2$ correctly.	M1
	$2^{3x+2} = 2^3 \Rightarrow 3x + 2 = 3 \Rightarrow x = \frac{1}{3}$	M1: Adds their powers of 2 on the lhs and puts this equal to 3 leading to a solution for x. A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	M1A1
			(4)
(b) Way 2	$4^{x+1} = 4 \times 4^x$	Obtains $4^{x+1}$ in terms of $4^x$ correctly	M1
	$2^x \times 4^x = 8^x$	Combines their $2^x$ and $4^x$ correctly	M1
	$4 \times 8^x = 8 \Rightarrow 8^x = 2 \Rightarrow x = \frac{1}{3}$	M1: Solves $8^x = k$ leading to a solution for x. A1: $x = \frac{1}{3}$ or $x = 0.\dot{3}$ or awrt 0.333	M1A1
			[5]



# Question 9

Question Number	Scheme		Marks
	$\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$	Multiplies top and bottom by a correct expression. This statement is sufficient.	M1
	(Allow to multiply top and bottom by $k(\sqrt{5}+1)$ )		
	$= \frac{\dots}{4}$	Obtains a denominator of 4 or sight of $(\sqrt{5}-1)(\sqrt{5}+1) = 4$	A1 cso
	<b>Note that M0A1 is not possible. The 4 must come from a correct method.</b>		
	$(7+\sqrt{5})(\sqrt{5}+1) = 7\sqrt{5} + 5 + 7 + \sqrt{5}$	An attempt to multiply the numerator by $(\pm\sqrt{5} \pm 1)$ and get 4 terms with at least 2 correct for their $(\pm\sqrt{5} \pm 1)$ . (May be implied)	M1
	$3 + 2\sqrt{5}$	Answer as written or $a = 3$ and $b = 2$ . (Allow $2\sqrt{5} + 3$ )	A1 cso
	<b>Correct answer with no working scores full marks</b>		
			[4]
Way 2	$\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{(-\sqrt{5}-1)}{(-\sqrt{5}-1)}$	Multiplies top and bottom by a correct expression. This statement is sufficient.	M1
	(Allow to multiply top and bottom by $k(-\sqrt{5}-1)$ )		
	$= \frac{\dots}{-4}$	Obtains a denominator of -4	A1 cso
	$(7+\sqrt{5})(-\sqrt{5}-1) = -7\sqrt{5} - 5 - 7 - \sqrt{5}$	An attempt to multiply the numerator by $(\pm\sqrt{5} \pm 1)$ and get 4 terms with at least 2 correct for their $(\pm\sqrt{5} \pm 1)$ . (May be implied)	M1
	$3 + 2\sqrt{5}$	Answer as written or $a = 3$ and $b = 2$	A1 cso
	<b>Correct answer with no working scores full marks</b>		
			[4]
	<b>Alternative using Simultaneous Equations:</b> $\frac{(7+\sqrt{5})}{\sqrt{5}-1} = a + b\sqrt{5} \Rightarrow 7 + \sqrt{5} = (a-b)\sqrt{5} + 5b - a$ M1 Multiplies and collects rational and irrational parts $a - b = 1, \quad 5b - a = 7$ A1 Correct equations $a = 3, \quad b = 2$ M1 for attempt to solve simultaneous equations A1 both answers correct		



# Question 10

Question Number	Scheme		Marks
(a)	$8^{\frac{1}{3}} = 2$ or $8^5 = 32768$	A correct attempt to deal with the $\frac{1}{3}$ or the 5. $8^{\frac{1}{3}} = \sqrt[3]{8}$ or $8^5 = 8 \times 8 \times 8 \times 8 \times 8$	M1
	$\left(8^{\frac{5}{3}}\right) = 32$	Cao	A1
	A correct answer with no working scores full marks		
	Alternative $8^{\frac{5}{3}} = 8 \times 8^{\frac{2}{3}} = 8 \times 2^2 = M1$ (Deals with the 1/3) $= 32$ A1		
			(2)
(b)	$\left(2x^{\frac{1}{2}}\right)^3 = 2^3 x^{\frac{3}{2}}$	One correct power either $2^3$ or $x^{\frac{3}{2}}$ . $\left(2x^{\frac{1}{2}}\right) \times \left(2x^{\frac{1}{2}}\right) \times \left(2x^{\frac{1}{2}}\right)$ on its own is not sufficient for this mark.	M1
	$\frac{8x^{\frac{3}{2}}}{4x^2} = 2x^{-\frac{1}{2}}$ or $\frac{2}{\sqrt{x}}$	M1: Divides coefficients of $x$ and subtracts their powers of $x$ . <b>Dependent on the previous M1</b>	dM1A1
		A1: Correct answer	
	Note that unless the power of $x$ implies that they have subtracted their powers you would need to see evidence of subtraction. E.g. $\frac{8x^{\frac{3}{2}}}{4x^2} = 2x^{\frac{1}{2}}$ would score dM0 unless you see some evidence that $3/2 - 2$ was intended for the power of $x$ .		
	Note that there is a misconception that $\frac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2} = \left(\frac{2x^{\frac{1}{2}}}{4x^2}\right)^3$ - this scores 0/3		
			(3)
			[5]



## Question 11

Question Number	Scheme	Marks
	<p>(a) <math>32^{\frac{1}{5}} = 2</math></p> <p>(b) For <math>2^{-2}</math> or <math>\frac{1}{4}</math> or <math>\left(\frac{1}{2}\right)^2</math> or 0.25 as coefficient of <math>x^k</math>, for any value of <math>k</math> including <math>k = 0</math></p> <p>Correct index for <math>x</math> so <math>Ax^{-2}</math> or <math>\frac{A}{x^2}</math> o.e. for any value of <math>A</math></p> <p><math>= \frac{1}{4x^2}</math> or <math>0.25x^{-2}</math></p>	<p>B1 (1)</p> <p>M1</p> <p>B1</p> <p>A1 cao (3)</p> <p><b>4 Marks</b></p>

### Notes

(a) B1 Answer 2 must be in part (a) for this mark

(b) Look at their final answer

M1 For  $2^{-2}$  or  $\frac{1}{4}$  or  $\left(\frac{1}{2}\right)^2$  or 0.25 in their answer as coefficient of  $x^k$  for numerical value of  $k$  (including  $k = 0$ ) so final answer  $\frac{1}{4}$  is M1 B0 A0

B1  $Ax^{-2}$  or  $\frac{A}{x^2}$  or equivalent e.g.  $Ax^{\frac{10}{5}}$  or  $Ax^{\frac{50}{25}}$  i.e. correct power of  $x$  seen in final answer  
May have a bracket provided it is  $(Ax)^{-2}$  or  $\left(\frac{A}{x}\right)^2$

A1  $\frac{1}{4x^2}$  or  $\frac{1}{4}x^{-2}$  or  $0.25x^{-2}$  oe but must be correct power and coefficient combined correctly and must not be followed by a different wrong answer.

**Poor bracketing:**  $2x^{-2}$  earns M0 B1 A0 as correct power of  $x$  is seen in this solution (They can recover if they follow this with  $\frac{1}{4x^2}$  etc )

**Special case**  $(2x)^{-2}$  as a final answer and  $\left(\frac{1}{2x}\right)^2$  can have M0 B1 A0 if the correct expanded answer is not seen

The correct answer  $\frac{1}{4x^2}$  etc. followed by  $\left(\frac{1}{2x}\right)^2$  or  $(2x)^{-2}$ , treat  $\frac{1}{4x^2}$  as final answer so M1 B1 A1 isw

But the correct answer  $\frac{1}{4x^2}$  etc clearly followed by the wrong  $2x^{-2}$  or  $4x^{-2}$ , gets M1 B1 A0 do not ignore subsequent wrong work here

## Question 12

Question Number	Scheme	Marks
	<p>(a) <math>80 = 5 \times 16</math>  <math>\sqrt{80} = 4\sqrt{5}</math></p> <p>Method 1</p> <p>(b) <math>\frac{\sqrt{80}}{\sqrt{5}+1}</math> or <math>\frac{c\sqrt{5}}{\sqrt{5}+1}</math>  <math>= \frac{\sqrt{80}}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1}</math> or <math>\frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}</math>  <math>= \frac{20-4\sqrt{5}}{4}</math> or <math>\frac{4\sqrt{5}-20}{-4}</math>  <math>= 5-\sqrt{5}</math></p> <p>Method 2</p> <p><math>(p+q\sqrt{5})(\sqrt{5}+1) = \sqrt{80}</math>  <math>p\sqrt{5}+q\sqrt{5}+p+5q = 4\sqrt{5}</math>  <math>p+5q = 0</math>  <math>p+q = 4</math>  <math>p = 5, q = -1</math></p>	<p>B1 (1)</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>A1cao</p> <p>(4)</p> <p>(5 marks)</p>

### Notes

(a) B1 Accept  $4\sqrt{5}$  or  $c = 4$  – no working necessary

(b)  
(Method 1)

B1ft Only ft on  $c$  See  $\frac{\sqrt{80}}{\sqrt{5}+1}$  or  $\frac{c\sqrt{5}}{\sqrt{5}+1}$

M1 State intention to multiply by  $\sqrt{5}-1$  or  $1-\sqrt{5}$  in the numerator and the denominator

A1 Obtain denominator of 4 (for  $\sqrt{5}-1$ ) or -4 (for  $1-\sqrt{5}$ ) or correct simplified numerator of  $20-4\sqrt{5}$  or  $4(5-\sqrt{5})$  or  $4\sqrt{5}-20$  or  $4(\sqrt{5}-5)$  So either numerator or denominator must be correct

A1 Correct answer only. Both numerator and denominator must have been correct and division of numerator and denominator by 4 has been performed.

Accept  $p=5, q=-1$  or accept  $5-\sqrt{5}$  or  $-\sqrt{5}+5$  Also accept  $5-1\sqrt{5}$

(Method 2)

B1ft Only ft on  $c$   $(p+q\sqrt{5})(\sqrt{5}+1) = \sqrt{80}$  or  $c\sqrt{5}$

M1 Multiply out the lhs and replace  $\sqrt{80}$  by  $c\sqrt{5}$

A1 Compare rational and irrational parts to give  $p+q=4$ , and  $p+5q=0$

A1 Solve equations to give  $p=5, q=-1$

Common error:

$\frac{\sqrt{80}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{4\sqrt{5}-20}{4} = \sqrt{5}-5$  gets B1 M1 A1 (for correct numerator – denominator is wrong for their product) then A0

Correct answer with no working – send to review – have they used a calculator?

Correct answer after trial and improvement with evidence that  $(5-\sqrt{5})(\sqrt{5}+1) = \sqrt{80}$  could earn all four marks



### Question 13

Question Number	Scheme	Marks
	$25x - 9x^3 = x(25 - 9x^2)$ $(25 - 9x^2) = (5+3x)(5-3x)$ $25x - 9x^3 = x(5+3x)(5-3x)$	B1 M1 A1 (3)

- B1 Take out a common factor, usually  $x$ , to produce  $x(25 - 9x^2)$ . Accept  $(x \pm 0)(25 - 9x^2)$  or  $-x(9x^2 - 25)$ . Must be correct.  
Other possible options are  $(5 + 3x)(5x - 3x^2)$  or  $(5 - 3x)(5x + 3x^2)$
- M1 For factorising their quadratic term, usually  $(25 - 9x^2) = (5+3x)(5-3x)$ . Accept sign errors.  
If  $(5 \pm 3x)$  has been taken out as a factor first, this is for an attempt to factorise  $(5x \mp 3x^2)$
- A1 cao  $x(5 + 3x)(5 - 3x)$  or any equivalent with three factors  
e.g.  $x(5 + 3x)(-3x + 5)$  or  $x(3x - 5)(-3x - 5)$  etc including  $-x(3x + 5)(3x - 5)$   
isw if they go on to show that  $x = 0, \pm \frac{5}{3}$

### Question 14

Question Number	Scheme	Marks
(a)	$81^{\frac{3}{2}} = (81^{\frac{1}{2}})^3 = 9^3 \quad \text{or} \quad 81^{\frac{3}{2}} = (81^3)^{\frac{1}{2}} = (531441)^{\frac{1}{2}} = 729$	M1 A1 (2)
(b)	$(4x^{\frac{1}{2}})^2 = 16x^{\frac{2}{2}} \text{ or } \frac{16}{x} \quad \text{or equivalent}$ $x^2(4x^{\frac{1}{2}})^2 = 16x$	M1 A1 (2) (4 marks)

- (a) M1 Dealing with either the 'cube' or the 'square root' first. A correct answer will imply this mark.  
Also accept a law of indices approach  $81^{\frac{3}{2}} = 81^1 \times 81^{\frac{1}{2}} = 81 \times 9$   
A1 Cao 729. Accept  $(\pm)729$
- (b) M1 For correct use of power 2 on both 4 and the  $x^{\frac{1}{2}}$  term.  
A1 Cao =  $16x$



# Question 15

Question Number	Scheme	Marks
<b>Method 1</b>	$x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}}$ $\times \sqrt{2} \Rightarrow x\sqrt{16} + 10\sqrt{2} = 6x$ $4x + 10\sqrt{2} = 6x \Rightarrow 2x = 10\sqrt{2}$ $x = 5\sqrt{2} \quad \text{or } a = 5 \text{ and } b = 2$	M1,A1 M1A1 (4)
<b>Method 2</b>	$x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}}$ $2\sqrt{2}x + 10 = 3\sqrt{2}x$ $\sqrt{2}x = 10 \Rightarrow x = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \quad \text{oe}$	M1A1 M1,A1 (4)

## Method 1

M1 For multiplying both sides by  $\sqrt{2}$  – allow a slip e.g.  $\sqrt{2}x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}} \times \sqrt{2}$  or

$\sqrt{2} \times 10 + x\sqrt{8} = \frac{6x}{\sqrt{2}} \times \sqrt{2}$ , where one term has an error or the correct  $\sqrt{2}(10 + x\sqrt{8}) = \frac{6x}{\sqrt{2}} \times \sqrt{2}$

NB  $x\sqrt{8} + 10 = 6x\sqrt{2}$  is M0

A1 A correct equation in  $x$  with no fractional terms. Eg  $x\sqrt{16} + 10\sqrt{2} = 6x$  oe.

M1 An attempt to solve their linear equation in  $x$  to produce an answer of the form  $a\sqrt{2}$  or  $a\sqrt{50}$

A1  $5\sqrt{2}$  oe (accept  $1\sqrt{50}$ )

## Method 2

M1 For writing  $\sqrt{8}$  as  $2\sqrt{2}$  or  $\frac{6}{\sqrt{2}}$  as  $3\sqrt{2}$

A1 A correct equation in  $x$  with no fractional terms. Eg  $2\sqrt{2}x + 10 = 3\sqrt{2}x$  or  $x\sqrt{8} + 10 = 3\sqrt{2}x$  oe.

M1 An attempt to solve their linear equation in  $x$  to produce an answer of the form  $a\sqrt{2}$  or  $a\sqrt{50}$

$$\sqrt{2}x = 10 \Rightarrow x = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$\text{or } \sqrt{2}x = 10 \Rightarrow 2x^2 = 100 \Rightarrow x^2 = 50 \Rightarrow x = \sqrt{50} \text{ or } 5\sqrt{2}$$

A1  $5\sqrt{2}$  oe Accept  $1\sqrt{50}$



# Question 16

Question Number	Scheme		Marks
(a)	20	Sight of 20. (4×5 is not sufficient)	B1
			(1)
(b)	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$	Multiplies top and bottom by a correct expression. This statement is sufficient. NB $2\sqrt{5}+3\sqrt{2} \equiv \sqrt{20}+\sqrt{18}$	M1
	(Allow to multiply top and bottom by $k(2\sqrt{5}+3\sqrt{2})$ )		
	$= \frac{\dots}{2}$	Obtains a denominator of 2 or sight of $(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})=2$ with no errors seen in this expansion. May be implied by $\frac{\dots}{2k}$	A1
	<b>Note that M0A1 is not possible. The 2 must come from a correct method.</b>		
	<b>Note that if M1 is scored there is no need to consider the numerator.</b>		
	e.g. $\frac{2(MR?)}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}} = \frac{\dots}{2}$ scores M1A1		
	Numerator = $\sqrt{2}(2\sqrt{5} \pm 3\sqrt{2}) = 2\sqrt{10} \pm 6$	An attempt to multiply the numerator by $\pm(2\sqrt{5} \pm 3\sqrt{2})$ and obtain an expression of the form $p+q\sqrt{10}$ where $p$ and $q$ are integers. This may be implied by e.g. $2\sqrt{10}+3\sqrt{4}$ or by their final answer.	M1
	(Allow attempt to multiply the numerator by $k(2\sqrt{5} \pm 3\sqrt{2})$ )		
	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{2\sqrt{10}+6}{2} = 3+\sqrt{10}$	Cso. For the answer as written or $\sqrt{10}+3$ or a statement that $a=3$ and $b=10$ . Score when first seen and ignore any subsequent attempt to 'simplify'. Allow $1\sqrt{10}$ for $\sqrt{10}$	A1
			(4)
			(5 marks)

Alternative for (b)			
	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} = \frac{1}{\sqrt{10}-3} \text{ or } \frac{2}{2\sqrt{10}-6}$	M1: Divides or multiplies top and bottom by $\sqrt{2}$ A1: $\frac{k}{k(\sqrt{10}-3)}$	M1A1
	$= \frac{1}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$	M1: Multiplies top and bottom by $\sqrt{10}+3$	M1
	$= 3+\sqrt{10}$		A1



## Question 17

Question Number	Scheme	Notes	Marks
	$9^{3x+1} =$ for example $3^{2(3x+1)}$ or $(3^2)^{3x+1}$ or $(3^{(3x+1)})^2$ or $3^{3x+1} \times 3^{3x+1}$ or $(3 \times 3)^{3x+1}$ or $3^2 \times (3^2)^{3x}$ or $(9^{\frac{1}{2}})^y$ or $9^{\frac{1}{2}y}$ or $y = 2(3x+1)$	Expresses $9^{3x+1}$ correctly as a power of 3 or expresses $3^y$ correctly as a power of 9 or expresses $y$ correctly in terms of $x$  (This mark is <u>not</u> for just $3^2 = 9$ )	M1
	$= 3^{6x+2}$ or $y = 6x + 2$ or $a = 6, b = 2$	Cao (isw if necessary)	A1
	Providing there is no incorrect work, allow sight of $6x + 2$ to score both marks Correct answer only implies both marks Special case: $3^{6x+1}$ only scores M1A0		
			[2]
	Alternative using logs		
	$9^{3x+1} = 3^y \Rightarrow \log 9^{3x+1} = \log 3^y$		
	$(3x+1)\log 9 = y\log 3$	Use power law correctly on both sides	M1
	$y = \frac{\log 9}{\log 3}(3x+1)$		
	$y = 6x + 2$	cao	A1
			2 marks



# Question 18

Question Number	Scheme	Notes	Marks
(a)	$\sqrt{50} - \sqrt{18} = 5\sqrt{2} - 3\sqrt{2}$	$\sqrt{50} = 5\sqrt{2}$ or $\sqrt{18} = 3\sqrt{2}$ and the other term in the form $k\sqrt{2}$ . This mark may be implied by the correct answer $2\sqrt{2}$	M1
	$= 2\sqrt{2}$	Or $a = 2$	A1
			[2]
(b) WAY 1	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}} = \frac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where $a$ is numeric. This is all that is required for this mark.	M1
	$= \frac{12\sqrt{3}}{"2"\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{6}}{4}$	Rationalises the denominator by a correct method e.g. multiplies numerator and denominator by $k\sqrt{2}$ to obtain a multiple of $\sqrt{6}$ . Note that multiplying numerator and denominator by $2\sqrt{2}$ or $-2\sqrt{2}$ is quite common and is acceptable for this mark. May be implied by a correct answer. <b>This is dependent on the first M1.</b>	dM1
	$= 3\sqrt{6}$ or $b = 3, c = 6$	Cao and cso	A1
			[3]
(b) WAY 2	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}} \times \frac{\sqrt{50}+\sqrt{18}}{\sqrt{50}+\sqrt{18}}$ or $\frac{12\sqrt{3}}{5\sqrt{2}-3\sqrt{2}} \times \frac{5\sqrt{2}+3\sqrt{2}}{5\sqrt{2}+3\sqrt{2}}$	For rationalising the denominator by a correct method i.e. multiplying numerator and denominator by $k(\sqrt{50} + \sqrt{18})$	M1
	$\frac{60\sqrt{6}+36\sqrt{6}}{50-18}$	For replacing numerator by $\alpha\sqrt{6} + \beta\sqrt{6}$ . <b>This is dependent on the first M1 and there is no need to consider the denominator for this mark.</b>	dM1
	$= 3\sqrt{6}$ or $b = 3, c = 6$	Cao and cso	A1
			[3]
(b) WAY 3	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}} = \frac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where $a$ is numeric. This is all that is required for this mark.	M1
	$= \frac{12\sqrt{3}}{2\sqrt{2}} = \frac{6\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{108}}{\sqrt{2}} = \sqrt{54} = \sqrt{9}\sqrt{6}$	Cancels to obtain a multiple of $\sqrt{6}$ . <b>This is dependent on the first M1.</b>	dM1
	$= 3\sqrt{6}$ Or $b = 3, c = 6$	Cao and cso	A1
			[3]
(b) WAY 4	$\frac{12\sqrt{3}}{\sqrt{50}-\sqrt{18}} = \frac{12\sqrt{3}}{"2"\sqrt{2}}$	Uses part (a) by replacing denominator by their $a\sqrt{2}$ where $a$ is numeric. This is all that is required for this mark.	M1
	$\left(\frac{12\sqrt{3}}{"2"\sqrt{2}}\right)^2 = \frac{432}{8}$		
	$\sqrt{54} = \sqrt{9}\sqrt{6}$	Obtains a multiple of $\sqrt{6}$ . <b>This is dependent on the first M1.</b>	dM1
	$= 3\sqrt{6}$ Or $b = 3, c = 6$	Cao and cso (do not allow $\pm 3\sqrt{6}$ )	A1
			5 marks



# Question 19

Question Number	Scheme		Marks
(a)	Replaces $2^{2x+1}$ with $2^{2x} \times 2$ or states $2^{2x+1} = 2^{2x} \times 2$ or states $(2^x)^2 = 2^{2x}$	Uses the addition <b>or</b> power law of indices on $2^{2x}$ or $2^{2x+1}$ . E.g. $2^x \times 2^x = 2^{2x}$ or $(2^x)^2 = 2^{2x}$ or $2^{2x+1} = 2 \times 2^{2x}$ or $2^{x+0.5} = 2^x \times \sqrt{2}$ or $2^{2x+1} = (2^{x+0.5})^2$ .	M1
	$2^{2x+1} - 17 \times 2^x + 8 = 0$ $\Rightarrow 2y^2 - 17y + 8 = 0^*$	Cso. Complete proof that includes explicit statements for the addition <b>and</b> power law of indices on $2^{2x+1}$ with no errors. The equation needs to be as printed including the " $= 0$ ". If they work backwards, they do not need to write down the printed answer first but must end with the version in $2^x$ including " $= 0$ ".	A1*

	<b>The following are examples of acceptable proofs.</b>	
	$2^{2x+1} = \left(2^{x+0.5}\right)^2 = \left(2^x \sqrt{2}\right)^2 = \left(y\sqrt{2}\right)^2 = 2y^2$ $\Rightarrow 2^{2x+1} - 17\left(2^x\right) + 8 = 2y^2 - 17y + 8 = 0$	
	$2y^2 = 2 \times 2^x \times 2^x = 2^{2x+1}$ $\Rightarrow 2^{2x+1} - 17\left(2^x\right) + 8 = 2y^2 - 17y + 8 = 0$	
	$2y^2 - 17y + 8 = 0 \Rightarrow 2\left(2^x\right)^2 - 17\left(2^x\right) + 8 = 0$ $\Rightarrow 2 \times 2^{2x} - 17\left(2^x\right) + 8 = 0 \Rightarrow 2^{2x+1} - 17\left(2^x\right) + 8 = 0$	
	$2^{2x+1} = 2 \times 2^{2x} \Rightarrow 2 \times 2^{2x} - 17\left(2^x\right) + 8 = 0$ $\Rightarrow 2y^2 - 17y + 8 = 0$ <p>Scores <b>M1A0</b> as <math>2^{2x} = \left(2^x\right)^2</math> has not been shown explicitly</p>	
	<p><b>Special Case:</b></p> $2^{2x+1} = 2^1 \times \left(2^x\right)^2 \text{ or } 2^{2x+1} = \left(2^x\right)^2 \times 2^1$ <p><b>With or without the multiplication signs and with no subsequent explicit evidence of the power law scores M1A0</b></p>	
	<p><b>Example of insufficient working:</b></p> $2^{2x+1} = 2\left(2^x\right)^2 = 2y^2$ <p><b>scores no marks as neither rule has been shown explicitly.</b></p>	
		(2)

(b)	$2y^2 - 17y + 8 = 0 \Rightarrow (2y - 1)(y - 8) = 0 \Rightarrow y = \dots$ <p>or</p> $2(2^x)^2 - 17(2^x) + 8 = 0 \Rightarrow (2(2^x) - 1)((2^x) - 8) = 0 \Rightarrow 2^x = \dots$ <p>Solves the <b>given</b> quadratic either in terms of <math>y</math> or in terms of <math>2^x</math> See General Principles for solving a 3 term quadratic</p> <p>Note that completing the square on e.g. <math>y^2 - \frac{17}{2}y + 4 = 0</math> requires</p> $\left(y \pm \frac{17}{4}\right)^2 \pm q \pm 4 = 0 \Rightarrow y = \dots$		M1
	$(y =) \frac{1}{2}, 8$ or $(2^x =) \frac{1}{2}, 8$	Correct values	A1
	$\Rightarrow 2^x = \frac{1}{2}, 8 \Rightarrow x = -1, 3$	<p>M1: Either finds one correct value of <math>x</math> for their <math>2^x</math> or obtains a correct numerical expression in terms of logs e.g. for <math>k &gt; 0</math></p> $2^x = k \Rightarrow x = \log_2 k \text{ or } \frac{\log k}{\log 2}$	M1 A1
		A1: $x = -1, 3$ only. Must be values of $x$ .	
			(4)
			(6 marks)