Integration - Edexcel Past Exam Questions 2 MARK SCHEME

Question 1

Question	Scheme	Marks
	$[\mathrm{f}(x)=]\dfrac{3x^3}{3}-\dfrac{3x^2}{2}+5x[+c]$ or $\left\{x^3-\dfrac{3}{2}x^2+5x(+c)\right\}$	M1A1
	$10=8-6+10+c$	M1
	$c=-2$	A1
	$\mathrm{f}(1)=1-\dfrac{3}{2}+5 \quad "-2"=\underline{\dfrac{5}{2}}$ (o.e.)	A1ft (5)
		5 marks
	Notes	
	1st M1 for attempt to integrate $x^n \to x^{n+1}$ 1st A1 all correct, possibly unsimplified. Ignore $+c$ here. 2nd M1 for using $x=2$ and $\mathrm{f}(2)=10$ to form a linear equation in c. Allow sign errors. They should be substituting into a changed expression 2nd A1 for $c=-2$ 3rd A1ft for $\dfrac{9}{2}+c$ Follow through their numerical c ($\neq 0$) This mark is dependent on 1st M1 and 1st A1 only.	

Question 2

Question Number	Scheme	Marks
	$\left\{\int\left(6x^2+\dfrac{2}{x^2}+5\right)\mathrm{d}x\right\}=\dfrac{6x^3}{3}+\dfrac{2x^{-1}}{-1}+5x(+c)$	M1 A1
	$=2x^3-2x^{-1};+5x+c$	A1; A1 **4**
	Notes	
	M1: for some attempt to integrate a term in x: $x^n \to x^{n+1}$ So seeing either $6x^2 \to \pm\lambda x^3$ or $\dfrac{2}{x^2}\to \pm\mu x^{-1}$ or $5 \to 5x$ is M1. **1st A1**: for a correct un-simplified x^3 or x^{-1} $\left(\text{or } \dfrac{1}{x}\right)$ term. **2nd A1**: for both x^3 and x^{-1} terms correct and simplified on the same line. Ie. $2x^3-2x^{-1}$ or $2x^3-\dfrac{2}{x}$. **3rd A1**: for $+5x+c$. Also allow $+5x^1+c$. This needs to be written on the same line. **Ignore the incorrect use of the integral sign in candidates' responses.** **Note**: If a candidate scores M1A1A1A1 and their answer is NOT ON THE SAME LINE then withhold the final accuracy mark.	

Question 3

Question Number	Scheme	Marks
	$P(4,-1)$ lies on C where $f'(x)=\frac{1}{2}x-\frac{6}{\sqrt{x}}+3,\ x>0$	
(a)	$f'(4)=\frac{1}{2}(4)-\frac{6}{\sqrt{4}}+3\ ;=2$	M1; A1
	T: $y--1=2(x-4)$	dM1
	T: $y=2x-9$	A1
		[4]
(b)	$f(x)=\frac{x^{1+1}}{2(2)}-\frac{6x^{-\frac{1}{2}+1}}{(\frac{1}{2})}+3x\,(+c)$ or equivalent.	M1 A1
	$\{f(4)=-1\Rightarrow\}\ \frac{16}{4}-12(2)+3(4)+c=-1$	dM1
	$\{4-24+12+c=-1 \quad\Rightarrow\ c=7\}$	
	So, $\{f(x)=\}\ \frac{x^2}{2(2)}-\frac{6x^{\frac{1}{2}}}{(\frac{1}{2})}+3x+7$	A1 cso
	$\left\{\text{NB: } f(x)=\frac{x^2}{4}-12\sqrt{x}+3x+7\right\}$	[4]
		8
	Notes	
(a)	**1st M1:** for clear attempt at $f'(4)$. **1st A1:** for obtaining 2 from $f'(4)$. **2nd dM1:** for $y--1=(\text{their } f'(4))(x-4)$ or $\frac{y--1}{x-4}=(\text{their } f'(4))$ or full method of $y=mx+c$, with $x=4,\ y=-1$ and their $f'(4)$ to find a value for c. **Note:** this method mark is dependent on the first method mark being awarded. **2nd A1:** for $y=2x-9$ or $y=-9+2x$ **Note:** This work needs to be contained in part (a) only.	
(b)	**1st M1:** for a clear attempt to integrate $f'(x)$ with at least one correct application of $x^n\to x^{n+1}$ on $f'(x)=\frac{1}{2}x-\frac{6}{\sqrt{x}}+3$. So seeing either $\frac{1}{2}x\to\pm\lambda x^{1+1}$ or $-\frac{6}{\sqrt{x}}\to\pm\mu x^{-\frac{1}{2}+1}$ or $3\to 3x^{0+1}$ is M1. **1st A1:** for correct un-simplified coefficients and powers (or equivalent) with or without $+c$. **2nd dM1:** for use of $x=4$ and $y=-1$ in an integrated equation to form a linear equation in c equal to -1. ie: applying $f(4)=-1$. This mark is dependent on the first method mark being awarded. **A1:** For $\{f(x)=\}\ \frac{x^2}{2(2)}-\frac{6x^{\frac{1}{2}}}{(\frac{1}{2})}+3x+7$ stated on one line where coefficients can be un-simplified or simplified, but must contain one term powers. Note this mark is for **correct solution only.** **Note: For a candidate attempting to find f(x) in part (a)** If it is clear that they understand that they are finding $f(x)$ in part (a); ie. by writing $f(x)=\ldots$ or $y=\ldots$ then you can give credit for this working in part (b).	

Question 4

Question Number	Scheme		Marks
	$\left(\frac{dy}{dx}=\right) \quad -x^3+"2"x^{-2}-"\left(\frac{5}{2}\right)"x^{-3}$		M1
	$(y=) \quad -\frac{1}{4}x^4+\frac{"2"x^{-1}}{(-1)}-"\left(\frac{5}{2}\right)"\frac{x^{-2}}{(-2)}(+c)$	Raises power correctly on any one term. Any two follow through terms correct.	M1 A1ft
	$(y=) \quad -\frac{1}{4}x^4+\frac{2x^{-1}}{(-1)}-\frac{5}{2}\frac{x^{-2}}{(-2)}(+c)$	This is not follow through – must be correct	A1
	Given that $y=7$, at $x=1$, then $7=-\frac{1}{4}-2+\frac{5}{4}+c \Rightarrow c=$		M1
	So, $(y=) \quad -\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+c,\ c=8$ **or** $(y=)-\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+8$		A1 [6]
			6 marks
	Notes		

Question 5

Question Number	Scheme		Marks
	$\left(\int=\right)\frac{10x^5}{5}-\frac{4x^2}{2},-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}$	M1: Some attempt to integrate: $x^n \to x^{n+1}$ on at least one term. (not for + c) (If they think $\frac{3}{\sqrt{x}}$ is $3x^{\frac{1}{2}}$ you can still award the method mark for $x^{\frac{1}{2}} \to x^{\frac{3}{2}}$ A1: $\frac{10x^5}{5}$ **and** $\frac{-4x^2}{2}$ or better A1: $-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}$ or better	M1A1, A1
	$= \underline{2x^5-2x^2-6x^{\frac{1}{2}}+c}$	Each term correct and simplified and the $+c$ all appearing together on the same line. Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$. Ignore any spurious integral or signs and/or dy/dx's.	A1
	Do **not** apply isw. If they obtain the correct answer and then e.g. divide by 2 they lose the last mark.		
			[4]

Question 6

Question Number	Scheme		Marks
(a)	$(3-x^2)^2 = 9-6x^2+x^4$	An attempt to expand the numerator obtaining an expression of the form $9 \pm px^2 \pm qx^4$, $p.q \neq 0$	M1
	$9x^{-2}+x^2$	Must come from $\frac{9+x^4}{x^2}$	A1
	-6	Must come from $\frac{-6x^2}{x^2}$	A1
	Alternative 1: Writes $\frac{(3-x^2)^2}{x^2}$ as $(3x^{-1}-x)^2$ and attempts to expand = M1 then A1A1 as in the scheme.		
	Alternative 2: Sets $(3-x^2)^2 = 9+Ax^2+Bx^4$, expands $(3-x^2)^2$ and compares coefficients = M1 then A1A1 as in the scheme.		
			(3)
	$(\mathrm{f}'(x) = 9x^{-2}-6+x^2)$		
(b)	$-18x^{-3}+2x$	M1: $x^n \to x^{n-1}$ on separate terms at least once. Do not award for $A \to 0$ (Integrating is M0) A1ft: $-18x^{-3}+2\text{"}B\text{"}x$ with a numerical B and no extra terms. (A may have been incorrect or even zero)	M1 A1ft
			(2)
(c)	$\mathrm{f}(x) = -9x^{-1}-6x+\frac{x^3}{3}(+c)$	M1: $x^n \to x^{n+1}$ on separate terms at least once. (Differentiating is M0) A1ft: $-9x^{-1}+Ax+\frac{Bx^3}{3}(+c)$ with numerical A and B, $A, B \neq 0$	M1A1ft
	$10 = \frac{-9}{-3}-6(-3)+\frac{(-3)^3}{3}+c$ so c $= \ldots$	Uses x = -3 and y =10 in what they think is f(x) (They may have differentiated here) but it must be a changed function i.e. not the original $\mathrm{f}'(x)$, to form a linear equation in c and attempts to find c. No $+ c$ gets M0 and A0 unless their method implies that they are correctly finding a constant.	M1
	c = -2	cso	A1
	$(\mathrm{f}(x) =) -9x^{-1}-6x+\frac{x^3}{3}+$ their c	Follow through their c in an otherwise (possibly un-simplified) **correct expression**. Allow $-\frac{9}{x}$ for $-9x^{-1}$ or even $\frac{9x^{-1}}{-1}$.	A1ft
	Note that if they integrate in (b), no marks there but if they then go on to use their integration in (c), the marks for integration are available.		
			(5)
			[10]

Question 7

Question Number	Scheme	Notes	Marks
	$\int 3x^2 - \frac{4}{x^2}\mathrm{d}x = 3\frac{x^3}{3} - 4\frac{x^{-1}}{-1}$	M1: $x^n \to x^{n+1}$ for either term. If they write $\frac{4}{x^2}$ *as* $4x^2$ allow $x^2 \to x^3$ here.	M1,A1,A1
		A1: $3\frac{x^3}{3}$ **or** $-4\frac{x^{-1}}{-1}$ (one correct term which may be un-simplified)	
		A1: $3\frac{x^3}{3}$ **and** $-4\frac{x^{-1}}{-1}$ (both terms correct which may be un-simplified)	
	Note that M1A0A1 is not possible		
	$= x^3 + \frac{4}{x} + c$ or $x^3 + 4x^{-1} + c$	Fully correct simplified answer with + c all appearing on the same line.	A1
			[4]

Question 8

Question Number	Scheme	Notes	Marks
(a)	$f'(x)=\frac{x+9}{\sqrt{x}}=\frac{x}{\sqrt{x}}+\frac{9}{\sqrt{x}}=x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$	M1: Correct attempt to split into 2 separate terms or fractions. May be implied by one correct term. Divides by $x^{\frac{1}{2}}$ or multiplies by $x^{-\frac{1}{2}}$. A1: $x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$ or equivalent	M1A1
	$f(x)=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+9\frac{x^{\frac{1}{2}}}{\frac{1}{2}}(+c)$	M1: Independent method mark for $x^n \to x^{n+1}$ **on separate terms** A1: Allow un-simplified answers. No requirement for + c here	M1A1
	$\frac{(9)^{\frac{3}{2}}}{\frac{3}{2}}+9\frac{(9)^{\frac{1}{2}}}{\frac{1}{2}}+c=0 \Rightarrow c=\ldots$	Substitutes $x=9$ and $y=0$ into their integrated expression leading to a value for c. If no c at this stage M0A0 follows unless their method implies that they are correctly finding a constant of integration.	M1
	$f(x)=\frac{2}{3}x^{\frac{3}{2}}+18x^{\frac{1}{2}}-72$	There is no requirement to simplify their f(x) so accept any correct un-simplified form.	A1
			(6)
(b)	$f'(x)=\frac{x+9}{\sqrt{x}}=10 \Rightarrow x+9=10\sqrt{x}$	Sets $f'(x)=\frac{x+9}{\sqrt{x}}=10$ and multiplies by $\sqrt{x}$. The terms in x must be in the numerator. E.g. allow $\frac{x+9}{10}=\sqrt{x}$	M1
	They must be setting either the **original** $f'(x)=10$ or an equivalent <u>**correct**</u> expression = 10		
	$(\sqrt{x}-9)(\sqrt{x}-1)=0 \Rightarrow \sqrt{x}=\ldots$	Correct attempt to solve a relevant 3TQ in $\sqrt{x}$ leading to solution for $\sqrt{x}$. **Dependent on the previous M1.**	dM1
	$x=81, x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a <u>correct</u> equation.	A1, B1
			(4)
			[10]
Alternative to part (b)	$(\frac{x+9}{\sqrt{x}})^2=10^2 \Rightarrow x^2+18x+81=100x$	Sets $\frac{x+9}{\sqrt{x}}=10$, squares and multiplies by x. They must be setting either the original $f'(x)=10$ or an equivalent <u>**correct**</u> expression = 10	M1
	$(x-81)(x-1)=0 \Rightarrow x=..$	Correct attempt to solve a relevant 3TQ leading to solution for x. **Dependent on the previous M1.**	dM1
	$x=81, x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a <u>correct</u> equation.	A1, B1

Question 9

Question Number	Scheme	Marks
	$\int (8x^3 + 4)\,dx = \frac{8x^4}{4} + 4x$	M1, A1
	$= 2x^4 + 4x + c$	A1
		(3 marks)

Notes

M1 $x^n \to x^{n+1}$ so $x^3 \to x^4$ or $4 \to 4x$ or $4x^1$

A1 This is for either term with coefficient unsimplified (power must be simplified)– so $\frac{8}{4}x^4$ or $4x$ (accept $4x^1$)

A1 Fully correct simplified solution with c i.e. $2x^4 + 4x + c$ [allow $2x^4 + 4x + cx^0$]

If the answer is given as $\int 2x^4 + 4x + c$, with an integral sign – having never been seen as the fully correct simplified answer without an integral sign – then give M1A1A0 but allow anything before the = sign e.g. $y = 2x^4 + 4x + c$, $f(x) = 2x^4 + 4x + c$, $\int = 2x^4 + 4x + c$, etc….

If this answer is followed by (for example) $x^4 + 2x + k$ then treat this as isw (ignore subsequent work)
If they follow it by finding a value for c, also isw, provided correct answer with c has been seen and credited

Question 10

Question Number	Scheme	Marks
	(a) $\mathrm{f}(x)=\int\left(\frac{3}{8}x^2-10x^{-\frac{1}{2}}+1\right)\mathrm{d}x$	
	$x^n \to x^{n+1} \Rightarrow \mathrm{f}(x)=\frac{3}{8}\times\frac{x^3}{3}-10\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+x(+c)$	M1, A1, A1
	Substitute $x=4, y=25 \Rightarrow 25=8-40+4+c \Rightarrow c=$	M1
	$(\mathrm{f}(x))=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$	A1
		(5)
	(b) Sub x=4 into $\mathrm{f}'(x)=\frac{3}{8}x^2-10x^{-\frac{1}{2}}+1$	M1
	$\Rightarrow \mathrm{f}'(4)=\frac{3}{8}\times4^2-10\times4^{-\frac{1}{2}}+1$	
	$\Rightarrow \mathrm{f}'(4)=2$	A1
	Gradient of tangent $=2 \Rightarrow$ Gradient of normal is $-1/2$	dM1
	Substitute $x=4, y=25$ into line equation with their changed gradient	
	e.g. $y-25=-\frac{1}{2}(x-4)$	dM1
	$\pm k(2y+x-54)=0$ o.e. (but must have integer coefficients)	A1cso
		(5)
		(10 Marks)

Notes

(a) M1 Attempt to integrate $x^n \to x^{n+1}$

A1 Term in x^3 or term in $x^{\frac{1}{2}}$ correct, coefficient need not be simplified, no need for $+x$ nor $+c$

A1 ALL three terms correct, coefficients need not be simplified, no need for $+c$

M1 For using $x=4$, $y=25$ in their f(x) to form a linear equation in c and attempt to find c

A1 $=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$ cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be f(x) or y). Need full expression with 53

These marks need to be scored in part (a)

(b) M1 Attempt to substitute $x=4$ into $\mathrm{f}'(x)$ must be in part (b)

A1 $\mathrm{f}'(x)=2$ at $x=4$

dM1 (Dependent on first method mark in part (b)) Using $m_1\times m_2=-1$ to find the gradient of the normal from their tangent gradient (Give mark if gradient of 1 becomes -1 as they will lose accuracy)

dM1 (Dependent on first method mark in part (b)) Attempt to find the equation of the normal (not tangent). Eg use x=4, y=25 in y= '$-1/2$'x+c to find a value of c or use '$-\frac{1}{2}$' $=\frac{y-25}{x-4}$ with their adapted gradient.

A1 cso $\pm k(2y+x-54)=0$ (where k is any integer)

Question 11

Question Number	Scheme	Marks
(a)	$y = 2x^5 + \dfrac{6}{\sqrt{x}}$	
	$x^n \to x^{n-1}$	M1
	$\dfrac{dy}{dx} = 10x^4 - 3x^{-\frac{3}{2}}$ oe	A1A1
		(3)
(b)	$\int 2x^5 + \dfrac{6}{\sqrt{x}} dx$	
	$x^n \to x^{n+1}$	M1
	$= \dfrac{x^6}{3} + 12x^{\frac{1}{2}} + c$	A1 A1
		(3)
		(6 marks)

(a) M1 For $x^n \to x^{n-1}$. ie. x^4 or $x^{-\frac{3}{2}}$ or $\left(\dfrac{1}{x^{\frac{3}{2}}}\right)$ seen

A1 For $2 \times 5x^4$ **or** $6 \times -\dfrac{1}{2} x^{-\frac{3}{2}}$ (oe). (Ignore $+c$ for this mark)

A1 For simplified expression $10x^4 - 3x^{-\frac{3}{2}}$ or $10x^4 - \dfrac{3}{x^{\frac{3}{2}}}$ o.e. and no $+c$

Apply ISW here and award marks when first seen.

(b) M1 For $x^n \to x^{n+1}$. ie. x^6 or $x^{\frac{1}{2}}$ or $\left(\sqrt{x}\right)$ seen

Do not award for integrating their answer to part (a)

A1 For either $2\dfrac{x^6}{6}$ **or** $6 \times \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}$ or simplified or unsimplified equivalents

A1 For fully correct and simplified answer with $+c$.

Question 12

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}$	
	$x\sqrt{x} = x^{\frac{3}{2}}$	B1
	$x^n \to x^{n+1}$	M1
	$y = \frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	A1, A1
	Use $x = 4$, $y = 37$ to give equation in c, $37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^5 + c$	M1
	$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2	A1
	$(y) = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$	A1
		(7 marks)

B1 $x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ oe in the subsequent work.

M1 $x^n \to x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both

A1 One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}}$ or $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$.
No need for $+c$

A1 Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or a simplified correct version

M1 Substitute $x = 4$, $y = 37$ to produce an equation in c.

A1 Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 cso $y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.
e.g. $y = \frac{1}{5}(60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{1}{5}$

Question 13

Question Number	Scheme		Marks
	$y = 4x^3 - \dfrac{5}{x^2}$		
(a)	$12x^2 + \dfrac{10}{x^3}$	M1: $x^n \to x^{n-1}$ e.g. Sight of x^2 or x^{-3} or $\dfrac{1}{x^3}$	M1A1A1
		A1: $3 \times 4x^2$ or $-5 \times -2x^{-3}$ (oe) (Ignore + c for this mark)	
		A1: $12x^2 + \dfrac{10}{x^3}$ or $12x^2 + 10x^{-3}$ <u>all on one line</u> and no + c	
	Apply ISW here and award marks when first seen.		
			(3)
(b)	$x^4 + \dfrac{5}{x} + c$ or $x^4 + 5x^{-1} + c$	M1: $x^n \to x^{n+1}$. e.g. Sight of x^4 or x^{-1} or $\dfrac{1}{x^1}$ **Do <u>not</u> award for integrating their answer to part (a)**	M1A1A1
		A1: $4\dfrac{x^4}{4}$ or $-5 \times \dfrac{x^{-1}}{-1}$	
		A1: For fully correct and simplified answer with + c <u>all on one line</u>. Allow $x^4 + 5 \times \dfrac{1}{x} + c$ Allow $1x^4$ for x^4	
	Apply ISW here and award marks when first seen. Ignore spurious integral signs for all marks.		
			(3)
			(6 marks)

Question 14

Question Number	Scheme		Marks
(a)	$\mathrm{f}(x)=x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x(+c)$	M1: $x^n \to x^{n+1}$ A1: Two terms in x correct, simplification is not required in coefficients or powers A1: All terms in x correct. Simplification not required in coefficients or powers and $+c$ is not required	M1A1A1
	Sub $x=4, y=9$ into $\mathrm{f}(x) \Rightarrow c = \ldots$	M1: Sub $x=4, y=9$ into $\mathrm{f}(x)$ to obtain a value for c. If no $+c$ then M0. Use of $x=9, y=4$ is M0.	M1
	$(\mathrm{f}(x)=)x^{\frac{3}{2}}-\frac{9}{2}x^{\frac{1}{2}}+2x+2$	Accept equivalents **but must be simplified** e.g. $\mathrm{f}(x)=x^{\frac{3}{2}}-4.5\sqrt{x}+2x+2$ Must be all 'on one line' **and simplified.** Allow $x\sqrt{x}$ for $x^{\frac{3}{2}}$	A1
			(5)
(b)	Gradient of normal is $-\frac{1}{2} \Rightarrow$ Gradient of tangent $=+2$	M1: Gradient of $2y+x=0$ is $\pm\frac{1}{2}(m) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{\pm\frac{1}{2}}$ A1: Gradient of tangent $=+2$ (May be implied)	M1A1
	The A1 may be implied by $\frac{-1}{\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2}=-\frac{1}{2}$		
	$\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2=2 \Rightarrow \frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}=0$	Sets the given $\mathrm{f}'(x)$ **or their** $\mathrm{f}'(x)$ = their **changed** m and **not** their m where m has come from $2y+x=0$	M1
	$\times 4\sqrt{x} \Rightarrow 6x-9=0 \Rightarrow x=..$	$\times 4\sqrt{x}$ or equivalent **correct algebraic** processing (**allow sign/arithmetic errors only**) and attempt to solve to obtain a value for x. If $\mathrm{f}'(x)\neq 2$ they need to be solving a three term quadratic in $\sqrt{x}$ correctly and square to obtain a value for x. **Must be using the given** $\mathrm{f}'(x)$ **for this mark.**	M1
	$x=1.5$	$x=\frac{3}{2}$ (1.5) Accept equivalents e.g . $x=\frac{9}{6}$ **If any 'extra' values are not rejected, score A0.**	A1
			(5)
	Beware $\frac{-1}{\frac{3\sqrt{x}}{2}-\frac{9}{4\sqrt{x}}+2}=-\frac{1}{2} \Rightarrow \frac{-2}{3\sqrt{x}}+\frac{4\sqrt{x}}{9}-\frac{1}{2}=-\frac{1}{2}$ etc. leads to the correct answer and could score M1A1M1M0(incorrect processing)A0		
			(10 marks)

Question 15

Question Number	Scheme	Notes	Marks
	$\int (2x^4 - \frac{4}{\sqrt{x}} + 3)\mathrm{d}x$		
	$\frac{2}{5}x^5 - \frac{4}{\frac{1}{2}}x^{\frac{1}{2}} + 3x$	M1: $x^n \to x^{n+1}$. One power increased by 1 but not for just $+c$. This could be for $3 \to 3x$ or for $x^n \to x^{n+1}$ on what they think $\frac{1}{\sqrt{x}}$ is as a power of x.	M1A1A1
		A1: One of these 3 terms correct. Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}$, $-\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$, $3x^1$	
		A1: Two of these 3 terms correct. Allow un-simplified e.g. $\frac{2x^{4+1}}{4+1}$, $-\frac{4x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$, $3x^1$	
	$= \frac{2}{5}x^5 - 8x^{\frac{1}{2}} + 3x + c$	**Complete fully correct simplified expression appearing all on one line with constant.** Allow 0.4 for $\frac{2}{5}$. Do not allow $3x^1$ for $3x$. Allow $\sqrt{x}$ or $x^{0.5}$ for $x^{\frac{1}{2}}$	A1
	Ignore any spurious integral signs and ignore subsequent working following a fully correct answer.		
			[4]
			4 marks

Question 16

Question Number	Scheme	Notes	Marks
	$y=3x^2+6x^{\frac{1}{3}}+\dfrac{2x^3-7}{3\sqrt{x}}$		
	$\dfrac{2x^3-7}{3\sqrt{x}}=\dfrac{2x^3}{3\sqrt{x}}-\dfrac{7}{3\sqrt{x}}=\dfrac{2}{3}x^{\frac{5}{2}}-\dfrac{7}{3}x^{-\frac{1}{2}}$	Attempts to split the fraction into 2 terms and obtains either $\alpha x^{\frac{5}{2}}$ or $\beta x^{-\frac{1}{2}}$. This may be implied by a correct power of x in their differentiation of one of these terms. But beware of $\beta x^{-\frac{1}{2}}$ coming from $\dfrac{2x^3-7}{3\sqrt{x}}=2x^3-7+3x^{-\frac{1}{2}}$	M1
	$x^n \to x^{n-1}$	Differentiates by reducing power by one for any of their powers of x	M1
	$\left(\dfrac{dy}{dx}=\right)\ 6x+2x^{-\frac{2}{3}}+\dfrac{5}{3}x^{\frac{3}{2}}+\dfrac{7}{6}x^{-\frac{3}{2}}$	A1: $6x$ Do not accept $6x^1$. Depends on second M mark only. Award when first seen and isw. A1: $2x^{-\frac{2}{3}}$. Must be simplified so do not accept e.g. $\dfrac{2}{1}x^{-\frac{2}{3}}$ but allow $\dfrac{2}{\sqrt[3]{x^2}}$. Depends on second M mark only. Award when first seen and isw. A1: $\dfrac{5}{3}x^{\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{2}{3}x^{1.5}$ or e.g. $\dfrac{5}{3}\sqrt{x^3}$. Award when first seen and isw. A1: $\dfrac{7}{6}x^{-\frac{3}{2}}$. Must be simplified but allow e.g. $1\dfrac{1}{6}x^{-1\frac{1}{2}}$ or e.g. $\dfrac{7}{6\sqrt{x^3}}$. Award when first seen and isw.	A1A1A1A1
	In an otherwise <u>fully correct solution</u>, penalise the presence of + c by deducting the final A1		
			[6]
	Use of Quotient Rule: First M1 and final A1A1 (Other marks as above)		
	$\dfrac{d\left(\dfrac{2x^3-7}{3\sqrt{x}}\right)}{dx}=\dfrac{3\sqrt{x}\left(6x^2\right)-\left(2x^3-7\right)\frac{3}{2}x^{-\frac{1}{2}}}{\left(3\sqrt{x}\right)^2}$	Uses <u>correct</u> quotient rule	M1
	$=\dfrac{10x^{\frac{5}{2}}+7x^{-\frac{1}{2}}}{6x}$	A1: Correct first term of numerator and correct denominator A1: All correct as simplified as shown	A1A1
	So $\dfrac{dy}{dx}=6x+2x^{-\frac{2}{3}}+\dfrac{10x^{\frac{5}{2}}+7x^{-\frac{1}{2}}}{6x}$ scores full marks		
			6 marks

Question 17

Question Number	Scheme		Marks
	$\int\left(2x^5 - \frac{1}{4}x^{-3} - 5\right)dx$		
	Ignore any spurious integral signs throughout		
	$x^n \to x^{n+1}$	Raises any of their powers by 1. E.g. $x^5 \to x^6$ or $x^{-3} \to x^{-2}$ or $k \to kx$ or $x^{\text{their}\,n} \to x^{\text{their}\,n+1}$. Allow the powers to be un-simplified e.g. $x^5 \to x^{5+1}$ or $x^{-3} \to x^{-3+1}$ or $kx^0 \to kx^{0+1}$.	M1
	$2\times\frac{x^{5+1}}{6}$ or $-\frac{1}{4}\times\frac{x^{-3+1}}{-2}$	Any one of the first two terms correct **simplified or un-simplified**.	A1
	Two of: $\frac{1}{3}x^6,\ \frac{1}{8}x^{-2},\ -5x$	Any two correct **simplified** terms. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Accept 0.125 for $\frac{1}{8}$ but $\frac{1}{3}$ would clearly need to be identified as 0.3 recurring.	A1
	$\frac{1}{3}x^6 + \frac{1}{8}x^{-2} - 5x + c$	All correct and simplified and including $+ c$ all on one line. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not x^1 for x. Apply isw here.	A1
			(4 marks)

Question 18

Question Number	Scheme		Marks
(a)	$f'(4) = 30 + \dfrac{6 - 5\times 4^2}{\sqrt{4}}$	Attempts to substitutes $x = 4$ into $f'(x) = 30 + \dfrac{6-5x^2}{\sqrt{x}}$ or their algebraically manipulated $f'(x)$	M1
	$f'(4) = -7$	Gradient = -7	A1
	$y-(-8) = "-7"\times(x-4)$ or $y = "-7"x + c \Rightarrow -8 = "-7"\times 4 + c$ $\Rightarrow c = ...$	Attempts an equation of a tangent using their numeric $f'(4)$ which has come from substituting $x = 4$ into the given $f'(x)$ or their algebraically manipulated $f'(x)$ and $(4, -8)$ with the 4 and -8 correctly placed. If using $y = mx + c$, must reach as far as $c = ...$	M1
	$y = -7x + 20$	Cao. Allow $y = 20 - 7x$ and allow the "$y =$" to become "detached" but it must be present at some stage. E.g. $y = ...$ $= -7x + 20$	A1
			(4)

(b)	**Allow the marks in (b) to score in (a) i.e. <u>mark (a) and (b) together</u>**		
	$\Rightarrow f(x) = 30x + 6\dfrac{x^{\frac{1}{2}}}{0.5} - 5\dfrac{x^{\frac{5}{2}}}{2.5} (+c)$	M1: $30 \to 30x$ or $\dfrac{6}{\sqrt{x}} \to \alpha x^{\frac{1}{2}}$ or $-\dfrac{5x^2}{\sqrt{x}} \to \beta x^{\frac{5}{2}}$ (these cases only)	M1A1A1
		A1: Any 2 correct terms which can be simplified or un-simplified. This includes the powers – so allow $-\frac{1}{2}+1$ for $\frac{1}{2}$ and allow $\frac{3}{2}+1$ for $\frac{5}{2}$ (With or without $+ c$)	
		A1: All 3 terms correct which can be simplified or un-simplified. (With or without $+ c$)	
	Ignore any spurious integral signs		
	$x = 4, f(x) = -8 \Rightarrow$ $-8 = 120 + 24 - 64 + c \Rightarrow c = ...$	Substitutes $x = 4, f(x) = -8$ into their f(x) (not $f'(x)$) i.e. a changed $f'(x)$ containing $+c$ and rearranges to obtain a value or numerical expression for c.	M1
	$\Rightarrow (f(x) =) 30x + 12x^{\frac{1}{2}} - 2x^{\frac{5}{2}} - 88$	Cao and cso (Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$ and e.g. $\sqrt{x^5}$ or $x^2\sqrt{x}$ for $x^{\frac{5}{2}}$). Isw here so as soon as you see the correct answer, award this mark. Note that the "f(x) =" is not needed.	A1
			(5)
			(9 marks)