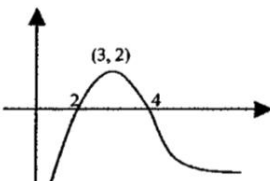
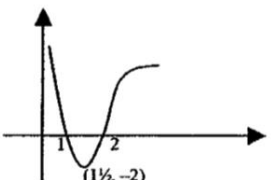
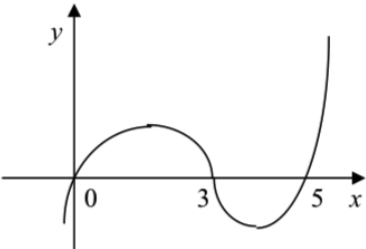


Graphs and Transformations - Edexcel Past Exam Questions **MARK SCHEME**

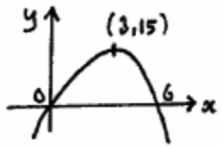
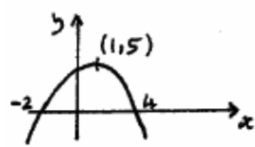
Question 1: Jan 05 Q6

Question number	Scheme	Marks
(a)	 <p>Reflection in <math>x</math>-axis 2 and 4 labelled (or (2, 0) and (4, 0) seen) Image of <math>P</math> (3, 2)</p>	<p>B1 B1 B1 (3)</p>
(b)	 <p>Stretch parallel to <math>x</math>-axis 1 and 2 labelled (or (1, 0) and (2, 0) seen) Image of <math>P</math> (<math>1\frac{1}{2}</math>, -2)</p>	<p>M1 A1 A1 (3) 6</p>

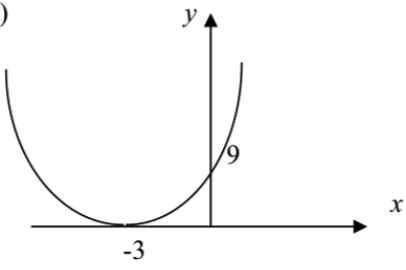
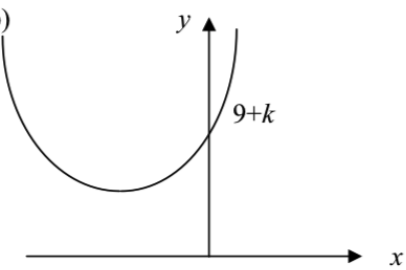
Question 2: June 06 Q9

Question number	Scheme	Marks
(a)	$f(x) = x[(x-6)(x-2)+3]$ or $x^3 - 6x^2 - 2x^2 + 12x + 3x = x(x^2 - 8x + 15)$ $b = -8$ or $c = 15$ both and $a = 1$	M1 A1 A1 (3)
(b)	$(x^2 - 8x + 15) = (x-5)(x-3)$ $f(x) = x(x-5)(x-3)$	M1 A1 (2)
(c)	 Shape their 3 <u>or</u> their 5 <u>both</u> their 3 <u>and</u> their 5 and (0,0) by implication	B1 B1f.t. B1f.t. (3)
		8
(a)	M1 for a correct method to get the factor of $x$ . $x($ as printed is the minimum. 1 <sup>st</sup> A1 for $b = -8$ or $c = 15$ . -8 comes from $-6-2$ and must be coefficient of $x$ , and 15 from $6 \times 2 + 3$ and must have no $x$ s. 2 <sup>nd</sup> A1 for $a = 1$ , $b = -8$ and $c = 15$ . Must have $x(x^2 - 8x + 15)$ .	
(b)	M1 for attempt to factorise their 3TQ from part (a). A1 for all 3 terms correct. They must include the $x$ . For part (c) they must have <u>at most</u> 2 non-zero roots of their $f(x) = 0$ to fit their 3 and their 5.	
(c)	1 <sup>st</sup> B1 for correct shape (i.e. from bottom left to top right and two turning points.) 2 <sup>nd</sup> B1f.t. for crossing at their 3 or their 5 indicated on graph or in text. 3 <sup>rd</sup> B1f.t. if graph passes through (0, 0) [needn't be marked] and both their 3 and their 5.	

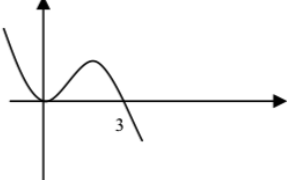
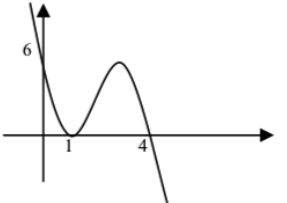
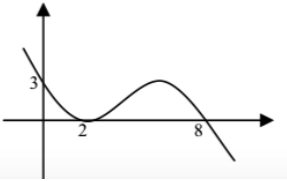
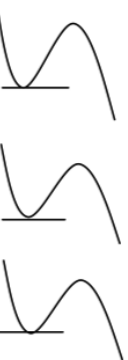
## Question 3: June 05 Q4

Question Number	Scheme	Marks
(a)		Shape Points B1 B1 (2)
(b)		M1
	-2 and 4 max	A1 A1 (3) (5)
Marks for shape: graphs must have curved sides and round top.		
(a)	1 <sup>st</sup> B1 for $\cap$ shape through $(0, 0)$ and $((k, 0)$ where $k > 0$ ) 2 <sup>nd</sup> B1 for max at $(3, 15)$ and 6 labelled or $(6, 0)$ seen Condone $(15, 3)$ if 3 and 15 are correct on axes. Similarly $(5, 1)$ in (b)	
(b)	M1 for $\cap$ shape <u>NOT</u> through $(0, 0)$ but must cut $x$ -axis twice. 1 <sup>st</sup> A1 for -2 and 4 labelled or $(-2, 0)$ and $(4, 0)$ seen 2 <sup>nd</sup> A1 for max at $(1, 5)$ . Must be clearly in 1 <sup>st</sup> quadrant	

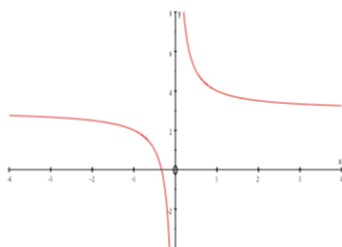
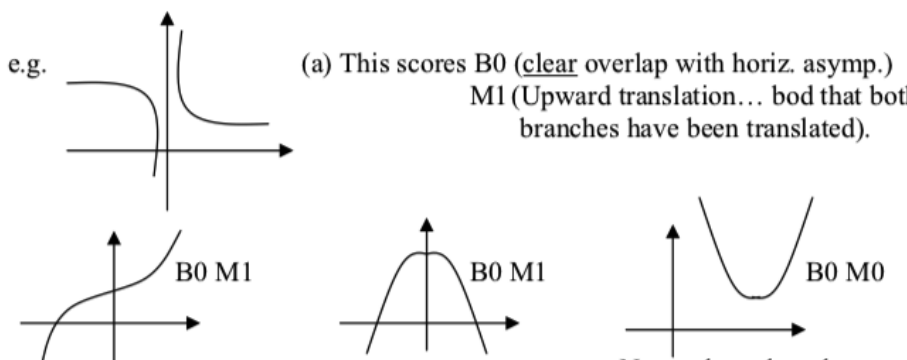
Question 4: June 06 Q3

Question number	Scheme		Marks
	<p>(a)</p> 	<p>U shape touching <math>x</math>-axis</p> <p><math>(-3, 0)</math></p> <p><math>(0, 9)</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
	<p>(b)</p> 	<p>Translated parallel to <math>y</math>-axis up</p> <p><math>(0, 9+k)</math></p>	<p>M1</p> <p>B1f.t.</p> <p>(2)</p>
			5
(a)	<p>2<sup>nd</sup> B1</p> <p>2<sup>nd</sup> B1 &amp; 3<sup>rd</sup> B1</p>	<p>They can score this even if other intersections with the <math>x</math>-axis are given.</p> <p>The -3 and 9 can appear on the sketch as shown</p>	
(b)	<p>M1</p> <p>B1f.t.</p>	<p>Follow their curve in (a) up only.</p> <p>If it is not obvious do not give it. e.g. if it cuts <math>y</math>-axis in (a) but doesn't in (b) then it is M0.</p> <p>Follow through their 9</p>	






Question 5: Jan 06 Q6

Question number	Scheme	Marks
(a)	 <p>(See below) Clearly through origin (or <math>(0, 0)</math> seen) 3 labelled (or <math>(3, 0)</math> seen)</p>	<p>M1 A1 A1 (3)</p>
(b)	 <p>Stretch parallel to <math>y</math>-axis 1 and 4 labelled (or <math>(1, 0)</math> and <math>(4, 0)</math> seen) 6 labelled (or <math>(0, 6)</math> seen)</p>	<p>M1 A1 A1 (3)</p>
(c)	 <p>Stretch parallel to <math>x</math>-axis 2 and 8 labelled (or <math>(2, 0)</math> and <math>(8, 0)</math> seen) 3 labelled (or <math>(0, 3)</math> seen)</p>	<p>M1 A1 A1 (3)</p>
(a) M1: (b) M1: (c) M1:	 <p>with at least two of: <math>(1, 0)</math> unchanged <math>(4, 0)</math> unchanged <math>(0, 3)</math> changed</p> <p>with at least two of: <math>(1, 0)</math> changed <math>(4, 0)</math> changed <math>(0, 3)</math> unchanged</p> <p><u>Beware:</u> Candidates may sometimes re-label the parts of their solution.</p>	

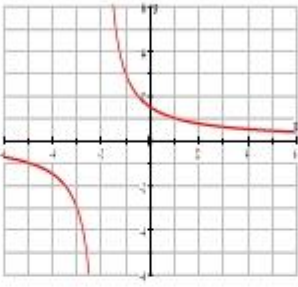
Question 6: Jan 07 Q3

Question number	Scheme	Marks
	<p>(a)</p>  <p>Shape of <math>f(x)</math>  Moved up <math>\uparrow</math>  Asymptotes: <math>y = 3</math>  <math>x = 0</math> (Allow “y-axis”)  (<math>y \neq 3</math> is B0, <math>x \neq 0</math> is B0).</p> <p>(b) <math>\frac{1}{x} + 3 = 0</math> No variations accepted.  <math>x = -\frac{1}{3}</math> (or <math>-0.33 \dots</math>) Decimal answer requires at least 2 d.p.</p>	<p>B1  M1  B1  B1 (4)    M1  A1 (2)  <b>6</b></p>
	<p>(a) B1: Shape requires both branches and no obvious “overlap” with the asymptotes (see below), but otherwise this mark is awarded generously. The curve may, e.g., bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both horizontal and vertical.  M1: Evidence of an upward translation parallel to the <math>y</math>-axis. The shape of the graph can be wrong, but the complete graph (both branches if they have 2 branches) must be translated upwards. This mark can be awarded generously by implication where the graph drawn is an upward translation of another standard curve (but <u>not</u> a straight line).  The B marks for asymptote equations are independent of the graph.  Ignore extra asymptote equations, if seen.</p> <p>(b) Correct answer with no working scores both marks.  The answer may be seen on the sketch in part (a).  Ignore any attempts to find an intersection with the <math>y</math>-axis.</p> <p>e.g.</p>  <p>(a) This scores B0 (<u>clear</u> overlap with horiz. asymp.)  M1 (Upward translation... bod that both branches have been translated).</p> <p>No marks unless the original curve is seen, to show upward translation.</p>	

Question 7: Jan 07 Q10

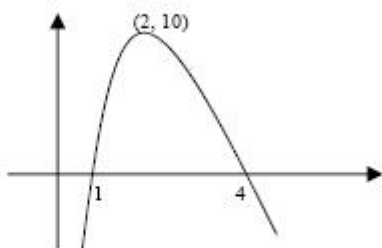
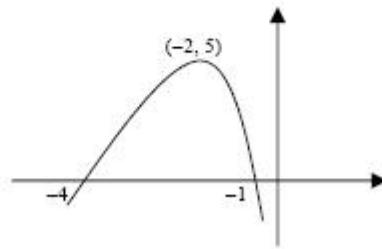

Question number	Scheme	Marks
(a)	 <p>(i) Shape  or  or   Max. at (0, 0).  (2, 0), (or 2 shown on x-axis).</p> <p>(ii) Shape   (It need not go below x-axis)  Through origin.  (6, 0), (or 6 shown on x-axis).</p>	<p>B1  B1  B1 (3)</p> <p>B1  B1  B1 (3)</p>
(b)	<p><math>x^2(x-2) = x(6-x)</math>  <math>x^3 - x^2 - 6x = 0</math> Expand to form 3-term cubic (or 3-term quadratic if divided by <math>x</math>), with all terms on one side. The “= 0” may be implied.</p> <p><math>x(x-3)(x+2) = 0</math> <math>x = \dots</math> Factor <math>x</math> (or divide by <math>x</math>), and solve quadratic.  <math>x = 3</math> and <math>x = -2</math>  <math>x = -2</math>: <math>y = -16</math> Attempt <math>y</math> value for a non-zero <math>x</math> value by substituting back into <math>x^2(x-2)</math> or <math>x(6-x)</math>.  <math>x = 3</math>: <math>y = 9</math> Both <math>y</math> values are needed for A1.  <math>(-2, -16)</math> and <math>(3, 9)</math>  <math>(0, 0)</math> This can just be written down. Ignore any ‘method’ shown. (But must be seen in part (b)).</p>	<p>M1  M1  M1  A1  M1  A1  B1 (7)  <b>13</b></p>
(a)	<p>(i) For the third ‘shape’ shown above, where a section of the graph coincides with the <math>x</math>-axis, the B1 for (2, 0) can still be awarded if the 2 is shown on the <math>x</math>-axis.</p> <p>For the final B1 in (i), and similarly for (6, 0) in (ii):  There must be a sketch.  If, for example (2, 0) is written <u>separately</u> from the sketch, the sketch must not clearly contradict this.  If (0, 2) instead of (2, 0) is shown <u>on the sketch</u>, allow the mark.  Ignore extra intersections with the <math>x</math>-axis.</p> <p>(ii) 2<sup>nd</sup> B is dependent on 1<sup>st</sup> B.  Separate sketches can score all marks.</p> <p>(b) Note the dependence of the first three M marks.  A common wrong solution is <math>(-2, 0)</math>, <math>(3, 0)</math>, <math>(0, 0)</math>, which scores M0 A0 B1 as the last 3 marks.  A solution using <u>no</u> algebra (e.g. trial and error), can score up to 3 marks:  M0 M0 M0 A0 M1 A1 B1. (The final A1 requires both <math>y</math> values).  Also, if the cubic is found but not solved algebraically, up to 5 marks:  M1 M1 M0 A0 M1 A1 B1. (The final A1 requires both <math>y</math> values).</p>	

Question 8: June 07 Q5

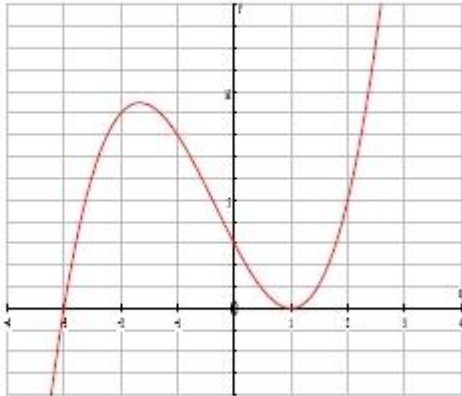

Question number	Scheme	Marks
S.C.	<p>(a)</p>  <p>Translation parallel to <math>x</math>-axis  Top branch intersects +ve <math>y</math>-axis  Lower branch has no intersections  No obvious overlap</p> <p><math>\left(0, \frac{3}{2}\right)</math> or <math>\frac{3}{2}</math> marked on <math>y</math>-axis</p>	<p>M1</p> <p>A1</p> <p>B1 (3)</p>
	<p>(b) <math>x = -2, y = 0</math></p> <p>[Allow ft on first B1 for <math>x = 2</math> when translated “the wrong way” but must be compatible with their sketch.]</p>	<p>B1, B1 (2)</p> <p>5</p>
(a)	<p>M1 for a horizontal translation – two branches with one branch cutting <math>y</math> – axis only.  If one of the branches cuts both axes (translation up and across) this is M0.</p> <p>A1 for a horizontal translation to left. Ignore any figures on axes for this mark.</p> <p>B1 for correct intersection on positive <math>y</math>-axis. More than 1 intersection is B0.</p> <p><math>x=0</math> and <math>y = 1.5</math> in a table alone is insufficient unless intersection of their sketch is with +ve <math>y</math>-axis.  A point marked on the graph overrides a point given elsewhere.</p>	
(b)	<p>1<sup>st</sup> B1 for <math>x = -2</math>. NB <math>x \neq -2</math> is B0.  Can accept <math>x = +2</math> if this is compatible with their sketch.  Usually they will have M1A0 in part (a) (and usually B0 too)</p> <p>2<sup>nd</sup> B1 for <math>y = 0</math>.</p>	
S.C.	<p>If <math>x = -2</math> and <math>y = 0</math> and some other asymptotes are also given award B1B0</p> <p>The asymptote equations should be clearly stated in part (b). Simply marking <math>x = -2</math> or <math>y = 0</math> on the sketch is insufficient <u>unless</u> they are clearly marked “asymptote <math>x = -2</math>” etc.</p>	



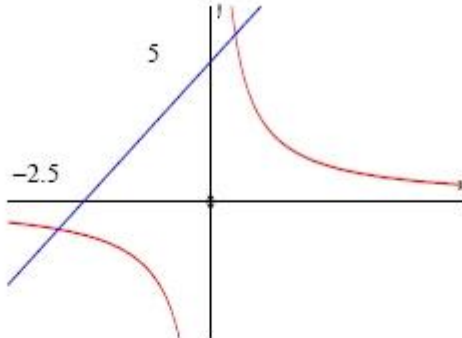
Question 9: Jan 08 Q6

Question number	Scheme	Marks
(a)	 <p>Shape: Max in 1<sup>st</sup> quadrant and 2 intersections on positive <math>x</math>-axis</p> <p>1 and 4 labelled (in correct place) or clearly stated as coordinates</p> <p>(2, 10) labelled or clearly stated</p>	B1 B1 B1 (3)
(b)	 <p>Shape: Max in 2<sup>nd</sup> quadrant and 2 intersections on negative <math>x</math>-axis</p> <p>-1 and -4 labelled (in correct place) or clearly stated as coordinates</p> <p>(-2, 5) labelled or clearly stated</p>	B1 B1 B1 (3)
(c)	<p>(a =) 2</p> <p>May be implicit, i.e. <math>f(x+2)</math></p> <p>Beware: The answer to part (c) may be seen on the first page.</p>	B1 (1)
<p>(a) and (b):</p> <p>1<sup>st</sup> B: 'Shape' is generous, providing the conditions are satisfied.</p> <p>2<sup>nd</sup> and 3<sup>rd</sup> B marks are dependent upon a sketch having been drawn.</p> <p>2<sup>nd</sup> B marks: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> the sketch is correct.</p> <p>Points must be labelled correctly and be in appropriate place (e.g. (-2, 5) in the first quadrant is B0).</p> <p>(b) <u>Special case:</u></p> <p>If the graph is reflected in the <math>x</math>-axis (instead of the <math>y</math>-axis), B1 B0 B0 can be scored. This requires shape and coordinates to be <u>fully correct</u>, i.e.</p> <p>Shape:  Minimum in 4<sup>th</sup> quadrant and 2 intersections on positive <math>x</math>-axis,</p> <p>1 and 4 labelled (in correct place) or clearly stated as coordinates,</p> <p>(2, -5) labelled or clearly stated.</p>		7

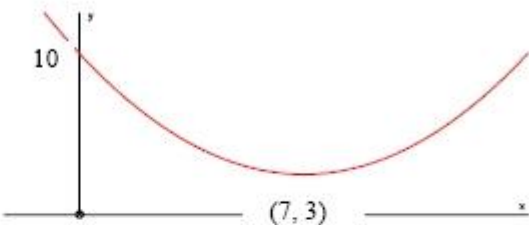
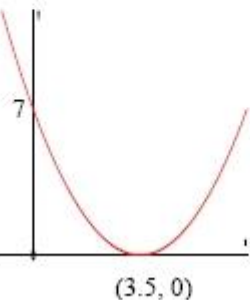
Question 10: Jan 08 Q10

Question number	Scheme	Marks
(a)	 <p>Shape  (drawn anywhere)</p> <p>Minimum at (1, 0) (perhaps labelled 1 on x-axis)</p> <p>(-3, 0) (or -3 shown on -ve x-axis)</p> <p>(0, 3) (or 3 shown on +ve y-axis)</p> <p>N.B. The max. can be anywhere.</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>(4)</p>
(b)	<p><math>y = (x+3)(x^2 - 2x + 1)</math></p> <p><math>= x^3 + x^2 - 5x + 3</math> (<math>k = 3</math>)</p> <p>Marks can be awarded if this is seen in part (a)</p>	<p>M1</p> <p>A1 cso</p> <p>(2)</p>
	<p>(a) The individual marks are independent, <u>but</u> the 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> B's are dependent upon a sketch having been attempted.</p> <p>B marks for coordinates: Allow (0, 1), etc. (coordinates the wrong way round) <u>if</u> marked in the correct place on the sketch.</p> <p>(b) M: Attempt to multiply out <math>(x-1)^2</math> and write as a product with <math>(x+3)</math>, or attempt to multiply out <math>(x+3)(x-1)</math> and write as a product with <math>(x-1)</math>, or attempt to expand <math>(x+3)(x-1)(x-1)</math> directly (at least 7 terms). The <math>(x-1)^2</math> or <math>(x+3)(x-1)</math> expansion must have 3 (or 4) terms, so should not, for example, be just <math>x^2 + 1</math>.</p> <p>A: It is not necessary to state explicitly '<math>k = 3</math>'. Condone missing brackets if the intention seems clear and a fully correct expansion is seen.</p>	

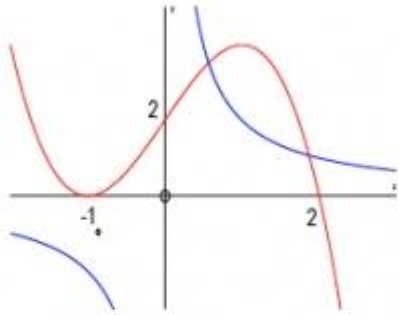




Question 11: June 08 Q6

Question Number	Scheme	Marks
(a)		<p>B1</p> <p>M1</p> <p>A1 (3)</p>
(b)	$2x + 5 = \frac{3}{x}$ $2x^2 + 5x - 3 [= 0] \quad \text{or} \quad 2x^2 + 5x = 3$ $(2x - 1)(x + 3) [= 0]$ $x = -3 \text{ or } \frac{1}{2}$ $y = \frac{3}{-3} \text{ or } 2 \times (-3) + 5 \quad \text{or} \quad y = \frac{3}{\frac{1}{2}} \text{ or } 2 \times \left(\frac{1}{2}\right) + 5$ <p>Points are <u><math>(-3, -1)</math> and <math>(\frac{1}{2}, 6)</math></u> (correct pairings)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 ft (6)</p> <p><b>(9 marks)</b></p>

Question 12: June 08 Q3

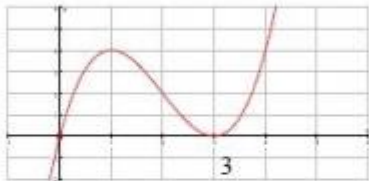

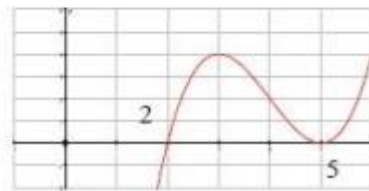
Question Number	Scheme	Marks
(a)		B1 B1 B1 (3)
(b)		B1 B1 (2) (5 marks)

Question 13: Jan 09 Q8

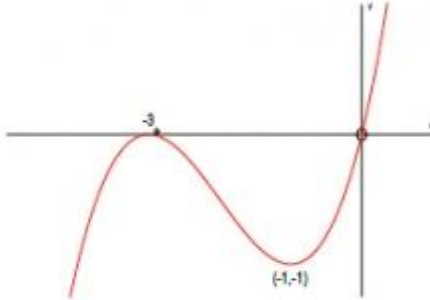

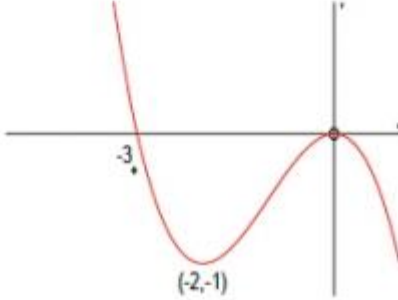

Question Number	Scheme	Marks
(a)	$(a =) (1+1)^2 (2-1) = \underline{4}$ (1, 4) or $y = 4$ is also acceptable	B1 (1)
(b)	 <p>(i) Shape  or  anywhere</p> <p>Min at <math>(-1, 0)</math> ... can be <math>-1</math> on <math>x</math>-axis.  Allow <math>(0, -1)</math> if marked on the <math>x</math>-axis.  Marked in the correct place, but 1, is B0.</p> <p><math>(2, 0)</math> and <math>(0, 2)</math> can be 2 on axes</p> <p>(ii)  Top branch in 1<sup>st</sup> quadrant with 2 intersections  Bottom branch in 3<sup>rd</sup> quadrant (ignore any intersections)</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1 (5)</p>
(c)	(2 intersections therefore) <u>2</u> (roots)	B1ft (1) [7]
(b)	<p>1<sup>st</sup> B1 for shape  or  Can be anywhere, but there must be one max. and one min. and no further max. and min. turning points.  Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min.</p> <p>2<sup>nd</sup> B1 for minimum at <math>(-1, 0)</math> (even if there is an additional minimum point shown)</p> <p>3<sup>rd</sup> B1 for the sketch meeting axes at <math>(2, 0)</math> and <math>(0, 2)</math>. They can simply mark 2 on the axes.  The marks for minimum and intersections are dependent upon having a sketch.  Answers on the diagram for min. and intersections take precedence over answers seen elsewhere.</p> <p>4<sup>th</sup> B1 for the branch fully within 1<sup>st</sup> quadrant having 2 intersections with (not just 'touching') the other curve. The curve can 'touch' the axes.  A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes, and when the curve looks like two straight lines with a small curve at the join.  Allow, for example, shapes like these:</p> <p>5<sup>th</sup> B1 for a branch fully in the 3<sup>rd</sup> quadrant (ignore any intersections with the other curve for this branch). The curve can 'touch' the axes.  A curve of (roughly) the correct shape is required, but be very generous, even when the arc appears to turn 'inwards' rather than approaching the axes.</p>	
(c)	<p>B1ft for a statement about the number of roots - compatible with their sketch. No sketch is B0.  The answer 2 <u>incompatible with the sketch</u> is B0 (ignore any algebra seen).  If the sketch shows the 2 correct intersections <u>and</u>, for example, one other intersection, the answer here should be 3, not 2, to score the mark.</p>	



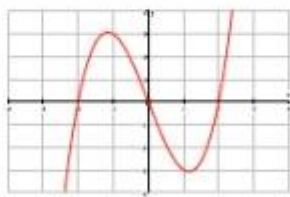

Question 14: June 09 Q10

Question Number	Scheme	Marks
Q (a)	$x(x^2 - 6x + 9)$ $= x(x - 3)(x - 3)$	B1 M1 A1 (3) B1
(b)	 <p>Shape </p> <p><u>Through</u> origin (<u>not</u> touching) Touching x-axis only once Touching at (3, 0), or 3 on x-axis [Must be on graph not in a table]</p>	B1 B1 B1ft (4)
(c)	 <p>Moved horizontally (either way) (2, 0) and (5, 0), or 2 and 5 on x-axis</p>	M1 A1 (2)
		[9]
(a)	<p>B1 for correctly taking out a factor of <math>x</math></p> <p>M1 for an attempt to factorize their 3TQ e.g. <math>(x + p)(x + q)</math> where <math> pq  = 9</math>. So <math>(x - 3)(x + 3)</math> will score M1 but A0</p> <p>A1 for a fully correct factorized expression - accept <math>x(x - 3)^2</math></p> <p>If they "solve" use ISW</p> <p>If the only correct linear factor is <math>(x - 3)</math>, perhaps from factor theorem, award B0M1A0</p> <p>Do not award marks for factorising in part (b)</p> <p><b>For the graphs</b></p> <p>"Sharp points" will lose the 1<sup>st</sup> B1 in (b) but otherwise be generous on shape Condone (0, 3) in (b) and (0, 2), (0, 5) in (c) if the points are marked in the correct places.</p>	
S.C.	<p>(b) 2<sup>nd</sup> B1 for a curve that starts or terminates at (0, 0) score B0</p> <p>4<sup>th</sup> B1ft for a curve that touches (not crossing or terminating) at (a, 0) where their <math>y = x(x - a)^2</math></p>	
(c)	<p>M1 for their graph moved horizontally (only) <u>or</u> a fully correct graph Condone a partial stretch if ignoring their values looks like a simple translation</p> <p>A1 for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only</p> <p>Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)</p>	

Question 15: Jan 09 Q5

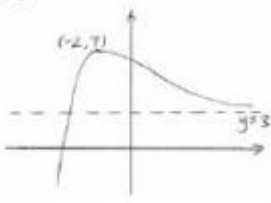
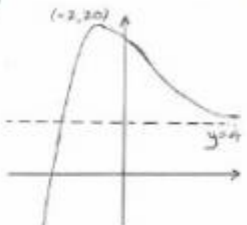
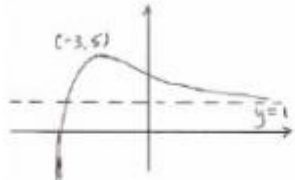
Question Number	Scheme	Marks
(a)	 <p>Shape , touching the x-axis at its maximum.</p> <p>Through (0,0) &amp; -3 marked on x-axis, or (-3,0) seen.</p> <p>Allow (0, -3) if marked on the x-axis.</p> <p>Marked in the correct place, but 3, is A0.</p> <p>Min at (-1, -1)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p>
(b)	 <p>Correct shape , (top left - bottom right)</p> <p>Through -3 and max at (0, 0).</p> <p>Marked in the correct place, but 3, is B0.</p> <p>Min at (-2, -1)</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p> <p>[6]</p>
(a)	<p>M1 as described above. Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min.</p> <p>1<sup>st</sup> A1 for curve passing through -3 and the origin. Max at (-3, 0)</p> <p>2<sup>nd</sup> A1 for minimum at (-1, -1). Can simply be indicated on sketch.</p>	
(b)	<p>1<sup>st</sup> B1 for the correct shape. A negative cubic passing from top left to bottom right. Shape: Be generous, even when the curve seems to be composed of straight line segments, but there must be a discernible 'curve' at the max. and min.</p> <p>2<sup>nd</sup> B1 for curve passing through (-3, 0) having a max at (0, 0) and no other max.</p> <p>3<sup>rd</sup> B1 for minimum at (-2, -1) and no other minimum.</p> <p>If in correct quadrant but labelled, e.g. (-2, 1), this is B0.</p> <p>In each part the (0, 0) does <u>not</u> need to be written to score the second mark... having the curve pass through the origin is sufficient.</p> <p>The last mark (for the minimum) in each part is dependent on a sketch being attempted, and the sketch must show the minimum in approximately the correct place (not, for example, (-2, -1) marked in the wrong quadrant).</p> <p>The mark for the minimum is <u>not</u> given for the coordinates just marked on the axes <u>unless</u> these are clearly linked to the minimum by vertical and horizontal lines.</p>	

Question 16: Jan 10 Q9

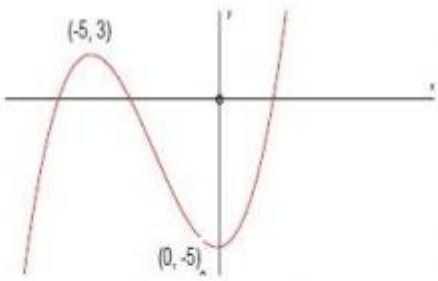
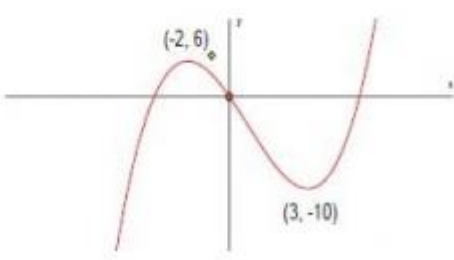
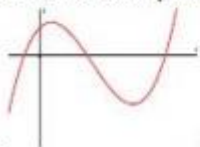
Question number	Scheme	Marks
	<p>(a) <math>x(x^2 - 4)</math> Factor <math>x</math> seen in a <u>correct</u> factorised form of the expression.  <math>= x(x - 2)(x + 2)</math> M: Attempt to factorise quadratic (general principles).            Accept <math>(x - 0)</math> or <math>(x + 0)</math> instead of <math>x</math> at any stage.            Factorisation must be seen in part (a) to score marks.</p>	<p>B1            M1 A1            (3)</p>
	<p>(b)</p>  <p>Shape  (2 turning points required)            Through (or touching) origin            Crossing x-axis or "stopping at x-axis" (not a turning point) at <math>(-2, 0)</math> and <math>(2, 0)</math>.            Allow <math>-2</math> and <math>2</math> on x-axis. Also allow <math>(0, -2)</math> and <math>(0, 2)</math> if marked on x-axis.            Ignore extra intersections with x-axis.</p>	<p>B1            B1            B1            (3)</p>



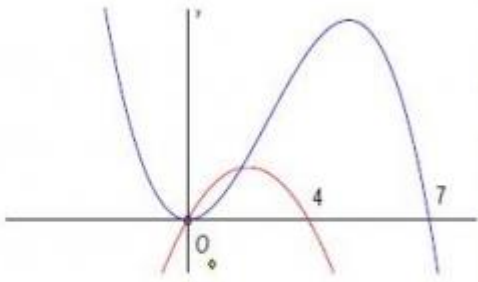
Question 17: Jan 10 Q8

Question number	Scheme	Marks
	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>(a)</p>  </div> <div style="text-align: center;"> <p>(b)</p>  </div> <div style="text-align: center;"> <p>(c)</p>  </div> </div>	
	<p>(a) <math>(-2, 7), y = 3</math> (Marks are dependent upon a sketch being attempted) See conditions below.</p>	<p>B1, B1 (2)</p>
	<p>(b) <math>(-2, 20), y = 4</math> (Marks are dependent upon a sketch being attempted) See conditions below.</p>	<p>B1, B1 (2)</p>
	<p>(c) Sketch: Horizontal translation (either way)... (There must be evidence that <math>y = 5</math> at the max and that the asymptote is still <math>y = 1</math>)</p> <p><math>(-3, 5), y = 1</math></p>	<p>B1 B1, B1 (3) [7]</p>
	<p><b>Parts (a) and (b):</b></p> <p>(i) If <u>only one</u> of the B marks is scored, there is <u>no penalty</u> for a wrong sketch.</p> <p>(ii) If both the maximum and the equation of the asymptote are correct, the sketch must be "correct" to score B1 B1. If the sketch is "wrong", award B1 B0. The (generous) conditions for a "correct" sketch are that the maximum must be in the 2<sup>nd</sup> quadrant and that the curve must not cross the positive x-axis... ignore other "errors" such as "curve appearing to cross its asymptote" and "curve appearing to have a minimum in the 1<sup>st</sup> quadrant".</p> <p><b>Special case:</b></p> <p>(b) Stretch <math>\frac{1}{4}</math> instead of 4: Correct shape, with <math>\left(-2, \frac{5}{4}\right), y = \frac{1}{4}</math> : B1 B0.</p> <p><b>Coordinates of maximum:</b></p> <p>If the coordinates are the wrong way round (e.g. <math>(7, -2)</math> in part (a)), or the coordinates are just shown as values on the x and y axes, penalise <u>only once in the whole question</u>, at first occurrence.</p> <p><b>Asymptote marks:</b></p> <p>If the <u>equation</u> of the asymptote is not given, e.g. in part (a), 3 is marked on the y-axis but <math>y = 3</math> is not seen, penalise <u>only once in the whole question</u>, at first occurrence.</p> <p><u>Ignore</u> extra asymptotes stated (such as <math>x = 0</math>).</p>	

Question 18: June 10 Q6

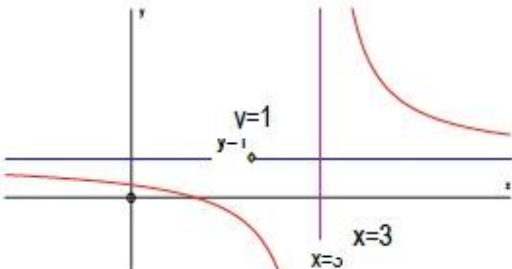
Question Number	Scheme	Marks
(a)	 <p>Horizontal translation of <math>\pm 3</math></p> <p><math>(-5, 3)</math> marked on sketch or in text</p> <p><math>(0, -5)</math> and min intentionally on y-axis Condone <math>(-5, 0)</math> if correctly placed on negative y-axis</p>	M1 B1 A1 (3)
(b)	 <p>Correct shape and intentionally through <math>(0, 0)</math> between the max and min</p> <p><math>(-2, 6)</math> marked on graph or in text</p> <p><math>(3, -10)</math> marked on graph or in text</p>	B1 B1 B1 (3)
(c)	$(a = ) \underline{\quad}$	B1 (1)
<b>Notes</b>		
<p>Turning points (not on axes) should have both co-ordinates given in form <math>(x,y)</math>. Do not accept points marked on axes e.g. <math>-5</math> on x-axis and <math>3</math> on y-axis is not sufficient. For repeated offenders apply this penalty <b>once only</b> at first offence and condone elsewhere.</p> <p>In (a) and (b) no graphs means no marks.</p> <p>In (a) and (b) the ends of the graphs do not need to cross the axes provided max and min are clear</p>  <p>(a) M1 for a horizontal translation of <math>\pm 3</math> so accept coordinates of <math>(1, 3)</math> or <math>(6, -5)</math> seen. i.e max in 1<sup>st</sup> quad <u>and</u> [Horizontal translation to the left should have a min <u>on</u> the y-axis] If curve passes through <math>(0,0)</math> then M0 (and A0) but they could score the B1 mark. A1 for minimum clearly on negative y-axis and at least <math>-5</math> marked on y-axis. Allow this mark if the minimum is very close and the point <math>(0, -5)</math> clearly indicated</p> <p>(b) 1<sup>st</sup> B1 Ignore coordinates for this mark Coordinates or points on sketch override coordinates given in the text. Condone <math>(y, x)</math> confusion for points on axes only. So <math>(-5, 0)</math> for <math>(0, -5)</math> is OK if the point is marked correctly but <math>(3, 10)</math> is B0 even if in 4<sup>th</sup> quadrant.</p> <p>(c) This may be at the bottom of a page or in the question...make sure you scroll up and down!</p>		

Question 19: June 10 Q10

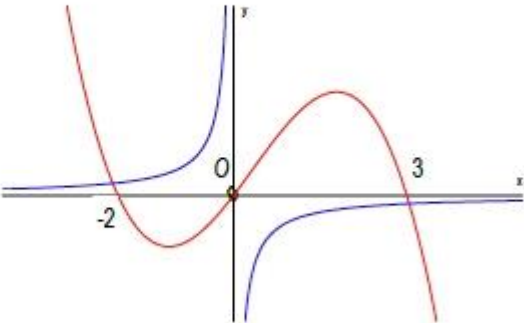
Question Number	Scheme	Marks
(a)	 <p>(i) <math>\cap</math> shape (anywhere on diagram)</p> <p>Passing through or stopping at (0, 0) and (4,0) only (Needn't be <math>\cap</math> shape)</p> <p>(ii) correct shape (-ve cubic) with a max and min drawn anywhere</p> <p>Minimum or maximum at (0,0)</p> <p>Passes through or stops at (7,0) but <u>NOT</u> touching.</p> <p>(7, 0) should be to right of (4,0) or B0</p> <p>Condone (0,4) or (0, 7) marked correctly on x-axis. Don't penalise poor overlap near origin.</p> <p>Points must be marked on the sketch...not in the text</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>(5)</p>
(b)	$x(4-x) = x^2(7-x) \quad (0=)x[7x-x^2-(4-x)]$ $(0=)x[7x-x^2-(4-x)] \quad (\text{o.e.})$ $0 = x(x^2 - 8x + 4) \quad *$	<p>M1</p> <p>B1ft</p> <p>A1 cso</p> <p>(3)</p>
(c)	$(0 = x^2 - 8x + 4 \Rightarrow) x = \frac{8 \pm \sqrt{64-16}}{2} \quad \text{or} \quad (x \pm 4)^2 - 4^2 + 4 (=0)$ $= \frac{8 \pm 4\sqrt{3}}{2} \quad \text{or} \quad (x-4)^2 = 12$ $x = 4 \pm 2\sqrt{3} \quad \text{or} \quad (x-4) = \pm 2\sqrt{3}$ <p>From sketch A is <math>x = 4 - 2\sqrt{3}</math></p> <p>So <math>y = (4 - 2\sqrt{3})(4 - [4 - 2\sqrt{3}]) \quad (\text{dependent on 1}^{\text{st}} \text{ M1})</math></p> $= -12 + 8\sqrt{3}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(7)</p>
<b>15</b>		
<b>Notes</b>		
(b)	<p>M1 for forming a suitable equation</p> <p>B1 for a common factor of <math>x</math> taken out legitimately. Can treat this as an M mark. Can fit their cubic = 0 found from an attempt at solving their equations e.g. <math>x^3 - 8x^2 - 4x = x(\dots)</math></p> <p>A1 also no incorrect working seen. The "= 0" is required but condone missing from some lines of working. Cancelling the <math>x</math> scores B0A0.</p>	
(c)	<p>1<sup>st</sup> M1 for some use of the correct formula or attempt to complete the square</p> <p>1<sup>st</sup> A1 for a fully correct expression: condone + instead of <math>\pm</math> or for <math>(x-4)^2 = 12</math></p> <p>B1 for simplifying <math>\sqrt{48} = 4\sqrt{3}</math> or <math>\sqrt{12} = 2\sqrt{3}</math>. Can be scored independently of this expression</p> <p>2<sup>nd</sup> A1 for correct solution of the form <math>p + q\sqrt{3}</math>: can be + or + or -</p> <p>2<sup>nd</sup> M1 for selecting their answer in the interval (0,4). If they have no value in (0,4) score M0</p> <p>3<sup>rd</sup> M1 for attempting <math>y = \dots</math> using their <math>x</math> in correct equation. An expression needed for M1A0</p> <p>3<sup>rd</sup> A1 for correct answer. If 2 answers are given A0.</p>	



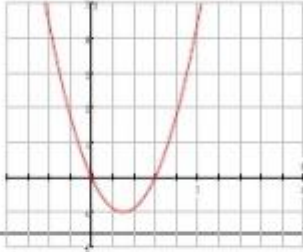

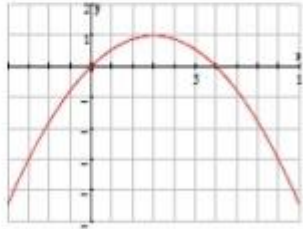

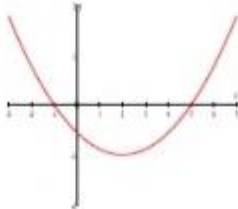

Question 20: Jan 11 Q5

Question Number	Scheme	Marks
(a)	 <p>Correct shape with a single crossing of each axis</p> <p><math>y = 1</math> labelled or stated</p> <p><math>x = 3</math> labelled or stated</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>(3)</p>
(b)	<p>Horizontal translation so crosses the x-axis at (1, 0)</p> <p>New equation is <math>(y =) \frac{x \pm 1}{(x \pm 1) - 2}</math></p> <p>When <math>x = 0</math> <math>y =</math></p> $= \frac{1}{3}$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>7</p>
<b>Notes</b>		
(b)	<p>B1 for point (1,0) identified - this may be marked on the sketch as 1 on x axis. Accept <math>x = 1</math>.</p> <p>1<sup>st</sup> M1 for attempt at new equation and either numerator or denominator correct</p> <p>2<sup>nd</sup> M1 for setting <math>x = 0</math> in their new equation and solving as far as <math>y = \dots</math></p> <p>A1 for <math>\frac{1}{3}</math> or exact equivalent. Must see <math>y = \frac{1}{3}</math> or <math>(0, \frac{1}{3})</math> or point marked on y-axis.</p> <p><b>Alternative</b></p> <p><math>f(-1) = \frac{-1}{-1-2} = \frac{1}{3}</math> scores M1M1A0 unless <math>x = 0</math> is seen or they write the point as <math>(0, \frac{1}{3})</math> or give <math>y = 1/3</math></p> <p>Answers only: <math>x = 1</math>, <math>y = 1/3</math> is full marks as is <math>(1, 0)</math> <math>(0, 1/3)</math></p> <p>Just 1 and <math>1/3</math> is B0 M1 M1 A0</p> <p>Special case : Translates 1 unit to left</p> <p>(a) B0, B1, B0</p> <p>(b) Mark (b) as before</p> <p>May score B0 M1 M1 A0 so 3/7 or may ignore sketch and start again scoring full marks for this part.</p>	

Question 21: Jan 11 Q10

Question Number	Scheme	Marks
(a)	 <p>(i) correct shape ( -ve cubic) Crossing at <math>(-2, 0)</math> Through the origin Crossing at <math>(3, 0)</math></p> <p>(ii) 2 branches in correct quadrants not crossing axes One intersection with cubic on each branch</p>	<p>B1 B1 B1 B1 B1 B1</p> <p>(6)</p>
(b)	<p>“2” solutions</p> <p>Since only “2” intersections</p>	<p>B1ft dB1ft</p> <p>(2) 8</p>
<b>Notes</b>		
(b)	<p>B1ft for a value that is compatible with their sketch dB1ft This mark is dependent on the value being compatible with their sketch. For a comment relating the number of solutions to the number of intersections.</p> <p>[ Only allow 0, 2 or 4]</p>	

Question 22: June 11 Q8

Question Number	Scheme	Marks
(a)	 <p>Shape  through (0, 0) (3, 0) (1.5, -1)</p>	<p>B1 B1 B1 (3)</p>
(b)	 <p>Shape  (0, 0) and (6, 0) (3, 1)</p>	<p>B1 B1 B1 (3)</p>
(c)	 <p>Shape , <u>not</u> through (0, 0) Minimum in 4<sup>th</sup> quadrant (-p, 0) and (6 - p, 0) (3 - p, -1)</p>	<p>M1 A1 B1 B1 (4) <b>10</b></p>
<b>Notes</b>		
<p>(a) B1: U shaped parabola through origin B1: (3,0) stated or 3 labelled on x axis B1: (1.5, -1) or equivalent e.g. (3/2, -1) (b) B1: Cap shaped parabola in any position  B1: through origin (may not be labelled) and (6,0) stated or 6 labelled on x - axis B1: (3,1) shown (c) M1: U shaped parabola not through origin A1: Minimum in 4<sup>th</sup> quadrant (depends on M mark having been given) B1: Coordinates stated or shown on x axis B1: Coordinates stated Note: If values are taken for <math>p</math>, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case all minima should be in fourth quadrant)</p>		