1. The curve $C_{1}$ has equation

$$
y=x^{2}(x+2) .
$$

(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(b) Sketch $C_{1}$, showing the coordinates of the points where $C_{1}$ meets the $x$-axis.
(c) Find the gradient of $C_{1}$ at each point where $C_{1}$ meets the $x$-axis.

The curve $C_{2}$ has equation

$$
y=(x-k)^{2}(x-k+2),
$$

where $k$ is a constant and $k>2$.
(d) Sketch $C_{2}$, showing the coordinates of the points where $C_{2}$ meets the $x$ and $y$ axes.
2.


Figure 2
Figure 2 shows a sketch of the curve $C$ with equation

$$
y=2-\frac{1}{x}, \quad x \neq 0 .
$$

The curve crosses the $x$-axis at the point $A$.
(a) Find the coordinates of $A$.
(b) Show that the equation of the normal to $C$ at $A$ can be written as

$$
\begin{equation*}
2 x+8 y-1=0 . \tag{6}
\end{equation*}
$$

The normal to $C$ at $A$ meets $C$ again at the point $B$, as shown in Figure 2 .
(c) Find the coordinates of $B$.
3.

$$
y=5 x^{3}-6 x^{\frac{4}{3}}+2 x-3
$$

(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$, giving each term in its simplest form.
(b) Find $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$.

June 12 Q4
4. The curve $C$ has equation

$$
y=2 x-8 \sqrt{ } x+5, \quad x \geq 0
$$

(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$, giving each term in its simplest form.

The point $P$ on $C$ has $x$-coordinate equal to $\frac{1}{4}$.
(b) Find the equation of the tangent to $C$ at the point $P$, giving your answer in the form $y=a x+b$, where $a$ and $b$ are constants.

The tangent to $C$ at the point $Q$ is parallel to the line with equation $2 x-3 y+18=0$.
(c) Find the coordinates of $Q$.
5.

$$
\mathrm{f}^{\prime}(x)=\frac{\left(3-x^{2}\right)^{2}}{x^{2}}, \quad x \neq 0
$$

(a) Show that $\mathrm{f}^{\prime}(x)=9 x^{-2}+A+B x^{2}$, where $A$ and $B$ are constants to be found.
(b) Find $\mathrm{f}^{\prime \prime}(x)$.

Given that the point $(-3,10)$ lies on the curve with equation $y=\mathrm{f}(x)$,
(c) find $\mathrm{f}(x)$.
6.


Figure 2
Figure 2 shows a sketch of the curve $H$ with equation $y=\frac{3}{x}+4, x \neq 0$.
(a) Give the coordinates of the point where $H$ crosses the $x$-axis.
(b) Give the equations of the asymptotes to $H$.
(c) Find an equation for the normal to $H$ at the point $P(-3,3)$.

This normal crosses the $x$-axis at $A$ and the $y$-axis at $B$.
(d) Find the length of the line segment $A B$. Give your answer as a surd.
7. Given $y=x^{3}+4 x+1$, find the value of $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $x=3$.
8. Differentiate with respect to $x$, giving each answer in its simplest form,
(a) $(1-2 x)^{2}$,
(b) $\frac{x^{5}+6 \sqrt{x}}{2 x^{2}}$.
9.


Figure 3
A sketch of part of the curve $C$ with equation

$$
y=20-4 x-\frac{18}{x}, \quad x>0
$$

is shown in Figure 3.
Point $A$ lies on $C$ and has an $x$ coordinate equal to 2 .
(a) Show that the equation of the normal to $C$ at $A$ is $y=-2 x+7$.

The normal to $C$ at $A$ meets $C$ again at the point $B$, as shown in Figure 3 .
(b) Use algebra to find the coordinates of $B$.
10. The curve $C$ has equation

$$
\begin{equation*}
y=\frac{\left(x^{2}+4\right)(x-3)}{2 x}, x \neq 0 . \tag{5}
\end{equation*}
$$

(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in its simplest form.
(b) Find an equation of the tangent to $C$ at the point where $x=-1$.

Give your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
June 15 Q6
11. The curve $C$ has equation $y=2 x^{3}+k x^{2}+5 x+6$, where $k$ is a constant.
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.

The point $P$, where $x=-2$, lies on $C$.
The tangent to $C$ at the point $P$ is parallel to the line with equation $2 y-17 x-1=0$.
Find
(b) the value of $k$,
(c) the value of the $y$ coordinate of $P$,
(d) the equation of the tangent to $C$ at $P$, giving your answer in the form $a x+b y+c=0$, where $a, b$ and $c$ are integers.
12. Given

$$
y=\sqrt{x}+\frac{4}{\sqrt{x}}+4, \quad x>0
$$

find the value of $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $x=8$, writing your answer in the form $a \sqrt{2}$, where $a$ is a rational number.
13.


Figure 2
Figure 2 shows a sketch of part of the curve $y=\mathrm{f}(x), x \in \mathbb{R}$, where

$$
\mathrm{f}(x)=(2 x-5)^{2}(x+3)
$$

(a) Given that
(i) the curve with equation $y=\mathrm{f}(x)-k, x \in \mathbb{R}$, passes through the origin, find the value of the constant $k$,
(ii) the curve with equation $y=\mathrm{f}(x+c), x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant $c$.
(b) Show that $\mathrm{f}^{\prime}(x)=12 x^{2}-16 x-35$

Points $A$ and $B$ are distinct points that lie on the curve $y=\mathrm{f}(x)$.
The gradient of the curve at $A$ is equal to the gradient of the curve at $B$.
Given that point $A$ has $x$ coordinate 3
(c) find the $x$ coordinate of point $B$.

