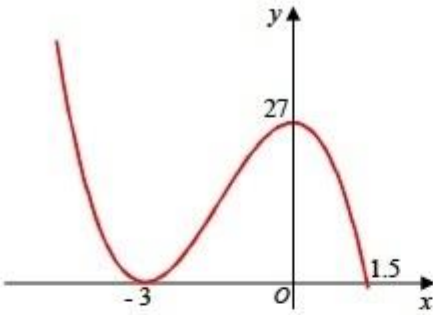
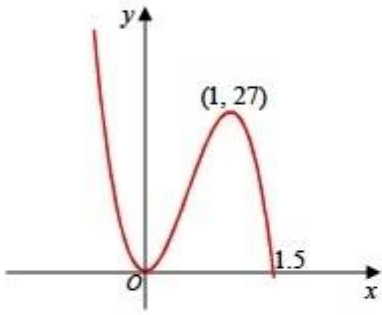
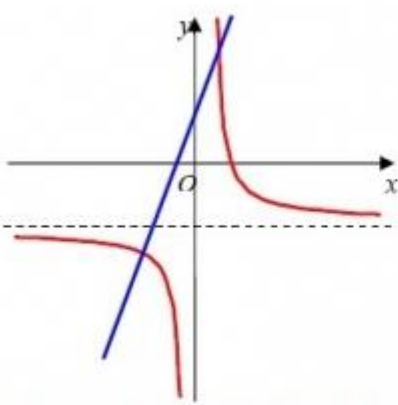


Graphs and Transformations - Edexcel Past Exam Questions 2 MARK SCHEME

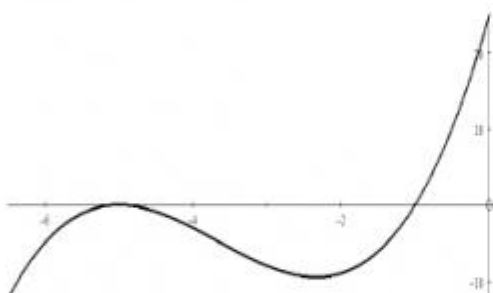
Question 1

Question Number	Scheme	Marks
(a)	{Coordinates of A are} (4.5, 0) See notes below	B1
(b)(i)	 <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p style="text-align: center;">Horizontal translation</p> <p style="text-align: center;">-3 and their ft 1.5 on positive x-axis</p> <p style="text-align: center;">Maximum at 27 marked on the y-axis</p> </div>	B1 [1] M1 A1 ft B1
(ii)	 <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p style="text-align: center;">Correct shape, minimum at (0, 0) and a maximum within the first quadrant.</p> <p style="text-align: center;">1.5 on x-axis</p> <p style="text-align: center;">Maximum at (1, 27)</p> </div>	B1 [3] M1 A1 ft B1
(c)	{k =} -17	B1 [3] [1] 8
Notes		
(a)	B1: For stating either $x = 4.5$ or $\frac{9}{2}$ or $\frac{18}{4}$ etc. or $A = 4.5$ or $\frac{9}{2}$ or (4.5, 0). Can be written on graph. Allow (0, 4.5) marked on curve for B1. Otherwise (0, 4.5) without reference to any of the above is B0.	
(b)(i)	M1: for any horizontal (left-right) translation where minimum is still on x-axis not at (0, 0). Ignore any values. A1ft: for -3 (NOT 3) and 1.5 (or their x in part (a) - 3) <i>evaluated</i> and marked on the positive x-axis. Allow (0, -3) and/or (0, ft 1.5) rather than (-3, 0) and (ft 1.5, 0) if marked in the "correct" place on the x-axis. Note: Candidate <i>cannot</i> gain this mark if their x in part (a) is less than 3.	
(ii)	B1: Maximum at 27 marked on the y-axis. Note: the maximum must be on the y-axis for this mark. M1: for correct shape with minimum still at (0, 0) and a maximum within the first quadrant. Ignore values. A1ft: for $\frac{\text{their } x \text{ in part (a)}}{3}$; as intercept on x-axis eg: $\frac{4.5}{3}$ or 1.5 or $\frac{3}{2}$ or $\frac{9}{6}$ Note: a generalised $\frac{A}{3}$ is A0. Allow (0, ft 1.5) rather than (ft 1.5, 0) if marked in the "correct" place on the x-axis.	
(c)	B1: Maximum at (1, 27) or allow 1 marked on the x-axis and the corresponding 27 marked on the y-axis. Note: Be careful to look at the correct graph. The candidate may draw another graph to help them to answer part (c). Note: You can recover (b)(i) (-3, 0) and (ft 1.5, 0) or in (b)(ii) (ft 1.5, 0) as <i>correct coordinates only</i> in candidate's working if these are not marked on their sketch(es).	
	B1: for $(k =) -17$ only. BEWARE: This could be written in the middle or at the bottom of a page.	

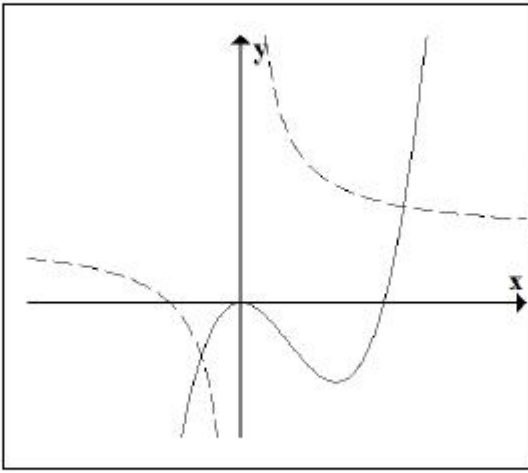
Question 2

Question Number	Scheme	Marks
(a)	 <p> $y = \frac{2}{x}$ is translated up or down. M1 $y = \frac{2}{x} - 5$ is in the correct position. A1 Intersection with x-axis at $(\frac{2}{5}, \{0\})$ only B1 Independent mark. $y = 4x + 2$: attempt at straight line, with positive gradient with positive y intercept. B1 Intersection with x-axis at $(-\frac{1}{2}, \{0\})$ and y-axis at $(\{0\}, 2)$. B1 </p> <p>Check graph in question for possible answers and space below graph for answers to part (b)</p>	[5]
(b)	<p>Asymptotes : $x = 0$ (or y-axis) and $y = -5$. An asymptote stated correctly. Independent of (a) B1 (Lose second B mark for extra asymptotes) These two lines only. Not fit their graph. B1</p>	[2]
(c)	<p>Method 1: $\frac{2}{x} - 5 = 4x + 2$ Method 2: $\frac{y-2}{4} = \frac{2}{y+5}$ M1</p> <p>$4x^2 + 7x - 2 = 0 \Rightarrow x =$ $y^2 + 3y - 18 = 0 \rightarrow y =$ dM1 $x = -2, \frac{1}{4}$ $y = -6, 3$ A1 When $x = -2, y = -6$, When $x = \frac{1}{4}, y = 3$ When $y = -6, x = -2$ When $y = 3, x = \frac{1}{4}$. M1A1</p>	[5]
Notes		12 marks
<p>(a) M1: Curve implies y axis as asymptote and does not change shape significantly. Changed curve needs horizontal asymptote (roughly) Asymptote(s) need not be shown but shape of curve should be implying asymptote(s) parallel to x axis. Curve should not remain where it was in the given figure. Both sections move in the same direction. There should be no reflection</p> <p>A1: Crosses positive x axis. Hyperbola has moved down. Both sections move by almost same amount. See sheet on page 19 for guidance.</p> <p>B1: Check diagram and text of answer. Accept $2/5$ or 0.4 shown on x-axis or $x = 2/5$, or $(2/5, 0)$ stated clearly in text or on graph. This is independent of the graph. Accept $(0, 2/5)$ if clearly on x axis. Ignore any intersection points with y axis. Do not credit work in table of values for this mark.</p> <p>B1: Must be attempt at a straight line, with positive gradient & with positive y intercept (need not cross x axis)</p> <p>B1: Accept $x = -1/2$, or -0.5 shown on x-axis or $(-1/2, 0)$ or $(-0.5, 0)$ in text or on graph and similarly accept 2 on y axis or $y = 2$ or $(0, 2)$ in text or on graph. Need not cross curve and allow on separate axes.</p> <p>(b) B1: For either correct asymptote equation. Second B1: For both correct (lose this if extras e.g. $x = \pm 1$ are given also). These asymptotes may follow correctly from equation after wrong graph in (a)</p> <p>Just $y = -5$ is B1 B0 This may be awarded if given on the graph. However for other B mark it must be clear that $x = 0$ (or the y-axis) is an asymptote. NB $x \neq 0, y \neq -5$ is B1B0</p> <p>(c) M1: Either of these equations is enough for the method mark (May appear labelled as part (b))</p> <p>dM1: Attempt to solve a 3 term quadratic by factorising, formula, completion of square or implied by correct answers. (see note 1) This mark depends on previous mark.</p> <p>A1: Need both correct x answers (Accept equivalents e.g. 0.25) or both correct y values (Method 2)</p> <p>M1: At least one attempt to find second variable (usually y) using their first variable (usually x) related to line meeting curve. Should not be substituting x or y values from part (a) or (b). This mark is independent of previous marks. Candidate may substitute in equation of line or equation of curve.</p> <p>A1: Need both correct second variable answers Need not be written as co-ordinates (allow as in the scheme)</p> <p>Note: Special case: Answer only with no working in part (c) can have 5 marks if completely correct, with both points found. If co-ordinates of just one of the points is correct – with no working – this earns M0 M0 A0 M1 A0 (i.e. 1 / 5)</p>		

Question 3

Question Number	Scheme		Marks
(a)		Horizontal translation – does not have to cross the y-axis on the right but must at least reach the x-axis.	B1
		Touching at (-5, 0). This could be stated anywhere or -5 could be marked on the x-axis. Or (0, -5) marked in the correct place . Be fairly generous with 'touching' if the intention is clear.	B1
		The right hand tail of their cubic shape crossing at (-1, 0). This could be stated anywhere or -1 could be marked on the x-axis. Or (0, -1) marked in the correct place . The curve must cross the x-axis and not stop at -1.	B1
			(3)
(b)	$(x + 5)^2(x + 1)$	Allow $(x + 3 + 2)^2(x - 1 + 2)$	B1
			(1)
(c)	When $x = 0$, $y = 25$	M1: Substitutes $x = 0$ into their expression in part (b) which is not $f(x)$. This may be implied by their answer. Note that the question asks them to use part (b) but allow independent methods.	M1 A1
		A1: $y = 25$ (Coordinates not needed)	
	If they expand <u>incorrectly</u> prior to substituting $x = 0$, score M1 A0 NB $f(x + 2) = x^3 + 11x^2 + 35x + 25$		
			(2)
			[6]

Question 4

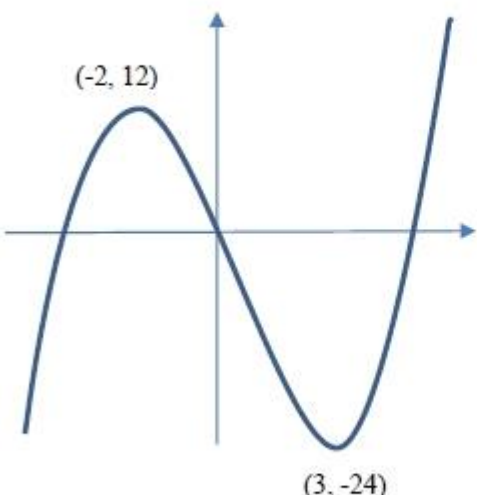
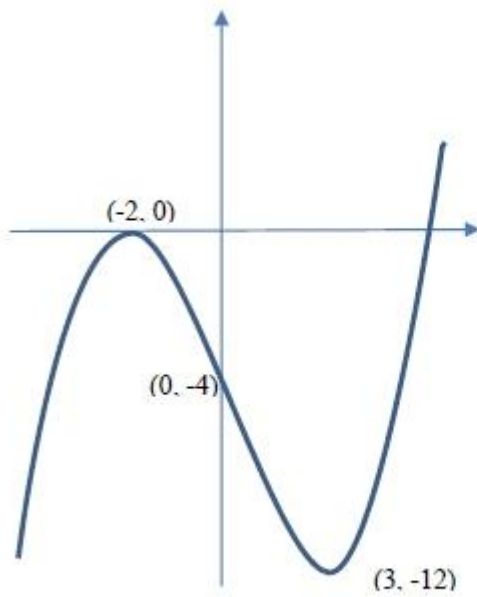
Question Number	Scheme	Marks
	<p>(a) -1 accept $(-1, 0)$</p> <p>(b)</p>  <p>(c) 2 solutions as curves cross twice</p>	<p>B1 (1)</p> <p>Shape Touches at $(0,0)$ Crosses at $(2,0)$ only</p> <p>B1 B1 B1</p> <p>(3)</p> <p>B1 ft (1) (5 marks)</p>

Notes

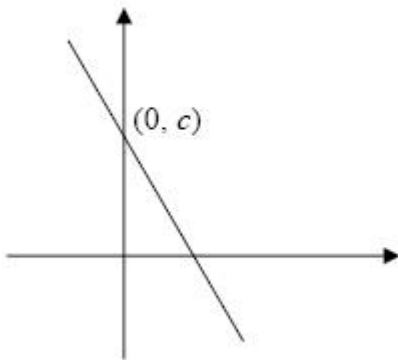
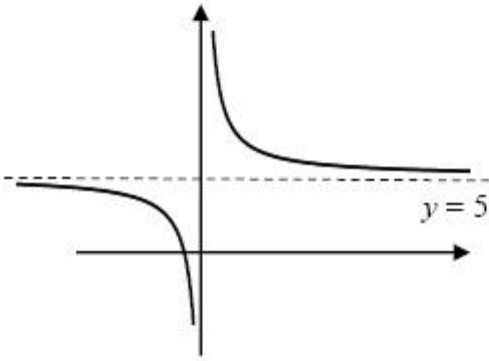
N.B. Check original diagram as answer may appear there.

- (a) B1 The x coordinate of A is -1 . Accept -1 or $(-1,0)$ on the diagram or stated with or without diagram. Allow $(0, -1)$ on the diagram if it is on the correct axis.
- (b) If no graph is drawn then no marks are available in part (b)
- B1 Correct shape. The position is not important for this mark but the curve must have two clear turning points and be a +ve x^3 curve (with a maximum and minimum)
- B1 The graph touches the origin. Accept touching as a maximum or minimum. There must be a sketch for this mark but sketch may be wrong and this mark is independent of previous mark. Origin is where axes cross and may not be labelled. This may be a quadratic or quartic curve for this mark.
- B1 The graph crosses the x -axis at the point $(2,0)$ only. If it crosses at $(2,0)$ and $(0,0)$ this is B0. Accept $(0,2)$ or 2 marked on the correct axis. Accept $(2, 0)$ in the text of the answer provided that the curve crosses the positive x axis. There must be a sketch for this mark. Do not give credit if $(2,0)$ appears only in a table with no indication that this is the intersection point. (If in doubt send to review) Graph takes precedence over text for third B mark.
- (c) B1ft Two (solutions) as there are two intersections (of the curves) N.B. Just states 2 with no reason is B0. If the answer states 2 roots and two intersections – or crosses twice this is enough for B1 BUT B0 If there is any wrong reason given – e.g. crosses x axis twice, or crosses asymptote twice. Isw – is not used for this mark so any wrong statement listed to follow a correct statement will result in B0. Allow ft – so if their graph crosses the hyperbola once – allow “one solution as there is one intersection” And if it crosses three times – allow “three solutions as there are three intersections” or four etc... If it does not cross at all (e.g. negative cubic) – allow “no solutions as there are no intersections” However in (c) if they have sketched a curve (even a fully correct one) but not extended it to intersect the hyperbola and they put “no points of intersection so no solutions” then this scores B0. Accept “lines or curves cross over twice, or touch twice, or meet twice...etc as explanation, but need some form of words)

Question 5

Question Number	Scheme	Notes	Marks
	Note original points are $A(-2, 4)$ and $B(3, -8)$		
(a)		<p>Similar shape to given figure passing through the origin. A cubic shape with a maximum in the second quadrant and a minimum in the 4th quadrant. There must be evidence of a change in at least one of the y-coordinates (inconsistent changes in the y-coordinates are acceptable) but not the x-coordinates.</p> <p>Maximum at $(-2, 12)$ and minimum at $(3, -24)$ with coordinates written the right way round. Condone missing brackets. The coordinates may appear on the sketch, or separately in the text (not necessarily referenced as A and B). If they are on the sketch, the x and y coordinates can be positioned correctly on the axes rather than given as coordinate pairs. In cases of ambiguity, the sketch has precedence. The origin does not need to be labelled. Nor do the x and y axes.</p>	<p>B1</p> <p>B1</p>
			[2]
(b)		<p>A positive cubic which does not pass through the origin with a maximum to the left of the y-axis and a minimum to the right of the y-axis.</p> <p>Maximum at $(-2, 0)$ and minimum at $(3, -12)$. Condone missing brackets. For the max allow just -2 or $(0, -2)$ if marked in the correct place. If the coordinates are in the text, they must appear as $(-2, 0)$ and must not contradict the sketch. The curve must touch the x-axis at $(-2, 0)$. For the min allow coordinates as shown or 3 and -12 to be marked in the correct places on the axes. In cases of ambiguity, the sketch has precedence.</p> <p>Crosses y-axis at $(0, -4)$. Allow just -4 (not $+4$) and allow $(-4, 0)$ if marked in the correct place. If the coordinates are in the text, they must appear as $(0, -4)$ and must not contradict the sketch. In cases of ambiguity, the sketch has precedence.</p>	<p>M1</p> <p>A1</p> <p>A1</p>
			[3]
			5 marks

Question 6

Question Number	Scheme		Marks
(a)(i)		B1: Straight line with negative gradient anywhere even with no axes.	B1
		B1: Straight line with an intercept at $(0, c)$ or just c marked on the positive y -axis provided the line passes through the positive y -axis. Allow $(c, 0)$ as long as it is marked in the correct place. Allow $(0, c)$ in the body of the script but in any ambiguity, the sketch has precedence. Ignore any intercepts with the x-axis.	B1
(a)(ii)		Either: For the shape of a $y = \frac{1}{x}$ curve in any position. It must have two branches and be asymptotic horizontally and vertically with no obvious “overlap” with the asymptotes, but otherwise be generous. The curve may bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both vertically and horizontally and the branches must approach the same asymptote Or the equation $y = 5$ seen independently i.e. whether the sketch has an asymptote here or not. Do not allow $y \neq 5$ or $x = 5$.	B1
		B1: Fully correct graph and with a horizontal asymptote on the positive y -axis. The asymptote does not have to be drawn but the equation $y = 5$ must be seen. The shape needs to be reasonably accurate with the “ends” not bending away significantly from the asymptotes and the branches must approach the same asymptote. Ignore $x = 0$ given as an asymptote.	B1
		Allow sketches to be on the same axes.	
			(4)

(b)	$\frac{1}{x} + 5 = -3x + c \Rightarrow 1 + 5x = -3x^2 + cx$ $\Rightarrow 3x^2 + 5x - cx + 1 = 0$	<p>Sets $\frac{1}{x} + 5 = -3x + c$, attempts to multiply by x and collects terms (to one side). Allow e.g. “>” or “<” for “=” . At least 3 of the terms must be multiplied by x, e.g. allow one slip. The ‘ = 0 ’ may be implied by subsequent work and provided correct work follows, full marks are still possible in (b).</p>	M1	
	$b^2 - 4ac = (5 - c)^2 - 4 \times 1 \times 3$	<p>Attempts to use $b^2 - 4ac$ with their a, b and c from their equation where $a = \pm 3$, $b = \pm 5 \pm c$ and $c = \pm 1$. This could be as part of the quadratic formula or as $b^2 < 4ac$ or as $b^2 > 4ac$ or as $\sqrt{b^2 - 4ac}$ etc. If it is part of the quadratic formula only look for use of $b^2 - 4ac$. There must be no x's.</p>	M1	
	$(5 - c)^2 > 12^*$	<p>Completes proof with no errors or incorrect statements and with the “>” appearing correctly before the final answer, which could be from $b^2 - 4ac > 0$. Note that the statement $3x^2 + 5x - cx + 1 > 0$ or starting with e.g. $\frac{1}{x} + 5 > -3x + c$ would be an error.</p>	A1*	
	<p>Note: A minimum for (b) could be,</p> $\frac{1}{x} + 5 = -3x + c \Rightarrow 3x^2 + 5x - cx + 1 (= 0) \text{ (M1)}$ $b^2 > 4ac \Rightarrow (5 - c)^2 > 12 \text{ (M1A1)}$ <p>If $b^2 > 4ac$ is not seen then $4 \times 3 \times 1$ needs to be seen explicitly.</p>			
				(3)

(c)	$(5-c)^2 = 12 \Rightarrow (c=) 5 \pm \sqrt{12}$ <p style="text-align: center;">or</p> $(5-c)^2 = 12 \Rightarrow c^2 - 10c + 13 = 0$ $\Rightarrow (c=) \frac{-10 \pm \sqrt{(-10)^2 - 4 \times 13}}{2}$	<p>M1: Attempts to find at least one critical value using the result in (b) or by expanding and solving a 3TQ (See General Principles) (the “= 0” may be implied)</p> <p>A1: Correct critical values in any form. Note that $\sqrt{12}$ may be seen as $2\sqrt{3}$.</p>	M1A1
	$c < "5 - \sqrt{12}", c > "5 + \sqrt{12}"$	<p>Chooses outside region. The ‘0 <’ can be ignored for this mark. So look for $c <$ their $5 - \sqrt{12}$, $c >$ their $5 + \sqrt{12}$. This could be scored from $5 + \sqrt{12} < c < 5 - \sqrt{12}$ or $5 - \sqrt{12} > c > 5 + \sqrt{12}$. Evidence is to be taken from their answers not from a diagram.</p>	M1
	$0 < c < 5 - \sqrt{12}, c > 5 + \sqrt{12}$	<p>Correct ranges including the ‘0 <’ e.g. answer as shown or each region written separately or e.g. $(0, 5 - \sqrt{12})$, $(5 + \sqrt{12}, \infty)$. The critical values may be un-simplified but must be at least $\frac{10 + \sqrt{48}}{2}$, $\frac{10 - \sqrt{48}}{2}$. Note that $0 < c < 5 - \sqrt{12}$ and $c > 5 + \sqrt{12}$ would score M1A0.</p>	A1
	<p>Allow the use of x rather than c in (c) but the final answer must be in terms of c.</p>		
			(4)
			(11 marks)