Circles - Edexcel Past Exam Questions 2 MARK SCHEME

Question 1

Question number	Scheme	Marks
	The equation of the circle is $(x+1)^2+(y-7)^2=(r^2)$	M1 A1
	The radius of the circle is $\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $(x+1)^2+(y-7)^2=50$ or equivalent	A1 (4)
		4
Notes	**M1** is for this expression on left hand side– allow *errors in sign* of 1 and 7. **A1** correct signs (just LHS) **M1** is for Pythagoras or substitution into equation of circle to give r or r^2 Giving this value as diameter is **M0** **A1**, cao for cartesian equation with numerical values but allow $(\sqrt{50})^2$ or $(5\sqrt{2})^2$ or any exact equivalent A correct answer implies a correct method – so answer given with no working earns all four marks for this question.	
Alternative method	Equation of circle is $x^2+y^2\pm 2x\pm 14y+c=0$	M1
	Equation of circle is $x^2+y^2+2x-14y+c=0$	A1
	Uses (0,0) to give $c=0$, **or** finds $r=\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $x^2+y^2+2x-14y=0$ or equivalent	A1

Question 2

Question number	Scheme	Marks
(a)	Obtain $\underline{(x \pm 10)^2}$ **and** $\underline{(y \pm 8)^2}$	M1
	Obtain $\underline{(x-10)^2}$ **and** $\underline{(y-8)^2}$	A1
	Centre is $(10, 8)$. N.B. This may be indicated on **diagram only** as (10, 8)	A1 (3)
(b)	See $\underline{(x \pm 10)^2} + \underline{(y \pm 8)^2} = 25 (= r^2)$ or $(r^2 =)\text{"}100\text{"}+\text{"}64\text{"}-139$	M1
	$r = 5$ * (this is a printed answer so need one of the above two reasons)	**A1** **(2)**
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$	**M1**
	e.g $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y =$ or $13^2 + y^2 - 20\times13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$	
	$y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	**A1, A1** (3)

Alternatives (a)	*Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$	M1
	Centre is $(-g, -f)$, **and so centre is** $(10, 8)$.	A1, A1
OR	*Method 3:* Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" **and** Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1 A1 A1 (3)
(b)	*Method 2:* Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)\text{"}100\text{"}+\text{"}64\text{"}-139$	M1
	$r = 5$ *	A1
OR	*Method 3:* Use point on circle with centre to find radius. Eg $\sqrt{(13-10)^2 + (12-8)^2}$	**M1**
	$r = 5$ *	**A1 cao** (2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - \text{"}10\text{"})^2 = h^2$, then evaluate $\text{"}8 \pm h\text{"}$ - (N.B. Could use 3,4,5 Triangle and 8 ± 4). Accuracy as before	**M1**
Notes (a) (b)	**Mark (a) and (b) together** **M1** as in scheme and can be implied by $(\pm10, \pm8)$. **Correct centre (10, 8) implies M1A1A1** **M1** for a correct method leading to $r = \ldots$, or $r^2 = \text{"}100\text{"}+\text{"}64\text{"}-139$ (not 139 – "100" – "64") or for using equation of circle in $\underline{(x \pm 10)^2} + \underline{(y \pm 8)^2} = k^2$ form to identify $r=$ **3rd A1** $r = 5$ **(NB This is a given answer so should follow** $k^2 = 25$ or $r^2 = 100 + 64 - 139$ **)** **Special case**: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow **M1A1** for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$	

Question 3

Question Number	Scheme		Marks
(a)			
(i)	The centre is at (10, 12)	B1: $x=10$ B1: $y=12$	B1 B1
(ii)	Uses $(x-10)^2+(y-12)^2=-195+100+144 \Rightarrow r=...$		M1
	Completes the square for both x and y in an attempt to find r. $(x\pm"10")^2 \pm a$ and $(y\pm"12")^2 \pm b$ and $+195=0, (a,b\neq 0)$ Allow errors in obtaining their r^2 but must find square root		
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r=7$	Not $r=\pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ i.e. (10, 12)	B1: $x=10$ B1: $y=12$	B1B1
	Uses $r=\sqrt{(\pm"10")^2+(\pm"12")^2-c}$		M1
	$r=\sqrt{10^2+12^2-195}$	A correct numerical expression for r	A1
	$r=7$		A1
			(5)
(b)	$MN=\sqrt{(25-"10")^2+(32-"12")^2}$	Correct use of Pythagoras	M1
	$MN\left(=\sqrt{625}\right)=25$		A1
			(2)
(c)	$NP=\sqrt{("25"^2-"7"^2)}$	$NP=\sqrt{(MN^2-r^2)}$	M1
	$NP\left(=\sqrt{576}\right)=24$		A1
			(2)
(c) Way 2	$\cos(NMP)=\frac{7}{"25"} \Rightarrow NP="25"\sin(NMP)$	Correct strategy for finding NP	M1
	$NP=24$		A1
			(2)
			[9]

Question 4

Question Number	Scheme	Marks
(a)	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, $k > 0$ Equation of form $(x - a)^2 + (y - b)^2 = 5^2$, with values for a and b $(x+5)^2 + (y-9)^2 = 25 = 5^2$	M1 M1 A1 (3)
(b)	$P(8, -7)$. Let centre of circle $= X(-5, 9)$ $PX^2 = (8 - "-5")^2 + (-7 - "9")^2$ or $PX = \sqrt{(8 - -5)^2 + (-7-9)^2}$ $(PX = \sqrt{425}$ or $5\sqrt{17})$ $PT^2 = (PX)^2 - 5^2$ with numerical PX $PT \left\{= \sqrt{400}\right\} = 20$ (allow 20.0)	M1 dM1 A1 cso (3) [6]
Alternative 2 for (a)	Equation of the form $x^2 + y^2 \pm 10x \pm 18y + c = 0$ Uses $a^2 + b^2 - 5^2 = c$ with their a and b or substitutes $(0, 9)$ giving $+9^2 \pm 2b \times 9 + c = 0$ $x^2 + y^2 + 10x - 18y + 81 = 0$	M1 M1 A1 (3)
Alternative 2 for (b)	An attempt to find the point T may result in pages of algebra, but solution needs to reach (-8, 5) or $\left(\frac{-8}{17}, 11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first) M1: Use either of the correct points with P (8, -7) and distance between two points formula A1: 20	M1 dM1 A1cso (3)
Alternative 3 for (b)	Substitutes (8, -7) into circle **equation** so $PT^2 = 8^2 + (-7)^2 + 10 \times 8 - 18 \times (-7) + 81$ Square roots to give $PT \left\{= \sqrt{400}\right\} = 20$	M1 dM1A1 (3)
	Notes for Question	
(a)	**The three marks in (a) each require a circle equation – (see special cases which are not circles)** M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be r^2 or $k > 0$ or a positive value) M1: Uses $r = 5$ to obtain RHS of circle equation as 25 or 5^2 A1: correct circle equation in any equivalent form **Special cases** $(x \pm 5)^2 + (x \pm 9)^2 = (5^2)$ is **not a circle** equation so M0M0A0 Also $(x \pm 5)^2 + (y - 9) = (5^2)$ **And** $(x \pm 5)^2 - (y \pm 9)^2 = (5^2)$ **are not circles** and gain M0M0A0 **But** $(x - 0)^2 + (y - 9)^2 = 5^2$ gains M0M1A0	
(b)	M1: Attempts to find distance from their **centre of circle** to P (or square of this value). If this is called PT and given as answer this is M0. Solution may use letter other than X, as centre was not labelled in the question. N.B. Distance from (0, 9) to (8, -7) is incorrect method and is M0, followed by M0A0. dM1: Applies the **subtraction** form of Pythagoras to find PT or PT^2 (depends on previous method mark for distance from **centre to** P) or uses appropriate complete method involving trigonometry A1: 20 cso	

Question 5

Question Number	Scheme		Marks
	Mark (a) and (b) together		
(a)	$OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{=14\}$	Uses the addition form of Pythagoras on $6\sqrt{5}$ and 4. Condone missing brackets on $\left(6\sqrt{5}\right)^2$ **(Working or 14 may be seen on the diagram)**	M1
	$y_Q = \sqrt{14^2 - 11^2}$	$y_Q = \sqrt{(\text{their } OQ)^2 - 11^2}$ **Must include √ and is dependent on the first M1 and requires OQ > 11**	dM1
	$= \sqrt{75}$ or $5\sqrt{3}$	$\sqrt{75}$ or $5\sqrt{3}$	A1cso
			[3]
(b)	$(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$	M1: $(x \pm 11)^2 + (y \pm \text{their } k)^2 = 4^2$ **Equation must be of this form and must use x and y not other letters. k could be their last answer to part (a). Allow their $k \neq 0$ or just the letter k.**	M1A1
		A1: $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ or $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 4^2$ **NB $5\sqrt{3}$ must come from correct work in (a) and allow awrt 8.66**	
	Allow in expanded form for the final A1 e.g. $x^2 - 22x + 121 + y^2 - 10\sqrt{3}y + 75 = 16$		
			[2]
			Total 5
	Watch out for:		
	(a) $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2} = \sqrt{46}$ M1 $y_Q = \sqrt{46 - 11^2}$ M0 (OQ < 11) $y_Q = \sqrt{75}$ A0 (b) $(x-11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$ M1A0		

Question 6

Question Number	Scheme		Marks
(a)	$A\left(\frac{-9+15}{2}, \frac{8-10}{2}\right) = A(3,-1)$	M1: A correct attempt to find the midpoint between P and Q. Can be implied by one of x or y-coordinates correctly evaluated.	M1A1
		A1: $(3,-1)$	
			[2]
(b)	$(-9-3)^2+(8+1)^2$ or $\sqrt{(-9-3)^2+(8+1)^2}$ or $(15-3)^2+(-10+1)^2$ or $\sqrt{(15-3)^2+(-10+1)^2}$ Uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius** and may be implied by their circle equation. Or $(15+9)^2+(-10-8)^2$ or $\sqrt{(15+9)^2+(-10-8)^2}$ Uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter** and may be implied by their circle equation. This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) **Allow this mark if there is a correct statement involving the radius or the diameter but <u>must be seen in (b)</u>**		M1
	$(x-3)^2+(y+1)^2=225$ $\left(\text{or } (15)^2\right)$	$(x\pm\alpha)^2+(y\pm\beta)^2=k^2$ where $A(\alpha,\beta)$ and k is their radius.	M1
	$(x-3)^2+(y+1)^2=225$	Allow $(x-3)^2+(y+1)^2=15^2$	A1
	Accept correct answer only		
			[3]
	Alternative using $x^2+2ax+y^2+2by+c=0$		
	Uses $A(\pm\alpha,\pm\beta)$ and $x^2+2ax+y^2+2by+c=0$ e.g. $x^2+2(-3)x+y^2+2(1)y+c=0$		M1
	Uses P or Q and $x^2+2ax+y^2+2by+c=0$ e.g. $(-9)^2+2(-3)(-9)+(8)^2+2(1)(8)+c=0 \Rightarrow c=-215$		M1
	$x^2-6x+y^2+2y-215=0$		A1
(c)	Distance $=\sqrt{15^2-10^2}$	$=\sqrt{(\text{their } r)^2-10^2}$ or a correct method for the distance e.g. their $r\times\cos\left[\sin^{-1}\left(\frac{10}{\text{their } r}\right)\right]$	M1
	$\left\{=\sqrt{125}\right\}=5\sqrt{5}$	$5\sqrt{5}$	A1
			[2]

Question Number	Scheme		Marks
(d)	$\sin\left(A\hat{R}Q\right) = \frac{20}{30}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\frac{10}{15}\right)$	$\sin\left(A\hat{R}Q\right) = \frac{20}{(2 \times \text{their } r)}$ or $\frac{10}{\text{their } r}$ or $A\hat{R}Q = 90 - \cos^{-1}\left(\frac{10}{\text{their } r}\right)$ or $A\hat{R}Q = \cos^{-1}\left(\frac{\text{Part}(c)}{\text{their } r}\right)$ or $A\hat{R}Q = 90 - \sin^{-1}\left(\frac{\text{Part}(c)}{\text{their } r}\right)$ or $20^2 = 15^2 + 15^2 - 2\times15\times15\cos(\mathbf{2}ARQ)$ or $15^2 = 15^2 + \left(10\sqrt{5}\right)^2 - 2\times15\times10\sqrt{5}\cos(ARQ)$ A fully correct method to find $A\hat{R}Q$, where their $r > 10$. Must be a **correct** statement involving angle ARQ	M1
	$A\hat{R}Q = 41.8103...$	awrt 41.8	A1
			[2]
			Total 9

Question 7

Question Number	Scheme		Marks
	Way 1	Way 2	
(a)	$(x \mp 2)^2 + (y \pm 1)^2 = k$, $k > 0$	$x^2 + y^2 \mp 4x \pm 2y + c = 0$	M1
	Attempts to use $r^2 = (4-2)^2 + (-5+1)^2$	$4^2 + (-5)^2 - 4\times 4 + 2\times -5 + c = 0$	M1
	Obtains $(x-2)^2 + (y+1)^2 = 20$	$x^2 + y^2 - 4x + 2y - 15 = 0$	A1 (3)
	N.B. Special case: $(x-2)^2 - (y+1)^2 = 20$ is not a circle equation but earns M0M1A0		
(b) Way 1	Gradient of radius from centre to (4, -5) = -2 (must be correct)		B1
	Tangent gradient = $-\dfrac{1}{\text{their numerical gradient of radius}}$		M1
	Equation of tangent is $(y+5) = '\tfrac{1}{2}'(x-4)$		M1
	So equation is $x - 2y - 14 = 0$ (or $2y - x + 14 = 0$ or other integer multiples of this answer)		A1
			(4)
b)Way 2	Quotes $xx' + yy' - 2(x + x') + (y + y') - 15 = 0$ and substitutes (4, -5)_		B1
	$4x - 5y - 2(x+4) + (y-5) - 15 = 0$ so $2x - 4y - 28 = 0$ (or alternatives as in Way 1)		M1,M1A1
			(4)
b)Way 3	Use differentiation to find expression for gradient of circle		
	Either $2(x-2) + 2(y+1)\dfrac{dy}{dx} = 0$ **or** states $y = -1 - \sqrt{20-(x-2)^2}$ so $\dfrac{dy}{dx} = \dfrac{(x-2)}{\sqrt{20-(x-2)^2}}$		B1
	Substitute $x = 4$, $y = -5$ after valid differentiation to give gradient =		M1
	Then as Way 1 above $(y+5) = '\tfrac{1}{2}'(x-4)$ so $x - 2y - 14 = 0$		M1 A1
			(4)
			[7]

Notes

(a) **M1**: Uses centre to write down equation of circle in one of these forms. There may be sign slips as shown.

M1: Attempts distance between two points **to establish** r^2 (independent of first M1)- allow one sign slip only using distance formula with -5 or -1, usually $(-5-1)$ in 2nd bracket. Must not identify this distance as diameter.

This mark may alternatively (e.g. way 2)be given for substituting (4, -5) into a **correct circle** equation with one unknown

Can be awarded for $r=\sqrt{20}$ or for $r^2=20$ stated or implied but not for $r^2=\sqrt{20}$ or $r=20$ or $r=\sqrt{5}$

A1: Either of the answers printed or correct equivalent e.g. $(x-2)^2+(y+1)^2=(2\sqrt{5})^2$ is A1 but $2\sqrt{5}^2$ (no bracket) is A0 unless there is recovery

Also $(x-2)^2+(y-(-1))^2=(2\sqrt{5})^2$ may be awarded M1M1A1as a correct equivalent.

N.B. $(x-2)^2+(y+1)^2=40$ commonly arises from one sign error evaluating r and earns M1M1A0

(b) **Way 1:**

B1: Must be correct answer -2 if evaluated (otherwise may be implied by the following work)

M1: Uses negative reciprocal of their gradient

M1: Uses $y-y_1=m(x-x_1)$ with (4,-5) and their **changed** gradient **or** uses $y=mx+c$ and (4, -5) with their changed gradient (not gradient of radius) to find c

A1: answers in scheme or multiples of these answers (must have " = 0"). NB Allow $1x-2y-14=0$

N.B. $(y+5)='\frac{1}{2}'(x-4)$ following gradient of is ½ after errors leads to $x-2y-14=0$ but is worth B0M0M0A0

Way 2: Alternative method (b) is rare.

Way 3: Some may use implicit differentiation to differentiate- others may attempt to make y the subject and use chain rule

B1: the differentiation must be accurate and the algebra accurate too. Need to take (-) root not (+)root in the alternative

M1: Substitutes into their gradient function but must follow valid accurate differentiation

M1: Must use "their" tangent gradient and $y+5=m(x-4)$ but allow over simplified attempts at differentiation for this mark.

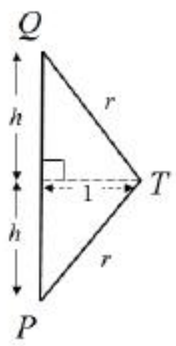
A1: As in Way 1

Question 8

Question Number	Scheme		Marks
(a)	$P(7, 8)$ and $Q(10, 13)$		
	$\{PQ =\}\ \sqrt{(7-10)^2+(8-13)^2}$ or $\sqrt{(10-7)^2+(13-8)^2}$	Applies distance formula. Can be implied.	M1
	$\{PQ\} = \sqrt{34}$	$\sqrt{34}$ or $\sqrt{17}.\sqrt{2}$	A1
			[2]
(b) Way 1	$(x-7)^2+(y-8)^2 = 34\ \left(\text{or } \left(\sqrt{34}\right)^2\right)$	$(x \pm 7)^2 + (y \pm 8)^2 = k$, where k is a positive value.	M1
		$(x-7)^2+(y-8)^2 = 34$	A1 oe
			[2]
(b) Way 2	$x^2+y^2-14x-16y+79=0$	$x^2+y^2 \pm 14x \pm 16y + c = 0$, where c is any value < 113.	M1
		$x^2+y^2-14x-16y+79=0$	A1 oe
			[2]
(c) Way 1	$\{\text{Gradient of radius}\} = \dfrac{13-8}{10-7}$ or $\dfrac{5}{3}$	This must be seen or implied in part (c).	B1
	Gradient of tangent $= -\dfrac{1}{m}\left(= -\dfrac{3}{5}\right)$	Using a perpendicular gradient method on their gradient. So Gradient of tangent $= -\dfrac{1}{\text{gradient of radius}}$	M1
	$y - 13 = -\dfrac{3}{5}(x-10)$	$y - 13 = (\text{their changed gradient})(x-10)$	M1
	$3x+5y-95=0$	$3x+5y-95=0$ o.e.	A1
			[4]
(c) Way 2	$2(x-7)+2(y-8)\dfrac{dy}{dx} = 0$	Correct differentiation (or equivalent). Seen or implied	B1
	$2(10-7)+2(13-8)\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{3}{5}$	Substituting **both** $x = 10$ and $y = 13$ into a valid differentiation to find a value for $\dfrac{dy}{dx}$	M1
	$y - 13 = -\dfrac{3}{5}(x-10)$	$y - 13 = (\text{their gradient})(x-10)$	M1
	$3x+5y-95=0$	$3x+5y-95=0$ o.e.	A1
			[4]
(c) Way 3	$10x+13y-7(x+10)-8(y+13)+79=0$	$10x+13y-7(x+10)-8(y+13)+79=0$	B1
		$10x+13y-7(x+10)-8(y+13)+c=0$ where c is any value <113	M2
	$3x+5y-95=0$	$3x+5y-95=0$ o.e.	A1
			[4]
			8

	Question Notes	
(a)	M1	Allow for $\{PQ =\}\ \sqrt{(7-10)^2+(8-13)^2}$ or for $\{PQ =\}\ \sqrt{3^2+5^2}$. Can be implied by answer.
	A1	Need to see $\sqrt{34}$. You can ignore subsequent work so $\sqrt{34}$ followed by 5.83 earns M1 A1, but $\{PQ =\}\ \sqrt{3^2+5^2} = 5.83$, with no exact value for the answer given, earns M1A0. Allow $\pm\sqrt{34}$ this time. NB Some use equation of circle to find this distance Achieving $\sqrt{34}$ gets M1A1 Others find half of their $\pm\sqrt{34}$. Do not isw here as it is an error – confusing d with diameter. Give M1A0
(b)	M1	Either of the correct approaches for equation of circle (as shown on scheme)
	A1	Correct equation (two are shown and any correct equivalent is acceptable)
(c)		A correct start to finding the gradient of the tangent (see each scheme)
	B1	Complete method for finding the gradient of the tangent (see each scheme) Where implicit differentiation has been used the only slips allowed here should be sign slips.
	1st M1	Correct attempt at line equation for tangent at correct point (10, 13) with **their tangent** gradient. If the $y = mx + c$ method is used to find the equation, this M1 is earned at the point where the x- and y-values are substituted to find c e.g. $13 = -3/5 \times 10 + c$
	2nd M1	
	A1	Accept any correct answer of the required format; so integer multiple of $3x+5y-95=0$ or $3x-95+5y=0$ or $-3x-5y+95=0$ (must include "=0") e.g. $6x+10y-190=0$ earns A1 Also allow $5y+3x-95=0$ etc
	Common error	$\frac{dy}{dx} = 2(x-7)+2(y-8) = 6+10 = 16$ so $(y-13)=16(x-10)$ is marked B0 M0 M1 A0 (Way 2)

Question 9

Question number	Scheme	Marks	
	$x^2 + y^2 - 10x + 6y + 30 = 0$		
(a)	Uses any appropriate method to find the coordinates of the centre, e.g achieves $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = \ldots$. Accept $(\pm 5, \pm 3)$ as indication of this.	M1	
	Centre is $(5, -3)$.	A1	(2)
(b) Way 1	Uses $\underline{(x \pm "5")^2 - "5^2"} + \underline{\underline{(y \pm "3")^2 - "3^2"}} + 30 = 0$ to give $r = \sqrt{"25"+"9"-30}$ or $r^2 = "25"+"9"-30$ (not $30 - 25 - 9$)	M1	
	$r = 2$	A1cao	(2)
Or Way 2	Using $\sqrt{g^2 + f^2 - c}$ from $x^2 + y^2 + 2gx + 2fy + c = 0$ (Needs formula stated or correct working)	M1	
	$r = 2$	A1	(2)
(c) Way 1	Use $x = 4$ in *an* equation of circle and obtain equation in y only	M1	
	e.g $(4-5)^2 + (y+3)^2 = 4$ or $4^2 + y^2 - 10 \times 4 + 6y + 30 = 0$		
	Solve their quadratic in y and obtain **two** solutions for y	dM1	
	e.g. $(y+3)^2 = 3$ or $y^2 + 6y + 6 = 0$ so $y = -3 \pm \sqrt{3}$	A1	(3)
Or Way 2	 Divide triangle PTQ and use Pythagoras with $"r"^2 - ("5"-4)^2 = h^2$,	M1	
	Find h and evaluate $"-3" \pm h$. May recognise $(1, \sqrt{3}, 2)$ triangle.	dM1	
	So $y = -3 \pm \sqrt{3}$	A1	(3) [7]

	Notes
(a)	**Parts (a) and (b) can be marked together** **M1** as in scheme and can be implied by $(\pm 5, \pm 3)$ May be awarded for writing LHS as $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = ...$ or by comparing with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ directly **A1:** (5, −3). **This correct answer implies M1A1**
(b)	**M1** for a **full** correct method leading to $r = \ldots$, or r^2 = with their 5, their −3, their 25 and their 9 and their "−30". Completion of square method errors result in **M0** here. Usually r = 4 or r = 16 imply M0A0 **A1** 2 cao Do not accept $r = \pm 2$ unless it is followed by (r =) 2 The correct answer with no wrong work seen implies M1A1 **Special case**: if centre is given as (-5, -3) or (5, 3) or (-5, 3) allow **M1A1** for r = 2 worked correctly. i.e. $r^2 = "25" + "9" - 30$
(c)	**M1**: *Way 1*: Use $x = 4$ in a circle equation (may have wrong centre and/or radius) to obtain an equation in y only or *Way* 2. Uses geometry to find equation in h (ft on their radius and centre) **dM1**: (needs first method mark) Solve their quadratic in y or *Way* 2. Uses their h and their y coordinate correctly **A1**: cao