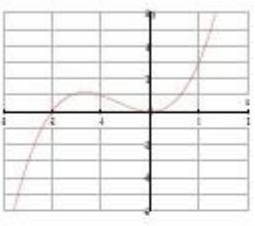

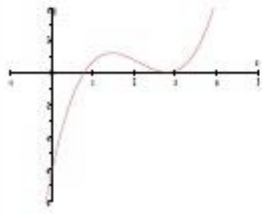


Modelling with Differentiation - Edexcel Past Exam Questions 2 **MARK SCHEME**

Question 1

Question	Scheme	Marks
(a)	$[y = x^3 + 2x^2]$ so $\frac{dy}{dx} = 3x^2 + 4x$	M1A1 (2)
(b)	 <p>Shape  Touching x-axis at origin Through and not touching or stopping at -2 on x-axis. Ignore extra intersections.</p>	B1 B1 B1 (3)
(c)	At $x = -2$: $\frac{dy}{dx} = 3(-2)^2 + 4(-2) = 4$ At $x = 0$: $\frac{dy}{dx} = 0$ (Both values correct)	M1 A1 (2)
(d)	 <p>Horizontal translation (touches x-axis still) $k - 2$ and k marked on positive x-axis $k^2(2 - k)$ (o.e) marked on negative y-axis</p>	M1 B1 B1 (3)
		10 marks
Notes		
Prod Rule	(a) M1 for attempt to multiply out and then some attempt to differentiate $x^n \rightarrow x^{n-1}$ Do not award for $2x(x + 2)$ or $2x(1 + 2)$ etc Award M1 for a correct attempt: 2 products with a + and at least one product correct A1 for both terms correct. (If +c or extra term is included score A0)	
	(b) 1 st B1 for correct shape (anywhere). Must have 2 clear turning points. 2 nd B1 for graph touching at origin (not crossing or ending) 3 rd B1 for graph passing through (not stopping or touching at) -2 on x axis and -2 marked on axis	
	SC B0B0B1 for $y = x^3$ or cubic with straight line between $(-2, 0)$ and $(0, 0)$	
	(c) M1 for attempt at $y'(0)$ or $y'(-2)$. Follow through their 0 or -2 and their $y'(x)$ or for a <u>correct</u> statement of zero gradient for an identified point on their curve that touches x -axis A1 for both correct answers	
(d)	For the M1 in part (d) ignore any coordinates marked – just mark the shape. M1 for a horizontal translation of their (b). Should still touch x – axis if it did in (b) Or for a graph of correct shape with min. and intersection in correct order on +ve x -axis	
	1 st B1 for k and $k - 2$ on the positive x -axis. Curve must pass through $k - 2$ and touch at k 2 nd B1 for a correct intercept on negative y -axis in terms of k . Allow $(0, 2k^2 - k^3)$ (o.e.) seen in script if curve passes through $-ve$ y -axis	

Question 2

Question	Scheme	Marks
(a)	$\left(\frac{1}{2}, 0\right)$	B1 (1)
(b)	$\frac{dy}{dx} = x^{-2}$ At $x = \frac{1}{2}$, $\frac{dy}{dx} = \left(\frac{1}{2}\right)^{-2} = 4$ ($= m$) Gradient of normal $= -\frac{1}{m} \left(= -\frac{1}{4} \right)$ Equation of normal: $y - 0 = -\frac{1}{4}\left(x - \frac{1}{2}\right)$ $2x + 8y - 1 = 0$ (*)	M1A1 A1 M1 M1
(c)	$2 - \frac{1}{x} = -\frac{1}{4}x + \frac{1}{8}$ $[= 2x^2 + 15x - 8 = 0]$ or $[8y^2 - 17y = 0]$ $(2x - 1)(x + 8) = 0$ leading to $x = \dots$ $x = \left[\frac{1}{2}\right]$ or -8 $y = \frac{17}{8}$ (or exact equivalent)	Alcs0 (6) M1 M1 A1 A1ft (4) 11 marks
Notes		
(a)	B1 accept $x = \frac{1}{2}$ if evidence that $y = 0$ has been used. Can be written on graph. Use ISW	
(b)	1 st M1 for kx^{-2} even if the '2' is not differentiated to zero. 1 st A1 for x^{-2} (o.e.) only 2 nd A1 for using $x = 0.5$ to get $m = 4$ (correctly) (or $m = 1/0.25$) To score final Alcs0 must see at least one intermediate equation for the line after $m = 4$ 2 nd M1 for using the perpendicular gradient rule on their m coming from their $\frac{dy}{dx}$ Their m must be a value not a letter. 3 rd M1 for using a changed gradient (based on y') and their A to find equation of line 3 rd Alcs0 for reaching printed answer with no incorrect working seen. Accept $2x + 8y = 1$ or equivalent equations with $\pm 2x$ and $\pm 8y$ Trial and improvement requires sight of first equation.	If no evidence of $\frac{dy}{dx}$ seen then 0/6
(c)	1 st M1 for attempt to form a suitable equation in one variable. Do not penalise poor use of brackets etc. 2 nd M1 for simplifying their equation to a 3TQ and attempting to solve. May be \Rightarrow by $x = -8$ 1 st A1 for $x = -8$ (ignore a second value). If found y first allow ft for x if $x < 0$ 2 nd A1ft for $y = \frac{17}{8}$ Follow through their x value in line or curve provided answer is > 0 This second A1 is dependent on <u>both</u> M marks	

Question 3

Question Number	Scheme	Marks
(a)	$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$ $\left\{ \frac{dy}{dx} = \right\} 5(3)x^2 - 6\left(\frac{4}{3}\right)x^{\frac{1}{3}} + 2$ $= 15x^2 - 8x^{\frac{1}{3}} + 2$	M1 A1 A1 A1 [4]
(b)	$\left\{ \frac{d^2y}{dx^2} = \right\} 30x - \frac{8}{3}x^{-\frac{2}{3}}$	M1 A1 [2] 6
Notes		
(a)	<p>M1: for an attempt to differentiate $x^n \rightarrow x^{n-1}$ to one of the first three terms of $y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$. So seeing either $5x^3 \rightarrow \pm \lambda x^2$ or $-6x^{\frac{4}{3}} \rightarrow \pm \mu x^{\frac{1}{3}}$ or $2x \rightarrow 2$ is M1.</p> <p>1st A1: for $15x^2$ only.</p> <p>2nd A1: for $-8x^{\frac{1}{3}}$ or $-8\sqrt[3]{x}$ only.</p> <p>3rd A1: for $+2$ (+c included in part (a) loses this mark). Note: $2x^0$ is A0 unless simplified to 2.</p>	
(b)	<p>M1: For differentiating $\frac{dy}{dx}$ again to give either</p> <ul style="list-style-type: none"> a correct follow through differentiation of their x^2 term or for $\pm \alpha x^{\frac{1}{3}} \rightarrow \pm \beta x^{-\frac{2}{3}}$. <p>A1: for any correct expression on the same line (accept un-simplified coefficients).</p> <p>For powers: $30x^{2-1} - \frac{8}{3}x^{\frac{1}{3}-1}$ is A0, but writing powers as one term eg: $(15 \times 2x) - \frac{8}{3}x^{-\frac{2}{3}}$ is ok for A1.</p> <p>Note: Candidates leaving their answers as $\left\{ \frac{dy}{dx} = \right\} 15x^2 - \frac{24}{3}x^{\frac{1}{3}} + 2$ and $\left(\frac{d^2y}{dx^2} = \right) 30x - \frac{24}{9}x^{-\frac{2}{3}}$ are awarded M1A1A0A1 in part (a) and M1A1 in part (b).</p> <p>Be careful: $30x - \frac{8}{3}x^{-\frac{1}{3}}$ will be A0.</p> <p>Note: For an extra term appearing in part (b) on the same line, ie $30x - \frac{8}{3}x^{-\frac{2}{3}} + 2$ is M1A0</p> <p>Note: If a candidate writes in part (a) $15x^2 - 8x^{\frac{1}{3}} + 2 + c$ and in part (b) $30x - \frac{8}{3}x^{-\frac{2}{3}} + c$ then award (a) M1A1A1A0 (b) M1A1</p>	

Question 4

Question Number	Scheme	Marks
(a)	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geq 0$ So, $y = 2x - 8x^{\frac{1}{2}} + 5$ $\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\} \quad (x > 0)$	M1 A1 A1 [3]
(b)	(When $x = \frac{1}{4}, y = 2(\frac{1}{4}) - 8\sqrt{(\frac{1}{4})} + 5$ so) $y = \frac{3}{2}$ (gradient = $\frac{dy}{dx} = 2 - \frac{4}{\sqrt{(\frac{1}{4})}} = -6$) Either: $y - \frac{3}{2} = -6(x - \frac{1}{4})$ or: $y = -6x + c$ and $\frac{3}{2} = -6(\frac{1}{4}) + c \Rightarrow c = 3$ So $y = -6x + 3$	B1 M1 dM1 A1 [4]
(c)	Tangent at Q is parallel to $2x - 3y + 18 = 0$ $(y = \frac{2}{3}x + 6 \Rightarrow)$ Gradient = $\frac{2}{3}$. so tangent gradient is $\frac{2}{3}$ So, $2 - \frac{4}{\sqrt{x}} = \frac{2}{3}$ Sets their gradient function = their numerical gradient. $\Rightarrow \frac{4}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9$ Ignore extra answer $x = -9$ When $x = 9, y = 2(9) - 8\sqrt{9} + 5 = -1$ Substitutes their found x into equation of curve. $y = -1$	B1 M1 A1 dM1 A1 [5]
Notes		
(a)	M1: Evidence of differentiation, so $x^n \rightarrow x^{n-1}$ at least once so $x^1 \rightarrow 1$ or x^0 or $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ not just $5 \rightarrow 0$ A1: Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified. A1: $2 - 4x^{-\frac{1}{2}}$ Both terms correct, and simplified. Do not need to include domain $x > 0$	
(b)	B1: Obtaining $y = 3/2$ or fractional or decimal equivalent (no working need be seen) M1: An attempt to substitute $x = \frac{1}{4}$ into $\frac{dy}{dx}$ to establish gradient. This may be implied by -6 or $m = -6$ but not $y = -6$. Can earn this M mark if they go on to use $m = \frac{1}{6}$ or use their numerical value of $\frac{dy}{dx}$. dM1: This depends on previous method mark. Complete method for obtaining the equation of the tangent, using their tangent gradient and their value for y_1 (obtained from $x = \frac{1}{4}$, allow slip) i.e. $y - y_1 = m_T(x - \frac{1}{4})$ with their tangent gradient and their y_1 or uses $y = mx + c$ with $(\frac{1}{4}, \text{their } y_1)$ and their tangent gradient. A1: $y = -6x + 3$ or $y = 3 - 6x$ or $a = -6$ and $b = 3$	
(c)	B1: For the value $2/3$ not $2/3 x$ not $-3/2$ M1: Sets their gradient function $dy/dx =$ their numerical gradient A1: Obtains $x = 9$ dM1: Substitutes their x (from gradient equation) into original equation of curve C i.e. original expression $y =$ A1: $(9, -1)$ or $x = 9, y = -1$, or just $y = -1$	
Special Cases: In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4		
In (c) Uses perpendicular instead of parallel then award B0 M1 A0 M1 A0 i.e. max 2/5 – see over		
12 marks		

Question 5

Question Number	Scheme		Marks
(a)	$(3 - x^2)^2 = 9 - 6x^2 + x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4$, $p, q \neq 0$	M1
	$9x^{-2} + x^2$	Must come from $\frac{9 + x^4}{x^2}$	A1
	-6	Must come from $\frac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\frac{(3 - x^2)^2}{x^2}$ as $(3x^{-1} - x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3 - x^2)^2 = 9 + Ax^2 + Bx^4$, expands $(3 - x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)
	$(f'(x) = 9x^{-2} - 6 + x^2)$		
(b)	$-18x^{-3} + 2x$	M1: $x^n \rightarrow x^{n-1}$ on separate terms at least once. Do not award for $A \rightarrow 0$ (Integrating is M0) A1ft: $-18x^{-3} + 2Bx$ with a numerical B and no extra terms. (A may have been incorrect or even zero)	M1 A1ft
			(2)
(c)	$f(x) = -9x^{-1} - 6x + \frac{x^3}{3} (+c)$	M1: $x^n \rightarrow x^{n+1}$ on separate terms at least once. (Differentiating is M0) A1ft: $-9x^{-1} + Ax + \frac{Bx^3}{3} (+c)$ with numerical A and B , $A, B \neq 0$	M1A1ft
	$10 = \frac{-9}{-3} - 6(-3) + \frac{(-3)^3}{3} + c$ so $c = \dots$	Uses $x = -3$ and $y = 10$ in what they think is $f(x)$ (They may have differentiated here) but it must be a changed function i.e. not the original $f'(x)$, to form a linear equation in c and attempts to find c . No $+c$ gets M0 and A0 unless their method implies that they are correctly finding a constant.	M1
	$c = -2$	cso	A1
	$(f(x) =) -9x^{-1} - 6x + \frac{x^3}{3} + \text{their } c$	Follow through their c in an otherwise (possibly un-simplified) correct expression. Allow $-\frac{9}{x}$ for $-9x^{-1}$ or even $\frac{9x^{-1}}{-1}$.	A1ft
	Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.		
			(5)
			[10]

Question 6

Question Number	Scheme		Marks
(a)	$\left(-\frac{3}{4}, 0\right)$. Accept $x = -\frac{3}{4}$		B1
			(1)
(b)	$y = 4$	B1: One correct asymptote	B1B1
	$x = 0$ or 'y-axis'	B1: Both correct asymptotes and no extra ones.	
	Special case $x \neq 0$ and $y \neq 4$ scores B1B0		
			(2)
(c)	$\frac{dy}{dx} = -3x^{-2}$	$\frac{dy}{dx} = kx^{-2}$ (Allow $\frac{dy}{dx} = kx^{-2} + 4$)	M1
	At $x = -3$, gradient of curve = $-\frac{1}{3}$	Cao (may be un-simplified but must be a fraction with no powers) e.g. $-3(-3)^{-2}$ scores A0 unless evaluated as e.g. $\frac{-3}{9}$ or is implied by their normal gradient.	A1
	Gradient of normal = $-1/m$	Correct perpendicular gradient rule applied to a numerical gradient that must have come from substituting $x = -3$ into their derivative. Dependent on the previous M1.	dM1
	Normal at P is $(y - 3) = 3(x + 3)$	M1: Correct straight line method using $(-3, 3)$ and a "changed" gradient. A wrong equation with no formula quoted is M0. Also dependent on the first M1.	dM1A1
		A1: Any correct equation	
			(5)
(d)	$(-4, 0)$ and $(0, 12)$.	Both correct (May be seen on a sketch)	B1
	So AB has length $\sqrt{160}$ or AB^2 has length 160	M1: Correct use of Pythagoras for their A and B one of which lies on the x-axis and the other on the y-axis, obtained from their equation in (c). A correct method for AB^2 or AB. A1: $\sqrt{160}$ or better e.g. $4\sqrt{10}$ with no errors seen	M1 A1 cso
			(3)
			[11]



Question 7

Question Number	Scheme	Notes	Marks
	$y = x^3 + 4x + 1 \Rightarrow \frac{dy}{dx} = 3x^2 + 4(+0)$	M1: $x^n \rightarrow x^{n-1}$ including $1 \rightarrow 0$	M1A1
		A1: Correct differentiation (Do not allow $4x^0$ unless $x^0 = 1$ is implied by later work)	
	substitute $x = 3 \Rightarrow \text{gradient} = 31$	M1: Substitutes $x = 3$ into their $\frac{dy}{dx}$ (not y)	M1A1
		Substitutes $x = 3$ into a “changed” function. They may even have integrated.	
		A1: cao	
			[4]

Question 8

Question Number	Scheme	Marks
	<p>(a) $(1-2x)^2 = 1-4x+4x^2$</p> $\frac{d}{dx}(1-2x)^2 = \frac{d}{dx}(1-4x+4x^2) = -4+8x \text{ o.e.}$	<p>M1</p> <p>M1A1</p> <p>(3)</p>
	Alternative method using chain rule: Answer of $-4(1-2x)$	<p>M1M1A1</p> <p>(3)</p>
	<p>(b) $\frac{x^5+6\sqrt{x}}{2x^2} = \frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2} = \frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$</p> <p>Attempts to differentiate $x^{-\frac{3}{2}}$ to give $kx^{-\frac{5}{2}}$</p> $= \frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}} \text{ o.e.}$ <p>Quotient Rule (May rarely appear) – See note below</p>	<p>M1,A1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>(7 marks)</p>

Notes

- (a) M1 Attempt to multiply out bracket. Must be 3 or 4 term quadratic and **must have constant term 1**
M1 $x^n \rightarrow x^{n-1}$. Follow through on any term in an incorrect expression. Accept a constant $\rightarrow 0$
A1 $-4+8x$ Accept $-4(1-2x)$ or equivalent. This is not cso and may follow error in the constant term
Following correct answer by $-2+4x$ – apply isw

Correct answer with no working – assume chain rule and give M1M1A1 i.e. 3/3

Common errors: $(1-2x)^2 = 2-4x+4x^2$ is M0, then allow M1A1 for $-4+8x$

$(1-2x)^2 = 1-4x^2$ is M0 then $-8x$ earns M1A0 or $(1-2x)^2 = 1-2x^2$ is M0 then $-4x$ earns M1A0

Use of Chain Rule:

M1M1: first M1 for complete method so $2 \times (\pm 2)(1-2x)$ second M1 for $(1-2x)$ (as power reduced)

Then A1 for $-4(1-2x)$ or for $-4+8x$

So (i) $2(1-2x)$ gets M0 M1A0 for reducing power and (ii) $2 \times 2(1-2x)$ gets M1 M1A0

- (b) M1 An attempt to divide by $2x^2$ first. This can be implied by the sight of the following

Some correct working e.g. $\frac{x^5}{2x^2} + 6\frac{\sqrt{x}}{2x^2}$ or $(x^5+6\sqrt{x})(2x^2)^{-1}$ leading to $ax^p + bx^q$ in either case

or can be implied by $\frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ (after no working) i.e. both coefficients correct and power 3 correct

Common error: $(x^5+6\sqrt{x})2x^{-2}$ is M0 (may earn next M mark for the differentiation $x^{-\frac{3}{2}} \rightarrow x^{-\frac{5}{2}}$)

A1 Writing the given expression as $\frac{1}{2}x^3 + 3x^{-\frac{3}{2}}$ or $0.5x^3 + \frac{6}{2}x^{-\frac{3}{2}}$ or $0.5x^3 + \frac{6}{2}x^{-\frac{1}{2}}$ or etc...

M1 $x^{-\frac{3}{2}} \rightarrow x^{-\frac{5}{2}}$ A1 Cao $\frac{3}{2}x^2 - \frac{9}{2}x^{-\frac{5}{2}}$ o.e. e.g. $\frac{3}{2}x^2 - \frac{9}{2x^2\sqrt{x}}$ then isw. Allow factorised form. Do not

penalise $+\frac{9}{2}x^{-\frac{5}{2}}$ used instead of $-\frac{9}{2}x^{-\frac{5}{2}}$

Use of Quotient Rule: M1,A1: Reaching $\frac{2x^2(5x^4+3x^{-\frac{1}{2}}) - 4x(x^5+6x^{\frac{1}{2}})}{4x^4} = \frac{6x^6-18x^{\frac{3}{2}}}{4x^4}$

Send to review if doubtful M1A1: Simplifying (e.g. dividing numerator and denominator by 2) to reach $\frac{3x^6-9x^{\frac{3}{2}}}{2x^4}$ o.e.

Question 9

Question Number	Scheme	Marks
(a)	<p>Substitutes $x = 2$ into $y = 20 - 4x - \frac{18}{x}$ and gets 3</p> $\frac{dy}{dx} = -4 + \frac{18}{x^2}$ <p>Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>Method 1</p> <p>States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)</p> <p>to deduce that $y = -2x + 7$ *</p> </div> <div style="width: 45%;"> <p>Method 2</p> <p>Or: Check that (2, 3) lies on the line $y = -2x + 7$</p> <p>Deduce equation of normal as it has the same gradient and passes through a common point</p> </div> </div>	<p>B1</p> <p>M1 A1</p> <p>dM1</p> <p>dM1</p> <p>A1*</p> <p>(6)</p>
(b)	<p>Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$</p> <p>Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$</p> <p>$(2x - 9)(x - 2) = 0$ so $x =$ or $(y - 3)(y + 2) = 0$ so $y =$</p> <p>$x = \frac{9}{2}, y = -2$</p>	<p>M1 A1</p> <p>dM1</p> <p>A1, A1</p> <p>(5)</p> <p>(11 marks)</p>

PTO for notes on this question.

Question 10

Question Number	Scheme		Marks
(a)	$(x^2 + 4)(x - 3) = x^3 - 3x^2 + 4x - 12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
	$\frac{x^3 - 3x^2 + 4x - 12}{2x} = \frac{x^2}{2} - \frac{3}{2}x + 2 - 6x^{-1}$	M1: Attempt to divide each term by $2x$. The powers of x of at least two terms must follow from their expansion. Allow an attempt to multiply by $2x^{-1}$ A1: Correct expression. May be un-simplified but powers of x must be combined e.g. $\frac{x^2}{2}$ not $\frac{x^3}{2x}$	M1A1
	$\frac{dy}{dx} = x - \frac{3}{2} + \frac{6}{x^2}$ oe e.g. $\frac{2x^3 - 3x^2 + 12}{2x^2}$	ddM1: $x^n \rightarrow x^{n-1}$ or $2 \rightarrow 0$ Dependent on both previous method marks. A1: $x - \frac{3}{2} + \frac{6}{x^2}$ oe and isw Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0 . If they lose the previous A1 because of an incorrect constant only then allow recovery here and in part (b) for a correct derivative.	ddM1A1
			(5)
	See appendix for alternatives using product/quotient rule		
	At $x = -1$, $y = 10$	Correct value for y	B1
(b)	$\left(\frac{dy}{dx}\right)_{-1} = -1 - \frac{3}{2} + \frac{6}{1} = 3.5$	M1: Substitutes $x = -1$ into their expression for dy/dx A1: 3.5 oe cso	M1A1
	$y - '10' = '3.5'(x - -1)$	Uses their tangent gradient which must come from calculus with $x = -1$ and their numerical y with a correct straight line method. If using $y = mx + c$, this mark is awarded for correctly establishing a value for c .	M1
	$2y - 7x - 27 = 0$	$\pm k(2y - 7x - 27) = 0$ cso	A1
			(5)
			(10 marks)

Appendix

(a)

Way 2 Quotient	$(x^2 + 4)(x - 3) = x^3 - 3x^2 + 4x - 12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
	$\frac{dy}{dx} = \frac{2x(3x^2 - 6x + 4) - 2(x^3 - 3x^2 + 4x - 12)}{(2x)^2}$	M1: Correct application of quotient rule A1: Correct derivative	M1A1
	$= \frac{4x^3}{4x^2} - \frac{6x^2}{4x^2} + \frac{24}{4x^2} = x - \frac{3}{2} + \frac{6}{x^2}$ oe e.g. $\frac{2x^3 - 3x^2 + 12}{2x^2}$	M1: Collects terms and divides by denominator. Dependent on both previous method marks. A1: $x - \frac{3}{2} + \frac{6}{x^2}$ oe and isw Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0 .	ddM1A1
Way 3 Product	$y = \left(\frac{x}{2} + \frac{2}{x}\right)(x - 3) \text{ or } (x^2 + 4)\left(\frac{1}{2} - \frac{3}{2x}\right)$	Divides one bracket by $2x$	M1
	$\frac{dy}{dx} = (x - 3)\left(\frac{1}{2} - \frac{2}{x^2}\right) + \left(\frac{x}{2} + \frac{2}{x}\right) \text{ or } \frac{dy}{dx} = (x^2 + 4)\frac{3}{2x^3} + 2x\left(\frac{1}{2} - \frac{3}{2x}\right)$	M1: Correct application of product rule A1: Correct derivative	M1A1
	$= \frac{3}{2} + \frac{6}{x^2} + x - 3 = x - \frac{3}{2} + \frac{6}{x^2}$ oe e.g. $\frac{2x^3 - 3x^2 + 12}{2x^2}$	M1: Expands and collects terms. Dependent on both previous method marks. A1: $x - \frac{3}{2} + \frac{6}{x^2}$ oe and isw Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0 .	ddM1A1

Way 4 Product	$(x^2 + 4)(x - 3) = x^3 - 3x^2 + 4x - 12$	Attempt to multiply out the numerator to get a cubic with 4 terms and at least 2 correct	M1
	$\frac{dy}{dx} = (x^3 - 3x^2 + 4x - 12) \times -\frac{1}{2}x^{-2} + \frac{1}{2}x^{-1}(3x^2 - 6x + 4)$ M1: Correct application of product rule A1: Correct derivative		M1A1
	$\frac{dy}{dx} = -\frac{x}{2} + \frac{3}{2} - \frac{2}{x} + \frac{6}{x^2} + \frac{3x}{2} - 3 + \frac{2}{x} = x - \frac{3}{2} + \frac{6}{x^2}$ ddM1: Expands and collects terms Dependent on both previous method marks. A1: $x - \frac{3}{2} + \frac{6}{x^2}$ oe e.g. $\frac{2x^3 - 3x^2 + 12}{2x^2}$ and isw. Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ and not x^0 .		ddM1A1
Way 5	$y = \left(\frac{x}{2} + \frac{2}{x}\right)(x - 3) \text{ or } (x^2 + 4)\left(\frac{1}{2} - \frac{3}{2x}\right)$	Divides one bracket by $2x$	M1
	$= \frac{x^2}{2} - \frac{3}{2}x + 2 - 6x^{-1}$	M1: Expands A1: Correct expression	M1A1
	$\frac{dy}{dx} = x - \frac{3}{2} + \frac{6}{x^2}$ oe e.g. $\frac{2x^3 - 3x^2 + 12}{2x^2}$	ddM1: $x^n \rightarrow x^{n-1}$ or $2 \rightarrow 0$ Dependent on both previous method marks. A1: $x - \frac{3}{2} + \frac{6}{x^2}$ oe and isw Accept $1x$ or even $1x^1$ but not $\frac{2x}{2}$ If they lose the previous A1 because of an incorrect constant only then allow recovery here for a correct derivative.	ddM1A1



Question 11

Question Number	Scheme	Marks
(a)	$y = 2x^3 + kx^2 + 5x + 6$	
	$\left(\frac{dy}{dx} = \right) 6x^2 + 2kx + 5$	M1: $x^n \rightarrow x^{n-1}$ for one of the terms including $6 \rightarrow 0$ A1: Correct derivative
		[2]
(b)	Gradient of given line is $\frac{17}{2}$	Uses or states $\frac{17}{2}$ or equivalent e.g. 8.5. Must be stated or used in (b) and not just seen as part of $y = \frac{17}{2}x + \frac{1}{2}$.
	$\left(\frac{dy}{dx}\right)_{x=-2} = 6(-2)^2 + 2k(-2) + 5$	Substitutes $x = -2$ into their derivative (not the curve)
	" $24 - 4k + 5 = \frac{17}{2}$ " $\Rightarrow k = \frac{41}{8}$	dM1: Puts their expression = their $\frac{17}{2}$ (Allow BOD for 17 or -17 but not the normal gradient) and solves to obtain a value for k . Dependent on the previous method mark. A1: $\frac{41}{8}$ or $5\frac{1}{8}$ or 5.125
	Note: $6x^2 + 2kx + 5 = \frac{17}{2}x + \frac{1}{2}$ scores no marks on its own but may score the first M mark if they substitute $x = -2$ into the lhs. If they rearrange this equation and then substitute $x = -2$, this scores no marks.	
		[4]
(c)	$y = -16 + 4k - 10 + 6 = 4"k" - 20 = \frac{1}{2}$	M1: Substitutes $x = -2$ and their numerical k into $y = \dots$ A1: $y = \frac{1}{2}$
	Allow the marks for part (c) to be scored in part (b).	
		[2]
(d)	$y - \frac{1}{2} = \frac{17}{2}(x - 2) \Rightarrow -17x + 2y - 35 = 0$ or $y = \frac{17}{2}x + c \Rightarrow c = \dots \Rightarrow -17x + 2y - 35 = 0$ or $2y - 17x = 1 + 34 \Rightarrow -17x + 2y - 35 = 0$	M1: Correct attempt at linear equation with their 8.5 gradient (not the normal gradient) using $x = -2$ and their $\frac{1}{2}$ A1: cao (allow any integer multiple)
		[2]
		10 marks

Question 12

Question Number	Scheme		Marks
	$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4 = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} + 4$		
	$x^n \rightarrow x^{n-1}$	Decreases any power by 1. Either $x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \rightarrow x^{-\frac{3}{2}}$ or $4 \rightarrow 0$ or $x^{\text{their } n} \rightarrow x^{\text{their } n-1}$ for fractional n .	M1
	$\left(\frac{dy}{dx}\right) = \frac{1}{2}x^{-\frac{1}{2}} + 4 \times -\frac{1}{2}x^{-\frac{3}{2}}$ $\left(= \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}}\right)$	Correct derivative, simplified or unsimplified including indices. E.g. allow $\frac{1}{2}-1$ for $-\frac{1}{2}$ and allow $-\frac{1}{2}-1$ for $-\frac{3}{2}$	A1
	$x = 8 \Rightarrow \frac{dy}{dx} = \frac{1}{2}8^{-\frac{1}{2}} + 4 \times -\frac{1}{2}8^{-\frac{3}{2}}$	Attempts to substitute $x = 8$ into their 'changed' (even integrated) expression that is clearly not y . If they attempt algebraic manipulation of their dy/dx before substitution, this mark is still available.	M1
	$= \frac{1}{2\sqrt{8}} - \frac{2}{(\sqrt{8})^3} = \frac{1}{2\sqrt{8}} - \frac{2}{8\sqrt{8}} = \frac{1}{8\sqrt{2}} = \frac{1}{16}\sqrt{2}$	B1: $\sqrt{8} = 2\sqrt{2}$ seen or implied anywhere, including from substituting $x = 8$ into y . May be seen explicitly or implied from e.g. $8^{\frac{3}{2}} = 16\sqrt{2}$ or $8^{\frac{5}{2}} = 128\sqrt{2}$ or $4\sqrt{8} = 8\sqrt{2}$ A1: cso $\frac{1}{16}\sqrt{2}$ or $\frac{\sqrt{2}}{16}$ and allow rational equivalents for $\frac{1}{16}$ e.g. $\frac{32}{512}$ Apply isw so award this mark as soon as a correct answer is seen.	B1A1
			(5 marks)

Question 13

Question Number	Scheme		Marks
(a)(i)	$k = (-5)^2 \times 3 = 75$	M1: Attempts to find the 'y' intercept. Accept as evidence $(-5)^2 \times 3$ with or without the bracket. If they expand $f(x)$ to polynomial form here then they must then select their constant to score this mark . May be implied by sight of 75 on the diagram.	M1A1
		A1: $k = 75$. Must clearly be identified as k . Allow this mark even from an incorrect or incomplete expansion as long as the constant $k = 75$ is obtained. Do not isw e.g. if 75 is seen followed by $k = -75$ score M1A0.	
(ii)	$c = \frac{5}{2}$ only	$c = \frac{5}{2}$ oe (and no other values). Do not award just from the diagram – must be stated as the value of c .	B1
			(3)
(b)	$f(x) = (2x-5)^2(x+3) = (4x^2 - 20x + 25)(x+3) = 4x^3 - 8x^2 - 35x + 75$ Attempts $f(x)$ as a cubic polynomial by attempting to square the first bracket and multiply by the linear bracket or expands $(2x-5)(x+3)$ and then multiplies by $2x-5$ Must be seen or used in (b) but may be done in part (a). Allow poor squaring e.g. $(2x-5)^2 = 4x^2 \pm 25$		M1
	$(f'(x) =) 12x^2 - 16x - 35^*$	M1: Reduces powers by 1 in all terms including any constant $\rightarrow 0$	M1A1*
		A1: Correct proof. Withhold this mark if there have been any errors including missing brackets earlier e.g. $(2x-5)^2(x+3) = 4x^2 - 20x + 25(x+3) = \dots$	
			(3)

(c)	$f'(3) = 12 \times 3^2 - 16 \times 3 - 35$	Substitutes $x = 3$ into their $f'(x)$ or the given $f'(x)$. Must be a changed function i.e. not into $f(x)$.	M1
	$12x^2 - 16x - 35 = '25'$	Sets their $f'(x)$ or the given $f'(x) =$ their $f'(3)$ with a consistent f' . Dependent on the previous method mark.	dM1
	$12x^2 - 16x - 60 = 0$	$12x^2 - 16x - 60 = 0$ or equivalent 3 term quadratic e.g. $12x^2 - 16x = 60$. (A correct quadratic equation may be implied by later work). This is cso so must come from correct work – i.e. they must be using the <u>given</u> $f'(x)$.	A1 cso
	$(x - 3)(12x + 20) = 0 \Rightarrow x = \dots$	Solves 3 term quadratic by suitable method – see General Principles. Dependent on both previous method marks.	ddM1
	$x = -\frac{5}{3}$	$x = -\frac{5}{3}$ or clearly identified. If $x = 3$ is also given and not rejected, this mark is withheld. (allow -1.6 recurring as long as it is clear i.e. a dot above the 6). This is cso and must come from correct work – i.e. they must be using the <u>given</u> $f'(x)$.	A1 cso
			(5)
			(11 marks)
Alt (b) Product rule.	$f(x) = (2x - 5)^2(x + 3) \Rightarrow f'(x) = (2x - 5)^2 \times 1 + (x + 3) \times 4(2x - 5)$ M1: Attempts product rule to give an expression of the form $p(2x - 5)^2 + q(x + 3)(2x - 5)$ M1: Multiplies out and collects terms A1: $f'(x) = 12x^2 - 16x - 35^*$		M1 M1A1*