

Algebraic Methods - Edexcel Past Exam Questions 2 MARK SCHEME

Question 1

Question number	Scheme	Marks
(a)	$f(-2) = 2(-2)^3 - 7(-2)^2 - 10(-2) + 24$ $= 0 \text{ so } (x+2) \text{ is a factor}$	M1 A1 (2)
(b)	$f(x) = (x+2)(2x^2 - 11x + 12)$ $f(x) = (x+2)(2x-3)(x-4)$	M1 A1 dM1 A1 (4)
6 marks		
Notes (a)	<p>M1 : Attempts $f(\pm 2)$ (Long division is M0) A1 : is for $=0$ and conclusion Note: Stating "hence factor" or "it is a factor" or a "✓" (tick) or "QED" is fine for the conclusion. Note also that a conclusion can be implied from a <u>preamble</u>, eg: "If $f(-2) = 0$, $(x+2)$ is a factor..." (Not just $f(-2)=0$)</p> <p>(b) 1st M1: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen as could be done "by inspection." Or <i>Alternative Method</i> : 1st M1: Use $(x+2)(ax^2 + bx + c) = 2x^3 - 7x^2 - 10x + 24$ with expansion and comparison of coefficients to obtain $a = 2$ and to obtain values for b and c 1st A1: For seeing $(2x^2 - 11x + 12)$. [Can be seen here in (b) after work done in (a)] 2nd M1: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and needs factors 2nd A1: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.)</p> <p>Note: Some candidates will go from $\{(x+2)\}(2x^2 - 11x + 12)$ to $\{x = -2\}$, $x = \frac{3}{2}$, 4, and not list all three factors. Award these responses M1A1M0A0.</p> <p>Finds $x = 4$ and $x = 1.5$ by factor theorem, formula or calculator and produces factors M1 $f(x) = (x+2)(2x-3)(x-4)$ or $f(x) = 2(x+2)(x-1.5)(x-4)$ o.e. is full marks $f(x) = (x+2)(x-1.5)(x-4)$ loses last A1</p>	



Question 2

Question Number	Scheme	Marks
(a)	Either (Way 1) : Attempt $f(3)$ or $f(-3)$ $f(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9^*$	Or (Way 2): Assume $a = -9$ and attempt $f(3)$ or $f(-3)$ $f(3) = 0$ so $(x - 3)$ is factor M1 A1 * cso (2)
	Or (Way 3): $(2x^3 - 5x^2 + ax + 18) \div (x - 3) = 2x^2 + px + q$ where p is a number and q is an expression in terms of a Sets the remainder $18 + 3a + 9 = 0$ and solves to give $a = -9$	M1 A1* cso (2)
(b)	Either (Way 1): $f(x) = (x - 3)(2x^2 + x - 6)$ $= (x - 3)(2x - 3)(x + 2)$	M1A1 M1A1 (4)
	Or (Way 2) Uses trial or factor theorem to obtain $x = -2$ or $x = 3/2$ Uses trial or factor theorem to obtain both $x = -2$ and $x = 3/2$ Puts three factors together (see notes below) Correct factorisation : $(x - 3)(2x - 3)(x + 2)$ or $(3 - x)(3 - 2x)(x + 2)$ or $2(x - 3)(x - \frac{3}{2})(x + 2)$ oe	M1 A1 M1 A1 (4)
	Or (Way 3) No working three factors $(x - 3)(2x - 3)(x + 2)$ otherwise need working	M1A1M1A1



Question 3

Question Number	Scheme		Marks
	If there is no labelling, mark (a) and (b) in that order		
	$f(x) = 2x^3 - 7x^2 + 4x + 4$		
(a)	$f(2) = 2(2)^3 - 7(2)^2 + 4(2) + 4$	Attempts $f(2)$ or $f(-2)$	M1
	$= 0$, and so $(x - 2)$ is a factor.	$f(2) = 0$ with no sign or substitution errors $(2(2)^3 - 7(2)^2 + 4(2) + 4 = 0$ is sufficient) and for conclusion. Stating "hence factor" or "it is a factor" or a "tick" or "QED" or "no remainder" or "as required" are fine for the conclusion but not $= 0$ just underlined and not hence $(2$ or $f(2))$ is a factor. Note also that a conclusion can be implied from a preamble, eg: "If $f(2) = 0$, $(x - 2)$ is a factor...."	A1
	Note: Long division scores no marks in part (a). The factor theorem is required.		
			[2]
(b)	$f(x) = (x - 2)(2x^2 - 3x - 2)$	M1: Attempts long division by $(x - 2)$ or other method using $(x - 2)$, to obtain $(2x^2 \pm ax \pm b)$, $a \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection." A1: $(2x^2 - 3x - 2)$	M1 A1
	$= (x - 2)(x - 2)(2x + 1)$ or $(x - 2)^2(2x + 1)$ or equivalent e.g. $= 2(x - 2)(x - 2)(x + \frac{1}{2})$ or $2(x - 2)^2(x + \frac{1}{2})$	dM1: Factorises a 3 term quadratic. (see rule for factorising a quadratic in the General Principles for Core Maths Marking). This is dependent on the previous method mark being awarded but there must have been no remainder. Allow an attempt to solve the quadratic to determine the factors. A1: cao – needs all three factors on one line . Ignore following work (such as a solution to a quadratic equation.)	dM1 A1
	Note $= (x - 2)(\frac{1}{2}x - 1)(4x + 2)$ would lose the last mark as it is not fully factorised		
	For correct answers only award full marks in (b)		
			[4]
			Total 6



Question 4

Question Number	Scheme		Marks
(a)	$f(x) = -4x^3 + ax^2 + 9x - 18$		
	$f(2) = -32 + 4a + 18 - 18 = 0$ $\Rightarrow 4a = 32 \Rightarrow a = 8$	Attempts $f(2)$ or $f(-2)$	M1
		cso	A1
			[2]
(a) Way 2	$f(x) = (x-2)(px^2 + qx + r)$		
	$= px^3 + (q-2p)x^2 + (r-2q)x - 2r$		
	$r=9 \Rightarrow q=0$ also $p=-4 \therefore a=-2p=8$	Compares coefficients leading to $-2p = a$	M1
	$a = 8$	cso	A1
(a) Way 3	$(-4x^3 + ax^2 + 9x - 18) \div (x-2)$		
	$Q = -4x^2 + (a-8)x + 2a - 7$ $R = 4a - 32$	Attempt to divide $\pm f(x)$ by $(x-2)$ to give a quotient at least of the form $\pm 4x^2 + g(a)x$ and a remainder that is a function of a	M1
	$4a - 32 = 0 \Rightarrow a = 8$	cso	A1
(b)	$f(x) = (x-2)(-4x^2 + 9)$	Attempts long division or other method, to obtain $(-4x^2 \pm ax \pm b)$, $b \neq 0$, even with a remainder. Working need not be seen as this could be done "by inspection."	M1
	$= (x-2)(3-2x)(3+2x)$	dM1: A <i>valid</i> attempt to factorise their quadratic – see General Principles. This is dependent on the previous method mark being awarded	



	<p>or equivalent e.g.</p> $= -(x - 2)(2x - 3)(2x + 3)$ <p>or</p> $= (x - 2)(2x - 3)(-2x - 3)$	<p>method using division, but there must have been no remainder.</p> <p>A1: cao – must have all 3 factors on the same line. Ignore subsequent work (such as a solution to a quadratic equation.)</p>	dM1A1
			[3]
(c)	$f\left(\frac{1}{2}\right) = -4\left(\frac{1}{8}\right) + 8\left(\frac{1}{4}\right) + 9\left(\frac{1}{2}\right) - 18 = -12$	<p>Attempts $f\left(\frac{1}{2}\right)$ or $f\left(-\frac{1}{2}\right)$</p> <p>Allow A1ft for the correct numerical value of $\frac{\text{their } a}{4} - 14$</p>	M1A1ft
			[2]
(c) Way 2	$\pm(-4x^3 + 8x^2 + 9x - 18) \div (2x - 1)$ $Q = -2x^2 + 3x + 6$ $R = -12$	<p>M1: Attempt long division to give a remainder that is independent of x</p> <p>A1: Allow A1ft for the correct numerical value of $\frac{\text{their } a}{4} - 14$.</p>	M1A1ft
			Total 7



Question 5

Question Number	Scheme	Marks
(a)	$f(-2) = 6(-2)^3 + 13(-2)^2 - 4$	Attempts $f(-2)$. M1
	$= 0$, and so $(x + 2)$ is a factor.	$f(-2) = 0$ with no sign or substitution errors and for conclusion. A1
		[2]
(b)	$f(x) = \{(x + 2)\}(6x^2 + x - 2)$	M1 A1
	$= (x + 2)(2x - 1)(3x + 2)$	M1 A1
		[4]

Question 6

Question Number	Scheme	Marks
(a)	Attempt $f(3)$ or $f(-3)$ Use of long division is M0A0 as factor theorem was required. $f(-3) = 162 - 63 - 120 + 21 = 0$ so $(x + 3)$ is a factor	M1 A1 (2)
(b)	Either (Way 1): $f(x) = (x + 3)(-6x^2 + 11x + 7)$ $= (x + 3)(-3x + 7)(2x + 1)$ or $-(x + 3)(3x - 7)(2x + 1)$	M1A1 M1A1 (4)
	Or (Way 2) Uses trial or factor theorem to obtain $x = -1/2$ or $x = 7/3$ Uses trial or factor theorem to obtain both $x = -1/2$ and $x = 7/3$ Puts three factors together (see notes below) Correct factorisation : $(x + 3)(7 - 3x)(2x + 1)$ or $-(x + 3)(3x - 7)(2x + 1)$ oe	M1 A1 M1 A1 (4)
	Or (Way 3) No working three factors $(x + 3)(-3x + 7)(2x + 1)$ otherwise need working	M1A1M1A1 (4)