**Exponential and Natural Logarithms - Edexcel Past Exam Questions 2 MARK SCHEME**

## Question 1

| Question No | Scheme | Marks |
|---|---|---|
| | (a) 20 ($mm^2$) | B1 (1) |
| | (b) $'40' = 20\ e^{1.5t} \quad \rightarrow \quad e^{1.5t} = c$<br>$e^{1.5t} = \frac{40}{20} = (2)$<br>Correct order $1.5t = ln'2' \rightarrow \quad t = \frac{lnc}{1.5}$<br>$t = \frac{ln2}{1.5} = (\, awrt\ 0.46)$<br>12.28 or 28 (minutes} | M1<br>A1<br>M1<br>A1<br>A1 (5) |
| | | (6 marks) |

**Question 2**

| Question Number | Scheme | Marks |
|---|---|---|
| | (a) (£) 19500 | B1 |
| | | (1) |
| | (b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ | |
| | $17e^{-0.25t} + 2e^{-0.5t} = 9$ | |
| | $(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$ | |
| | $0 = 9e^{0.5t} - 17e^{0.25t} - 2$ | M1 |
| | $0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$ | M1 |
| | $e^{0.25t} = 2$ | A1 |
| | $t = 4\ln(2)$ *oe* | A1 |
| | | (4) |
| | (c ) | |
| | $(\frac{dV}{dt}) = -4250e^{-0.25t} - 1000e^{-0.5t}$ | M1A1 |
| | When $t$=8 Decrease = 593 (£/year) | M1A1 |
| | | (4) |
| | | **(9 marks)** |

(a) B1 19500. The £ sign is not important for this mark

(b) M1 Substitute V=9500, collect terms and set on 1 side of an equation =0. Indices must be correct
Accept $17000e^{-0.25t}+2000e^{-0.5t}-9000=0$ and $17000x+2000x^2-9000=0$ where $x=e^{-0.25t}$

M1 Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$
For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t}-1)(e^{-0.25t}+9)=0$
Alternatively let $'x'=e^{0.25t}$ or otherwise and factorise a quadratic equation in $x$

A1 Correct solution of the quadratic. Either $e^{0.25t}=2$ or $e^{-0.25t}=\frac{1}{2}$ oe.

A1 Correct exact value of t. Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}$, $\frac{\ln(2)}{0.25}$, $-4\ln(\frac{1}{2})$

.(c ) M1 Differentiates $V=17000e^{-0.25t}+2000e^{-0.5t}+500$ by the chain rule.
Accept answers of the form $(\frac{dV}{dt})=\pm Ae^{-0.25t}\pm Be^{-0.5t}$ $A,B$ are constants $\neq 0$

A1 Correct derivative $(\frac{dV}{dt})=-4250e^{-0.25t}-1000e^{-0.5t}$.
There is no need for it to be simplified so accept

$$(\frac{dV}{dt})=17000\times-0.25e^{-0.25t}+2000\times-0.5e^{-0.5t} \quad oe$$

M1 Substitute $t$=8 into their $\frac{dV}{dt}$.
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept $t$=8 in $V$

A1 ±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. **This would not be isw. Be aware that sub t=8 into V first and then differentiating can achieve 593. This is M0A0M0A0.**

## Question 3

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $\ln(4-2x)(9-3x), = \ln(x+1)^2$ | M1, M1 |
| | So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$ | A1 |
| | Solve $5x^2-32x+35=0$ to give $x=\frac{7}{5}$ oe (Ignore the solution $x=5$) | M1A1 |
| | | **(5)** |
| (b) | Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$ | M1 |
| | $x\ln 2 + (3x+1)\ln e = \ln 10$ | M1 |
| | $x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$ | dM1 |
| | and uses $\ln e = 1$ | M1 |
| | $x = \frac{-1+\ln 10}{3+\ln 2}$ | A1 |
| | | **(5)** |
| | **Note that the 4th M mark may occur on line 2** | **(10 marks)** |

**Notes for Question**

(a)

M1 Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.

M1 Uses power rule for logs write the $2\ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets

A1 Undoes the logs to obtain the 3TQ =0. $5x^2-32x+35=0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.

M1 Solves a quadratic by any allowable method.
The quadratic cannot be a version of $(4-2x)(9-3x)=0$ however.

A1 Deduces $x=1.4$ or equivalent. Accept both $x$=1.4 and $x$=5. Candidates do not have to eliminate $x=5$.
You may ignore any other solution as long as it is not in the range $-1<x<2$. Extra solutions in the range scores A0.

| Notes for Question Continued |
|---|
| (b)<br><br>M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law.<br><br>M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1<br><br>$\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x\ln 2 \times (3x+1)\ln e = \ln 10$<br><br>dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in $x$. The terms in $x$ must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to $x = \ldots$<br><br>M1 Uses ln e = 1. This could appear in line 2, but it must be part of their equation and not just a statement.<br><br>Another example where it could be awarded is $e^{3x+1} = \frac{10}{2^x} \Rightarrow 3x+1 = \ldots$<br><br>A1 Obtains answer $x = \frac{-1+\ln 10}{3+\ln 2} = \left(\frac{\ln 10 - 1}{3+\ln 2}\right) = \left(\frac{\log_e 10 - 1}{3+\log_e 2}\right)$ *oe*. **DO NOT ISW HERE**<br><br>Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5.<br><br>Answer $= x = \frac{-\log e + \log 10}{3\log e + \log 2} = \left(\frac{-\log e + 1}{3\log e + \log 2}\right)$<br><br>Note 2: If the candidate writes $x = \frac{-1+\log 10}{3+\log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5. |

| Question Number | Scheme | Marks |
|---|---|---|
| **Alt 1 to (b)** | Writes lhs in e's $2^x e^{3x+1} = 10 \Rightarrow e^{x\ln 2} e^{3x+1} = 10$ | 1st M1 |
| | $\Rightarrow e^{x\ln 2+3x+1} = 10, \quad x\ln 2 + 3x + 1 = \ln 10$ | 2nd M1, 4th M1 |
| | $x(\ln 2+3) = \ln 10 - 1 \Rightarrow x = ..$ | dM1 |
| | $x = \dfrac{-1+\ln 10}{3+\ln 2}$ | A1 **(5)** |

**Notes for Question Alt 1**

M1 Writes the lhs of the expression in e's. Seeing $2^x = e^{x\ln 2}$ in their equation is sufficient

M1 Uses the addition law on the lhs to produce a single exponential

dM1 Takes ln's of both sides to produce and attempt to solve a linear equation in $x$
You may condone slips in signs for this mark but the process must be correct leading to $x=$ ..

M1 Uses ln e = 1. This could appear in line 2

## Question 4

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $t=0 \Rightarrow P=\dfrac{8000}{1+7}=1000$ cao | M1A1 |
| | | **(2)** |
| (b) | $t \to \infty \quad P \to \dfrac{8000}{1}=8000$ | B1 |
| | | **(1)** |
| (c ) | $t=3,\ P=2500 \Rightarrow 2500=\dfrac{8000}{1+7e^{-3k}}$ | B1 |
| | $e^{-3k}=\dfrac{2.2}{7}=(0.31..)$ oe | M1,A1 |
| | $k=-\dfrac{1}{3}\ln\left(\dfrac{2.2}{7}\right)=$ awrt 0.386 | M1A1 |
| | | **(5)** |
| (d) | Sub t=10 into $P=\dfrac{8000}{1+7e^{-0.386t}} \Rightarrow P=6970$ cao | M1A1 |
| | | **(2)** |
| (e) | $\dfrac{dP}{dt}=-\dfrac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$ | M1,A1 |
| | Sub t=10 $\left.\dfrac{dP}{dt}\right\|_{t=10}=346$ | A1 |
| | | **(3)** |
| | | **(13 marks)** |

**Notes for Question**

(a)

M1 Sets $t$=0, giving $e^{-k\times 0}=1$. Award if candidate attempts $\frac{8000}{1+7\times 1}$, $\frac{8000}{8}$

A1 Correct answer only 1000. Accept 1000 for both marks as long as no incorrect working is seen.

(b)

B1 8000. Accept $P<8000$. Condone $P\leqslant 8000$ but not $P>8000$

(c )

B1 Sets both $t=3$, and $P=2500 \Rightarrow 2500=\frac{8000}{1+7e^{-3k}}$

This may be implied by a subsequent correct line.

M1 Rearranges the equation to make $e^{\pm 3k}$ the subject. They need to multiply by the $1+7e^{-3k}$ term, and proceed to $e^{\pm 3k}=A,\quad A>0$

A1 The correct intermediate answer of $e^{-3k}=\frac{2.2}{7},\frac{11}{35}$ or equivalent. Accept awrt 0.31..

Alternatively accept $e^{3k}=\frac{35}{11}$, 3.18.. or equivalent.

M1 Proceeds from $e^{\pm 3k}=A,\quad A>0$ by correctly taking $ln$'s and then making $k$ the subject of the formula.

Award for $e^{-3k}=A \Rightarrow -3k=\ln(A) \Rightarrow k=\frac{\ln(A)}{-3}$

If $e^{3k}$ was found accept $e^{3k}=C \Rightarrow 3k=\ln C \Rightarrow k=\frac{\ln C}{3}$ As with method 1, $C$>0

A1 Awrt $k=0.386$ 3dp

(d)

M1 Substitutes t=10 into $P=\frac{8000}{1+7e^{-kt}}$ with their numerical value of $k$ to find $P$

A1 $(P=)6970$ or other exact equivalents like $6.97\times 10^{3}$

(e)

M1 Differentiates using the chain rule to a form $\frac{dP}{dt}=\frac{C}{(1+7e^{-kt})^2}\times e^{-kt}$

Accept an application of the quotient rule to achieve $\frac{(1+7e^{-kt})\times 0-C\times -e^{-kt}}{(1+7e^{-kt})^2}$

A1 A correct un simplified $\frac{dP}{dt}=-\frac{8000}{(1+7e^{-kt})^2}\times -7ke^{-kt}$.

The derivative can be given in terms of $k$. If a numerical value is used you may follow through on incorrect values.

A1 Awrt $346$. Note that M1 must have been achieved. Just the answer scores 0

**Question 5**

| Question Number | Scheme | Marks |
|---|---|---|
| **(a)** | $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$ | M1 |
| | $\Rightarrow x=\dfrac{e^5-1}{2}$ | A1 |
| | | (2) |
| **(b)** | $3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$ | |
| | $\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ | M1,M1 |
| | $x(\ln 3+4)=7 \Rightarrow x=\ldots$ | dM1 |
| | $x=\dfrac{7}{(\ln 3+4)}$ oe | A1 |
| | | (4) |
| | | **6 marks** |
| **Alt 1 (b)** | $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$ | |
| | $3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$ | M1,M1 |
| | $x(\ln 3+4)=7 \Rightarrow x=\ldots$ | dM1 |
| | $x=\dfrac{7}{(\ln 3+4)}$ | A1 |
| | | (4) |
| **Alt 2 (b) Using logs** | $3^x e^{4x}=e^7 \Rightarrow \log(3^x e^{4x})=\log e^7$ | |
| | $\log 3^x+\log e^{4x}=\log e^7 \Rightarrow x\log 3+4x\log e=7\log e$ | M1, M1 |
| | $x(\log 3+4\log e)=7\log e \Rightarrow x=\ldots$ | dM1 |
| | $x=\dfrac{7\log e}{(\log 3+4\log e)}$ | A1 |
| | | (4) |
| **Alt 3 (b) Using $\log_3$** | $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$ | |
| | $3^x=e^{7-4x} \Rightarrow x=(7-4x)\log_3 e$ | M1,M1 |
| | $x(1+4\log_3 e)=7\log_3 e \Rightarrow x=\ldots$ | dM1 |
| | $x=\dfrac{7\log_3 e}{(1+4\log_3 e)}$ | A1 |
| | | (4) |
| **Alt 4 (b) Using $3^x=e^{x\ln 3}$** | $3^x e^{4x}=e^7 \Rightarrow e^{x\ln 3}e^{4x}=e^7$ | |
| | $\Rightarrow e^{x\ln 3+4x}=e^7, \Rightarrow x\ln 3+4x=7$ | M1,M1 |
| | $x(\ln 3+4)=7 \Rightarrow x=\ldots \qquad x=\dfrac{7}{(\ln 3+4)}$ | dM1 A1 |
| | | (4) |

(a)

M1 Proceeds from $2\ln(2x+1)-10=0$ to $\ln(2x+1)=5$ before taking exp's to achieve $x$ in terms of $e^5$

Accept for M1 $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow x=f(e^5)$

Alternatively they could use the power law before taking exp's to achieve $x$ in terms of $\sqrt{e^{10}}$

$2\ln(2x+1)=10 \Rightarrow \ln(2x+1)^2=10 \Rightarrow (2x+1)^2=e^{10} \Rightarrow x=g\left(\sqrt{e^{10}}\right)$

A1 cso. Accept $x=\frac{e^5-1}{2}$ or other exact simplified alternatives such as $x=\frac{e^5}{2}-\frac{1}{2}$. Remember to isw.

The decimal answer of 73.7 will score M1A0 unless the exact answer has also been given.

The answer $\frac{\sqrt{e^{10}}-1}{2}$ does not score this mark unless simplified. $x=\frac{\pm e^5-1}{2}$ is M1A0

(b)

M1 Takes ln's or logs of both sides and applies the addition law.

$\ln(3^x e^{4x})=\ln 3^x+\ln e^{4x}$ or $\ln(3^x e^{4x})=\ln 3^x+4x$ is evidence for the addition law

If the $e^{4x}$ was 'moved' over to the right hand side score for either $e^{7-4x}$ or the subtraction law.

$\ln\frac{e^7}{e^{4x}}=\ln e^7-\ln e^{4x}$ or $3^x e^{4x}=e^7 \Rightarrow 3^x=\frac{e^7}{e^{4x}} \Rightarrow 3^x=e^{7-4x}$ is evidence of the subtraction law

M1 Uses the power law of logs (seen at least once in a term with x as the index Eg $3^x, e^{4x}$ or $e^{7-4x}$ ).

$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ is an example after the addition law

$3^x=e^{7-4x} \Rightarrow x\log 3=(7-4x)\log e$ is an example after the subtraction law.

It is possible to score M0M1 by applying the power law after an incorrect addition/subtraction law

For example $3^x e^{4x}=e^7 \Rightarrow \ln(3^x)\times\ln(e^{4x})=\ln e^7 \Rightarrow x\ln 3\times 4x\ln e=7\ln e$

dM1 This is dependent upon **both** previous M's. Collects/factorises out term in $x$ and proceeds to $x=$ .
Condone sign slips for this mark. An unsimplified answer can score this mark.

A1 If the candidate has taken ln's then they must use $\ln e=1$ and achieve $x=\frac{7}{(\ln 3+4)}$ or equivalent.

If the candidate has taken log's they must be writing log as oppose to ln and achieve

$x=\frac{7\log e}{(\log 3+4\log e)}$ or other exact equivalents such as $x=\frac{7\log e}{\log 3e^4}$.

## Question 6

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $P=\frac{800e^{0}}{1+3e^{0}},=\frac{800}{1+3}=200$ | M1,A1 |
| | | (2) |
| (b) | $250=\frac{800e^{0.1t}}{1+3e^{0.1t}}$ | |
| | $250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250,\ \Rightarrow e^{0.1t}=5$ | M1,A1 |
| | $t=\frac{1}{0.1}\ln(5)$ | M1 |
| | $t=10\ln(5)$ | A1 |
| | Note: This part is for Year 13 | (4) |
| (c ) | $P=\frac{800e^{0.1t}}{1+3e^{0.1t}}\Rightarrow\frac{dP}{dt}=\frac{(1+3e^{0.1t})\times800\times0.1e^{0.1t}-800e^{0.1t}\times3\times0.1e^{0.1t}}{(1+3e^{0.1t})^{2}}$ | M1,A1 |
| | At $t$=10 | |
| | $\frac{dP}{dt}=\frac{(1+3e)\times80e-240e^{2}}{(1+3e)^{2}}=\frac{80e}{(1+3e)^{2}}$ | M1,A1 |
| | | (4) |
| (d) | $P=\frac{800e^{0.1t}}{1+3e^{0.1t}}=\frac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\frac{800}{3}=266$. Hence P cannot be 270 | B1 |
| | | (1) |
| | | **(11 marks)** |

(a)

M1 Sub $t = 0$ into $P$ **and** use $e^0 = 1$ in at least one of the two cases. Accept $P = \frac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub $P$=250 into $P = \frac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t} = B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t} = D$

A1 $e^{0.1t} = 5$ or $e^{-0.1t} = 0.2$

M1 Dependent upon gaining $e^{0.1t} = E$ , for taking ln's of both sides and proceeding to $t$=…

Accept $e^{0.1t} = E \Rightarrow 0.1t = \ln E \Rightarrow t = ...$It could be implied by $t =$ awrt 16.1

A1 $t = 10\ln(5)$

Accept exact equivalents of this as long as $a$ and $b$ are integers. Eg. $t = 5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that

$\frac{d}{dt}e^{0.1t} = ke^{0.1t}$ and NOT $kte^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u = 800e^{0.1t}, v = 1+3e^{0.1t}, u' = pe^{0.1t}, v' = qe^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form

$$\frac{(1+3e^{0.1t})\times pe^{0.1t} - 800e^{0.1t}\times qe^{0.1t}}{(1+3e^{0.1t})^2}$$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$

For the chain rule look for

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow \frac{dP}{dt} = 800\times\left(e^{-0.1t}+3\right)^{-2}\times -0.1e^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$$

M1 For substituting $t = 10$ into their $\frac{dP}{dt}$, NOT $P$

Accept numerical answers for this. 2.59 is the numerical value if $\frac{dP}{dt}$ was correct

A1 $\frac{dP}{dt} = \frac{80e}{(1+3e)^2}$ or equivalent such as $\frac{dP}{dt} = 80e(1+3e)^{-2}, \frac{80e}{1+6e+9e^2}$

Note that candidates who substitute $t = 10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow 0.1t = \ln(-27)$ which has no answers.

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow e^{0.1t} / e^{x}$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of $t$ ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

**Question 7**

| Question Number | Scheme | Marks |
| --- | --- | --- |
| (a) | $(\theta =)20$ | B1 (1) |
| (b) | Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$ | |
| | $\Rightarrow e^{-40\lambda} = 0.5$ | M1A1 |
| | $\Rightarrow \lambda = \frac{\ln 2}{40}$ | M1A1 |
| | | (4) |
| (c) | $\theta = 100 \Rightarrow T = \frac{\ln 0.2}{-\text{their}\ '\lambda'}$ | M1 |
| | $T = \text{awrt } 93$ | A1 |
| | | (2) |
| | | (7 marks) |
| Alt (b) | Sub $t = 40, \theta = 70 \Rightarrow 100e^{-40\lambda} = 50$ | |
| | $\Rightarrow \ln 100 - 40\lambda = \ln 50$ | M1A1 |
| | $\Rightarrow \lambda = \frac{\ln 100 - \ln 50}{40} = \frac{\ln 2}{40}$ | M1A1 |
| | | (4) |

(a)

B1 Sight of $(\theta =)20$

(b)

M1 Sub $t = 40, \theta = 70 \Rightarrow 70 = 120 - 100e^{-40\lambda}$ and proceed to $e^{\pm 40\lambda} = A$ where $A$ is a constant. Allow sign slips and copying errors.

A1 $e^{-40\lambda} = 0.5$, or $e^{40\lambda} = 2$ or exact equivalent

M1 For undoing the e's by taking ln's and proceeding to $\lambda = ..$

May be implied by the correct decimal answer awrt 0.017 or $\lambda = \frac{\ln 0.5}{-40}$

A1 cso $\lambda = \frac{\ln 2}{40}$

Accept equivalents in the form $\frac{\ln a}{b}$, $a, b \in \mathbb{Z}$ such as $\lambda = \frac{\ln 4}{80}$

(c)

M1 Substitutes $\theta = 100$ and their numerical value of $\lambda$ into $\theta = 120 - 100e^{-\lambda t}$ and proceed to $T = \pm\frac{\ln 0.2}{\text{their } '\lambda'}$ or $T = \pm\frac{\ln 5}{\text{their } '\lambda'}$ Allow inequalities here.

A1 awrt $T = 93$

Watch for candidates who lose the minus sign in (b) and use $\lambda = \frac{\ln \frac{1}{2}}{40}$ in (c). Many then reach $T = -93$ and ignore the minus. This is M1 A0

**Question 8**

| Question | Scheme | Marks |
|---|---|---|
| (a) | Subs $D = 15$ and $t = 4$ $x = 15e^{-0.2\times 4} = 6.740\ (mg)$ | M1A1 |
| | | (2) |
| (b) | $15e^{-0.2\times 7} + 15e^{-0.2\times 2} = 13.754(mg)$ | M1A1* |
| | | (2) |
| (c) | $15e^{-0.2\times T} + 15e^{-0.2\times (T+5)} = 7.5$ | M1 |
| | $15e^{-0.2\times T} + 15e^{-0.2\times T}e^{-1} = 7.5$ | |
| | $15e^{-0.2\times T}(1+e^{-1}) = 7.5 \Rightarrow e^{-0.2\times T} = \frac{7.5}{15(1+e^{-1})}$ | dM1 |
| | $T = -5\ln\left(\frac{7.5}{15(1+e^{-1})}\right) = 5\ln\left(2+\frac{2}{e}\right)$ | A1, A1 |
| | | (4) |
| | | (8 marks) |

(a)

M1 Attempts to substitute both $D=15$ and $t=4$ in $x=De^{-0.2t}$

It can be implied by sight of $15e^{-0.8}$, $15e^{-0.2\times4}$ or awrt 6.7

Condone slips on the power. Eg you may see -0.02

A1 CAO 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1 Attempt to find the sum of two expressions with $D=15$ in both terms with $t$ values of 2 and 7

Evidence would be $15e^{-0.2\times7}+15e^{-0.2\times2}$ or similar expressions such as $\left(15e^{-1}+15\right)e^{-0.2\times2}$

Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**

Alternatively finds the amount after 5 hours, $15e^{-1}$ = awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $e^{-0.4}$ to get awrt **13.75**.

Sight of $5.52+15=20.52\rightarrow 13.75$ is fine.

A1* cso so both the expression $15e^{-0.2\times7}+15e^{-0.2\times2}$ and $13.754(mg)$ are required

Alternatively both the expression $\left(15e^{-0.2\times5}+15\right)\times e^{-0.2\times2}$ and $13.754(mg)$ are required.

Sight of just the numbers is not enough for the A1*

(c)

M1 Attempts to write down a correct equation involving $T$ or $t$. Accept with or without correct bracketing

Eg. accept $15e^{-0.2\times T}+15e^{-0.2\times(T\pm5)}=7.5$ or similar equations $\left(15e^{-1}+15\right)e^{-0.2\times T}=7.5$

dM1 Attempts to solve their equation, dependent upon the previous mark, by proceeding to $e^{-0.2\times T}=\ldots$

An attempt should involve an attempt at the index law $x^{m+n}=x^m\times x^n$ and taking out a factor of $e^{-0.2\times T}$ Also score for candidates who make $e^{+0.2\times T}$ the subject using the same criteria

A1 Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15\left(1+e^{-1}\right)}\right)$

A1 CSO $T=5\ln\left(2+\dfrac{2}{e}\right)$ Condone $t$ appearing for $T$ throughout this question.

Alt (c) using lns

| | | |
|---|---|---|
| (c) | $15e^{-0.2\times T}+15e^{-0.2\times(T+5)}=7.5$<br>$15e^{-0.2\times T}+15e^{-0.2\times T}e^{-1}=7.5$<br>$e^{-0.2\times T}(1+e^{-1})=0.5\Rightarrow -0.2\times T+\ln(1+e^{-1})=\ln 0.5$<br>$\Rightarrow T=\dfrac{\ln 0.5-\ln(1+e^{-1})}{-0.2}, \Rightarrow T=5\ln\left(2+\dfrac{2}{e}\right)$ | M1<br><br>dM1<br>A1, A1<br>**(4)**<br>**(8 marks)** |

You may see numerical attempts at part (c).

Such an attempt can score a maximum of two marks.

This can be achieved either by

Method One

1st Mark (Method): $15e^{-0.2\times T}+\text{awrt } 5.52e^{-0.2\times T}=7.5\Rightarrow e^{-0.2\times T}=\text{awrt } 0.37$

2nd Mark (Accuracy): $T=-5\ln(\text{awrt } 0.37)$ or awrt 5.03 or $T=-5\ln\left(\dfrac{7.5}{\text{awrt } 20.52}\right)$

Method Two

1st Mark (Method ): $13.754e^{-0.2\times T}=7.5\Rightarrow T=-5\ln\left(\dfrac{7.5}{13.754}\right)$ or equivalent such as 3.03

2nd Mark (Accuracy): 3.03+2=5.03 Allow $-5\ln\left(\dfrac{7.5}{13.754}\right)+2$

Method Three (by trial and improvement)

1st Mark (Method): $15e^{-0.2\times 5}+15e^{-0.2\times 10}=7.55$ or $15e^{-0.2\times 5.1}+15e^{-0.2\times 10.1}=7.40$ or any value between

2nd Mark (Accuracy): Answer $T$=5.03.

## Question 9

| Question Number | Scheme | Marks |
|---|---|---|
| **(a)** | $e^{3x-9} = 8 \Rightarrow 3x - 9 = \ln 8$ | M1 |
| | $\Rightarrow x = \dfrac{\ln 8 + 9}{3}, = \ln 2 + 3$ | A1, A1 |
| | | (3) |
| **(b)** | $\ln(2y+5) = 2 + \ln(4-y)$ | |
| | $\ln\left(\dfrac{2y+5}{4-y}\right) = 2$ | M1 |
| | $\left(\dfrac{2y+5}{4-y}\right) = e^2$ | M1 |
| | $2y + 5 = e^2(4-y) \Rightarrow 2y + e^2 y = 4e^2 - 5 \Rightarrow y = \dfrac{4e^2 - 5}{2 + e^2}$ | dM1, A1 |
| | | (4) |
| | | **7 marks** |

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\frac{\ln 8+9}{3}$ or equivalent such as $\frac{\ln 8e^9}{3}$, $3+\ln\left(\sqrt[3]{8}\right)$, $\frac{\log 8}{3\log e}+3$ or even 3.69

A1 cso $\ln 2+3$. Accept $\ln 2e^3$

Alt I (a)

$e^{3x-9}=8 \Rightarrow \frac{e^{3x}}{e^9}=8 \Rightarrow e^{3x}=8e^9 \Rightarrow 3x=\ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3}=\sqrt[3]{8} \Rightarrow x-3=\ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9}=2 \Rightarrow x-9=\ln 2$)

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2+\ln(4-y)=\ln\left(e^2(4-y)\right)$ or $\ln(2y+5)-\ln(4-y)=\ln\left(\frac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the $e^2$

M1 Scored for an attempt to undo the ln's to get an equation in $y$ This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y))=2 \Rightarrow g(y)=e^2$ and can be scored eg where $g(y)=2y+5-(4-y)$

It cannot be awarded for just $2y+5=e^2+4-y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making $y$ the subject. Expect to see both terms in $y$ collected and factorised (may be implied) before reaching $y=$. Condone slips, for eg, on signs. $y=2.615$ scores this.

A1 $y=\frac{4e^2-5}{2+e^2}$ or equivalent such as $y=4-\frac{13}{2+e^2}$ ISW after you see the correct answer.

Special Case: $\ln(2y+5)-\ln(4-y)=2 \Rightarrow \frac{\ln(2y+5)}{\ln(4-y)}=2 \Rightarrow \frac{2y+5}{4-y}=e^2 \Rightarrow$ Correct answer score M0 M1 M1 A0