

Area and Definite Integrals - Edexcel Past Exam Questions 2 MARK SCHEME

Question 1

Question number	Scheme	Marks																
(a)	<table><tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td><td>3.5</td><td>4</td></tr><tr><td>y</td><td>16.5</td><td>7.361</td><td>4</td><td>2.31</td><td>1.278</td><td>0.556</td><td>0</td></tr></table>	x	1	1.5	2	2.5	3	3.5	4	y	16.5	7.361	4	2.31	1.278	0.556	0	B1, B1 (2)
x	1	1.5	2	2.5	3	3.5	4											
y	16.5	7.361	4	2.31	1.278	0.556	0											
(b)	$\frac{1}{2} \times 0.5, \{(16.5 + 0) + 2(7.361 + 4 + 2.31 + 1.278 + 0.556)\}$ = 11.88 (or answers listed below in note)	B1, M1A1ft A1 (4)																
(c)	$\int_1^4 \frac{16}{x^2} - \frac{x}{2} + 1 \, dx = \left[-\frac{16}{x} - \frac{x^2}{4} + x \right]_1^4$ $= [-4 - 4 + 4] - [-16 - \frac{1}{4} + 1]$ $= 11\frac{1}{4} \text{ or equivalent}$	M1 A1 A1 dM1 A1 (5) 11																
Notes	<p>(a) B1 for 4 or any correct equivalent e.g. 4.000 B1 for 2.31 or 2.310</p> <p>(b) B1: Need 0.25 or $\frac{1}{2}$ of 0.5</p> <p>M1: requires first bracket to contain first y value plus last y value (0 may be omitted or be at end) and second bracket to include no additional y values from those in the scheme. They may however omit one value as a slip.</p> <p>N.B. Special Case - Bracketing mistake</p> <p>$\frac{1}{2} \times 0.5(16.5 + 0) + 2(7.361 + 4 + 2.31 + 1.278 + 0.556)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks)</p> <p>A1ft: This should be correct but ft their 4 and 2.31</p> <p>A1: Accept 11.8775 or 11.878 or 11.88 only</p> <p>(c) M1 Attempt to integrate ie power increased by 1 or 1 becomes x ,</p> <p>A1 two correct terms, next A1 all three correct unsimplified (ignore +c)</p> <p>(Allow $-16x^{-1} - 0.25x^2 + 1x$ or equivalent)</p> <p>dM1 (This cannot be earned if previous M mark has not been awarded) Uses limits 4 and 1 in their integrated expression and subtracts (either way round)</p> <p>A1 11.25 or $11\frac{1}{4}$ or $45/4$ or equivalent (penalise negative final answer here)</p>																	
Alternative Method for (b)	Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2}h(a+b)$ used 5 or 6 times (and A1ft all correct for their “4” and “2.31”) final A1 for 11.88 etc. as before																	
	In part (b) Need to use trapezium rule – answer only (with no working) is 0/4 -any doubts send to review In part (c) need to see integration																	



Question 2

(a)	<p>Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic</p> <p>Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give $x =$</p> <p>Obtains $x = 2$, $x = 9$ (may be on diagram or in part (b) in limits)</p> <p>Substitutes their x into a given equation to give $y =$ (may be on diagram)</p> <p>$y = 8, y = 1$</p>	<p>Or puts $y = 10(10 - y) - (10 - y)^2 - 8$</p> <p>and rearranges to give three term quadratic</p> <p>Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give $y =$</p> <p>Obtains $y = 8, y = 1$ (may be on diagram)</p> <p>Substitutes their y into a given equation to give $x =$ (may be on diagram or in part (b))</p> <p>$x = 2, x = 9$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1 A1</p> <p>dM1</p> <p>B1</p> <p>M1A1</p> <p>cao</p> <p>(7)</p> <p>12 marks</p>
(b)	$\int (10x - x^2 - 8) dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \{ + c \}$ $\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x \right]_2^9 = (\dots) - (\dots)$ $= 90 - \frac{4}{3} = 88\frac{2}{3} \text{ or } \frac{266}{3}$ <p>Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$</p> <p>So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6}$ or $\frac{343}{6}$</p>		

Question 3

Question Number	Scheme	Marks
(a)	Seeing -4 and 2.	B1 (1)
(b)	$x(x+4)(x-2) = x^3 + 2x^2 - 8x \quad \text{or} \quad x^3 - 2x^2 + 4x^2 - 8x \text{ (without simplifying)}$ $\int (x^3 + 2x^2 - 8x) dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \{+c\} \quad \text{or} \quad \frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^3}{3} - \frac{8x^2}{2} \{+c\}$ $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \right]_{-4}^0 = (0) - \left(64 - \frac{128}{3} - 64 \right) \quad \text{or} \quad \left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \right]_0^2 = \left(4 + \frac{16}{3} - 16 \right) - (0)$ <p>One integral = $\pm 42\frac{2}{3}$ (42.6 or awrt 42.7) or other integral = $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7)</p> <p>Hence Area = "their $42\frac{2}{3}$" + "their $6\frac{2}{3}$" or Area = "their $42\frac{2}{3}$" - "their $6\frac{2}{3}$"</p> <p>= $49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)</p> <p>(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)</p>	B1 M1A1ft dM1 A1 dM1 A1 (7)
Notes for Question		
(a)	B1: Need both -4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0,2) or $A = -4, B = 2$ or indeed any indication of -4 and 2 – check graph also	
(b)	B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here) M1: Tries to integrate their expansion with $x^n \rightarrow x^{n+1}$ for at least one of the terms A1ft: completely correct integral following through from their CUBIC expansion (if only quadratic or quartic this is A0) dM1: (dependent on previous M) substituting EITHER -a and 0 and subtracting either way round OR similarly for 0 and b. If their limits -a and b are used in ONE integral, apply the Special Case below. A1: Obtain either $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7) from the integral from -4 to 0 or $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) from the integral from 0 to 2; NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ or $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0. dM1 (depends on first method mark) Correct method to obtain shaded area so adds two positive numbers (areas) together or uses their positive value minus their negative value, obtained from two separate definite integrals. A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with no errors seen. For full marks on this question there must be two definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen. (Trapezium rule gets no marks after first two B marks)	
(b)	Special Case: one integral only from -a to b: B1M1A1 available as before, then $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \right]_{-4}^2 = \left(4 + \frac{16}{3} - 16 \right) - \left(64 - \frac{128}{3} - 64 \right) = -6\frac{2}{3} + 42\frac{2}{3} = \dots$ dM1 for correct use of their limits -a and b and subtracting either way round. A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)	

Question 4

Question Number	Scheme	Marks
(a)	$x^2 + 2x + 2 = 10 \Rightarrow x^2 + 2x - 8 = 0$ (so $(x+4)(x-2) = 0$) $\Rightarrow x = \dots\dots$	M1
	$x = -4, 2$	A1 (2)
(b) Way 1	$\int (x^2 + 2x + 2) dx = \frac{x^3}{3} + \frac{2x^2}{2} + 2x (+C)$	M1A1A1
	$\left[\frac{x^3}{3} + \frac{2x^2}{2} + 2x \right]_{-4}^{2} = \left(\frac{8}{3} + \frac{8}{2} + 4 \right) - \left(-\frac{64}{3} + \frac{32}{2} - 8 \right) (= 24)$	M1
	Rectangle: $10 \times (2 - -4) = 60$	B1 cao
	$R = "60" - "24"$	M1
	$= 36$	A1 (7)
		Total 9
(b) Way 2	$\int (8 - x^2 - 2x) dx = 8x - \frac{x^3}{3} - \frac{2x^2}{2} (+C)$	M1 A1ft A1
	$\left[8x - \frac{x^3}{3} - \frac{2x^2}{2} \right]_{-4}^{2} = \left(16 - \frac{8}{3} - 4 \right) - \left(-32 + \frac{64}{3} - 16 \right) = (9.3 - (-26.7))$	M1
	Implied by final answer of 36 after correct work	B1
	$10 - (x^2 + 2x + 2) = 8 - x^2 - 2x, = 36$	M1, A1
Notes for Question		
(a)	M1 Set the curve equation equal to 10 and collect terms. Solves quadratic to $x = \dots$	
	A1 cao : Both values correct – allow $A = -4, B = 2$	
(b)	M1: One correct integration	
	A1: Two correct integrations(ft slips subtracting in Way 2)	
	A1: All 3 terms correct (penalise subtraction errors here in Way 2)	
	M1: Substitute their limits from (a) into the integrated function and subtract (either way round)	
	B1: Way 1: Find area under the line by integration or area of rectangle – should be 60 here (no follow through)	
	Way 2: (implied by final correct answer in second method)	
	M1: Subtract one area from the other (implied by subtraction of functions in second method)- award even after differentiation	
	A1: Must be 36 not -36.	
	<i>Special case 1:</i> Combines both methods. Uses Way 2 integration, but continues after reaching “36” to subtract “36” from rectangle giving answer as “24” This loses final M1 A1	
	<i>Special case 2:</i> Integrates $(x^2 + 2x - 8)$ between limits -4 and 2 to get -36 and then changes sign and obtains 36. Do not award final A mark – so M1A1A1M1B1M1A0	
	If the answer is left as -36, then M1A1A1M1B0M1A0	
	N.B. Allow full marks for modulus used earlier in working e.g. $\left \int_{-4}^2 x^2 + 2x - 2 dx - \int_{-4}^2 10 dx \right $	

Question 5

Question Number	Scheme		Marks
	$\left\{ \int \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx \right\} = \frac{x^4}{6(4)} + \frac{x^{-1}}{(3)(-1)}$	M1: $x^n \rightarrow x^{n+1}$	M1A1A1
		A1: At least one of either $\frac{x^4}{6(4)}$ or $\frac{x^{-1}}{(3)(-1)}$.	
		A1: $\frac{x^4}{6(4)} + \frac{x^{-1}}{(3)(-1)}$ or equivalent.	
		e.g. $\frac{x^4}{6} + \frac{x^{-1}}{-1}$ (they will lose the final mark if they cannot deal with this correctly)	
	<p>Note that some candidates may change the function prior to integrating e.g.</p> $\int \frac{x^3}{6} + \frac{1}{3x^2} dx = \int 3x^5 + 6 dx$ <p>in which case allow the M1 if $x^n \rightarrow x^{n+1}$ for their changed function and allow the M1 for limits if scored</p>		
	$\left\{ \int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx \right\} = \left(\frac{(\sqrt{3})^4}{24} + \frac{(\sqrt{3})^{-1}}{-1(3)} \right) - \left(\frac{(1)^4}{24} + \frac{(1)^{-1}}{-1(3)} \right)$		dM1
	<p>2nd dM1: For using limits of $\sqrt{3}$ and 1 on an integrated expression and subtracting the correct way round. The 2nd M1 is dependent on the 1st M1 being awarded.</p>		
	$= \left(\frac{9}{24} - \frac{1}{3\sqrt{3}} \right) - \left(\frac{1}{24} - \frac{1}{3} \right) = \frac{2}{3} - \frac{1}{9}\sqrt{3}$	$\frac{2}{3} - \frac{1}{9}\sqrt{3}$ or $a = \frac{2}{3}$ and $b = -\frac{1}{9}$. Allow equivalent fractions for a and/or b and 0.6 recurring and/or 0.1 recurring but do not allow $\frac{6-\sqrt{3}}{9}$	A1cso
This final mark is cao and cso – there must have been no previous errors			
			Total 5
Common Errors (Usually 3 out of 5)			
	$\left\{ \int \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx \right\} = \int \left(\frac{x^3}{6} + 3x^{-2} \right) dx = \frac{x^4}{6(4)} + \frac{3x^{-1}}{(-1)}$ $\left\{ \int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx \right\} = \left(\frac{(\sqrt{3})^4}{24} + \frac{3(\sqrt{3})^{-1}}{-1} \right) - \left(\frac{(1)^4}{24} + \frac{3(1)^{-1}}{-1} \right)$ $= \left(\frac{9}{24} - \frac{3}{\sqrt{3}} \right) - \left(\frac{1}{24} + \frac{3}{-1} \right) = \frac{10}{3} - \sqrt{3}$		M1A1A0 dM1 A0
	$\left\{ \int \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx \right\} = \int \left(\frac{x^3}{6} + (3x)^{-2} \right) dx = \frac{x^4}{6(4)} + \frac{(3x)^{-1}}{(-1)}$ $\left\{ \int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx \right\} = \left(\frac{(\sqrt{3})^4}{24} + \frac{(3\sqrt{3})^{-1}}{-1} \right) - \left(\frac{(1)^4}{24} + \frac{(3 \times 1)^{-1}}{-1} \right)$ $= \left(\frac{9}{24} - \frac{1}{3\sqrt{3}} \right) - \left(\frac{1}{24} - \frac{1}{3} \right) = \frac{2}{3} - \frac{\sqrt{3}}{9}$		M1A1A0 dM1 A0
Note this is the correct answer but follows incorrect work.			

Question 6

Question Number	Scheme		Marks
	$\int \left(\frac{1}{8}x^3 + \frac{3}{4}x^2 \right) dx = \frac{x^4}{32} + \frac{x^3}{4} \{+c\}$	M1: $x^n \rightarrow x^{n+1}$ on either term	M1A1
		A1: $\frac{x^4}{32} + \frac{x^3}{4}$. Any correct simplified or un-simplified form. (+ c not required)	
	$\left[\frac{x^4}{32} + \frac{x^3}{4} \right]_{-4}^2 = \left(\frac{16}{32} + \frac{8}{4} \right) - \left(\frac{256}{32} + \frac{(-64)}{4} \right)$ <p style="text-align: center;">or</p> $\left[\frac{x^4}{32} + \frac{x^3}{4} \right]_{-4}^0 = (0) - \left(\frac{(-4)^4}{32} + \frac{(-4)^3}{4} \right) \text{ added to } \left[\frac{x^4}{32} + \frac{x^3}{4} \right]_0^2 = \left(\frac{(2)^4}{32} + \frac{(2)^3}{4} \right) - (0)$		dM1
	Substitutes limits of 2 and -4 into an “integrated function” and subtracts either way round. Or substitutes limits of 0 and -4 and 2 and 0 into an “integrated function” and subtracts either way round and adds the two results.		
	$= \frac{21}{2}$	$\frac{21}{2}$ or 10.5	A1
	{At $x = -4$, $y = -8 + 12 = 4$ or at $x = 2$, $y = 1 + 3 = 4$ }		
	<p style="text-align: center;">Area of Rectangle = $6 \times 4 = 24$</p> <p style="text-align: center;">or</p> <p style="text-align: center;">Area of Rectangles = $4 \times 4 = 16$ and $2 \times 4 = 8$</p>		M1
	<p style="text-align: center;">Evidence of $(4 - -2) \times$ their y_{-4} or $(4 - -2) \times$ their y_2</p> <p style="text-align: center;">or</p> <p style="text-align: center;">Evidence of $4 \times$ their y_{-4} and $2 \times$ their y_2</p>		
	<p style="text-align: center;">So, $\text{area(R)} = 24 - \frac{21}{2} = \frac{27}{2}$</p>	dddM1: Area rectangle – integrated answer. Dependent on all previous method marks and requires: Rectangle > integration > 0	dddM1A1
		A1: $\frac{27}{2}$ or 13.5	
			[7]
			Total 7

<u>Alternative:</u>		
$\pm \int "their4" - \left(\frac{1}{8}x^3 + \frac{3}{4}x^2 \right) dx$	Line – curve. Condone missing brackets and allow either way round.	4 th M1
$= 4x - \frac{x^4}{32} - \frac{x^3}{4} \{+ c\}$	M1: $x^n \rightarrow x^{n+1}$ on either curve term	1 st M1, 1 st A1ft
	A1ft: " $-\frac{x^4}{32} - \frac{x^3}{4}$." Any correct simplified or un-simplified form of their curve terms, follow through sign errors. (+ c not required)	
$[]_{-4}^2 = \left(8 - \frac{16}{32} - \frac{8}{4} \right) - \left(-16 - \frac{256}{32} - \frac{(-64)}{4} \right)$	2 nd M1 Substitutes limits of 2 and -4 into an "integrated curve" and subtracts either way round.	2 nd M1, 3 rd M1 2 nd A1
	3 rd M1 for $\pm ("8" - "-16")$ Substitutes limits into the 'line part' and subtracts either way round.	
	2 nd A1 for correct \pm (underlined expression). Now needs to be correct but allow \pm the correct expression.	
$= \frac{27}{2}$	A1: $\frac{27}{2}$ or 13.5	3 rd A1
If the final answer is -13.5 you can withhold the final A1 If -13.5 then "becomes" +13.5 allow the A1		



Question 7

Question Number	Scheme	Marks
(a)	May mark (a) and (b) together Expands to give $10x^{\frac{3}{2}} - 20x$ Integrates to give $\frac{10}{\frac{5}{2}}x^{\frac{5}{2}} + \frac{-20x^2}{2} (+c)$ Simplifies to $4x^{\frac{5}{2}} - 10x^2 (+c)$	B1 M1 A1ft A1cao (4)
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted) Use limits 4 and 9 either way round on their integrated function Obtains either ± 32 or ± 194 needs at least one of the previous M marks for this to be awarded (So area = $\left \int_0^4 y dx \right + \int_4^9 y dx$) i.e. $32 + 194 = 226$	M1 dM1 A1 ddM1, A1 (5) [9]
Notes		
<p>(a) B1: Expands the bracket correctly M1: Correct integration process on at least one term after attempt at multiplication. (Follow correct expansion or one slip resulting in $10x^k - 20x$ where k may be $\frac{1}{2}$ or $\frac{5}{2}$ or resulting in $10x^{\frac{3}{2}} - Bx$, where B may be 2 or 5) So $x^{\frac{3}{2}} \rightarrow \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or $x^{\frac{5}{2}} \rightarrow \frac{x^{\frac{7}{2}}}{\frac{7}{2}}$ or $x^{\frac{3}{2}} \rightarrow \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ and/or $x \rightarrow \frac{x^2}{2}$. A1: Correct unsimplified follow through for both terms of their integration. Does not need $(+c)$ A1: Must be simplified and correct– allow answer in scheme or $4x^{\frac{5}{2}} - 10x^2$. Does not need $(+c)$</p> <p>(b) M1: (does not depend on first method mark) Attempt to substitute 4 into their integral (however obtained but must not be differentiated) or seeing their evaluated number (usually 32) is enough – do not need to see minus zero. dM1: (depends on first method mark in (a)) Attempt to subtract either way round using the limits 4 and 9 $A \times 9^{\frac{5}{2}} - B \times 9^2$ with $A \times 4^{\frac{5}{2}} - B \times 4^2$ is enough – or seeing $162 - (-32)$ {but not $162 - 32$} A1: At least one of the values (32 and 194) correct (needs just one of the two previous M marks in (b)) or may see $162 + 32 + 32$ or $162 + 64$ or may be implied by correct final answer if not evaluated until last line of working ddM1: Adds 32 and 194 (may see $162 + 32 + 32$ or may be implied by correct final answer if not evaluated until last line of working). This depends on everything being correct to this point. A1cao: Final answer of 226 not (-226)</p> <p>Common errors: $4 \times 4^{\frac{5}{2}} - 10 \times 4^2 + 4 \times 9^{\frac{5}{2}} - 10 \times 9^2 = 4 \times 4^{\frac{5}{2}} - 10 \times 4^2 = \pm 162$ obtains M1 M1 A0 (neither 32 nor 194 seen and final answer incorrect) then M0 A0 so 2/5 Uses correct limits to obtain $-32 + 162 + 32 = +/-162$ is M1 M1 A1 (32 seen) M0 A0 so 3/5</p> <p>Special case: In part (b) Uses limits 9 and 0 = $972 - 810 - 0 = 162$ M0 M1 A0 M0A0 scores 1/5 This also applies if 4 never seen.</p>		

Question 8

Question Number	Scheme		Marks
(a)	$\left\{ \int \left(3x - x^{\frac{3}{2}} \right) dx \right\} = \frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2} \right)} \{ + c \}$	Either	M1
		$3x \rightarrow \pm \lambda x^2 \text{ or } x^{\frac{3}{2}} \rightarrow \pm \mu x^{\frac{5}{2}}, \lambda, \mu \neq 0$	
		At least one term correctly integrated	A1
		Both terms correctly integrated	A1
[3]			
(b)	$0 = 3x - x^{\frac{3}{2}} \Rightarrow 0 = 3 - x^{\frac{1}{2}} \text{ or } 0 = x \left(3 - x^{\frac{1}{2}} \right) \Rightarrow x = \dots$	Sets $y = 0$, in order to find	M1
		the correct $x^{\frac{1}{2}} = 3$ or $x = 9$	
	$\left\{ \text{Area}(S) = \left[\frac{3x^2}{2} - \frac{2}{5}x^{\frac{5}{2}} \right]_0^9 \right\}$		
	$= \left(\frac{3(9)^2}{2} - \left(\frac{2}{5} \right)(9)^{\frac{5}{2}} \right) - \{0\}$	Applies the limit 9 on an integrated function with no wrong lower limit.	ddM1
	$\left\{ = \left(\frac{243}{2} - \frac{486}{5} \right) - \{0\} \right\} = \frac{243}{10} \text{ or } 24.3$	$\frac{243}{10} \text{ or } 24.3$	A1 oe
[3]			
6			
Question Notes			
(a)	M1	Either $3x \rightarrow \pm \lambda x^2$ or $x^{\frac{3}{2}} \rightarrow \pm \mu x^{\frac{5}{2}}, \lambda, \mu \neq 0$	
	1 st A1	At least one term correctly integrated. Can be simplified or un-simplified but power must be simplified. Then isw.	
	2 nd A1	Both terms correctly integrated. Can be un-simplified (as in the scheme) but the $n+1$ in each denominator and power should be a single number. (e.g. 2 – not 1+1) Ignore subsequent work if there are errors simplifying. Ignore the omission of “+ c”. Ignore integral signs in their answer.	
	(b)	1 st M1	Sets $y = 0$, and reaches the correct $x^{\frac{1}{2}} = 3$ or $x = 9$ (isw if $x^{\frac{1}{2}} = 3$ is followed by $x = \sqrt{3}$)
			Just seeing $x = \sqrt{3}$ without the correct $x^{\frac{1}{2}} = 3$ gains M0. May just see $x = 9$.
			Use of trapezium rule to find area is M0A0 as hence implies integration needed.
	ddM1	This mark is dependent on the two previous method marks and needs both to have been awarded. Sees the limit 9 substituted in an integrated function. (Do not follow through their value of x) Do not need to see MINUS 0 but if another value is used as lower limit – this is M0. This mark may be implied by 9 in the limit and a correct answer.	
	A1	$\frac{243}{10} \text{ or } 24.3$	
	Common Error	Common Error $0 = 3x - x^{\frac{3}{2}} \Rightarrow x^{\frac{1}{2}} = 3$ so $x = \sqrt{3}$ Then uses limit $\sqrt{3}$ etc gains M1 M0 A0 so 1/3	

Question 9

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 12x^2 + 18x - 30$	M1
(b) Way 1	<div> <div>Either</div> <div>Substitute $x = 1$ to give $\frac{dy}{dx} = 12 + 18 - 30 = 0$</div> <div>So turning point (all correct work so far)</div> </div>	A1
	<div> <div>Or</div> <div>Solve $\frac{dy}{dx} = 12x^2 + 18x - 30 = 0$ to give $x =$</div> <div>Deduce $x = 1$ from correct work</div> </div>	A1cso (3)
	When $x = 1$, $y = 4 + 9 - 30 - 8 = -25$	B1
	Area of triangle $ABP = \frac{1}{2} \times 1 \times 25 = 12.5$ (Where P is at $(1, 0)$)	B1
	Way 1: $\int (4x^3 + 9x^2 - 30x - 8)dx = x^4 + \frac{9}{3}x^3 - \frac{30x^2}{2} - 8x \{+ c\}$ or $x^4 + 3x^3 - 15x^2 - 8x \{+ c\}$	M1A1
	$\left[x^4 + 3x^3 - 15x^2 - 8x \right]_{-4}^1 = (1 + 3 - 15 - 8) - \left(\left(-\frac{1}{4} \right)^4 + 3 \left(-\frac{1}{4} \right)^3 - 15 \left(-\frac{1}{4} \right)^2 - 8 \left(-\frac{1}{4} \right) \right)$	dM1
	$= (-19) - \frac{261}{256} \text{ or } -19 - 1.02$	
	So Area = "their 12.5" + "their $20 \frac{5}{256}$ " or "12.5" + "20.02" or "12.5" + "their $\frac{5125}{256}$ "	ddM1
	= 32.52 (NOT -32.52)	A1 (7)
	Less efficient alternative methods for first two marks in part (b) with Way 1 or 2	[10]
	For first mark: Finding equation of the line AB as $y = 25x - 50$ as this implies the -25	B1
	For second mark: Integrating to find triangle area	
	$\int_1^2 (25x - 50)dx = \left[\frac{25}{2}x^2 - 50x \right]_1^2 = -50 + 37.5 = -12.5$	so area is 12.5
	Then mark as before if they use Method in original scheme	B1

<p>(b) Way 2</p>	<p>Way 2: Those who use area for original curve between -1/4 and 2 and subtract area between line and curve between 1 and 2 have a correct (long) method .</p> <p>The first B1 (if $y=-25$ is not seen) is for equation of straight line $y = 25x - 50$</p> <p>The second B1 may be implied by final answer correct, or 4.5 seen for area of "segment shaped" region between line and curve, or by area between line and axis/triangle found as 12.5</p> $\int (4x^3 + 9x^2 - 55x + 42) dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x \{+ c\} \quad (\text{or integration as in Way 1})$ <p>The dM1 is for correct use of the different correct limits for each of the two areas: i.e.</p> $\left[x^4 + 3x^3 - 15x^2 - 8x \right]_{-\frac{1}{4}}^2 = (16 + 24 - 60 - 16) - \left(\left(-\frac{1}{4} \right)^4 + 3 \left(-\frac{1}{4} \right)^3 - 15 \left(-\frac{1}{4} \right)^2 - 8 \left(-\frac{1}{4} \right) \right)$ <p>And $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x \right]_1^2 = 16 + 24 - 110 + 84 - (1 + 3 - 27.5 + 42)$</p> <p>So Area = their $\left[x^4 + 3x^3 - 15x^2 - 8x \right]_{-\frac{1}{4}}^2$ minus their $\left[x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x \right]_1^2$</p> <p>i.e. "their 37.0195" - "their 4.5" (with both sets of limits correct for the integral)</p> <p>Reaching = 32.52 (NOT - 32.52)</p> <p>See over for special case with wrong limits</p> <p>NB: Those who attempt curve - line wrongly with limits -1/4 to 2 may earn M1A1 for correct integration of their cubic. Usually e.g.</p> $\int (4x^3 + 9x^2 - 55x + 42) dx = x^4 + \frac{9}{3}x^3 - \frac{55x^2}{2} + 42x \{+ c\}$ <p>(They will not earn any of the last 3 marks)</p> <p>They may also get first B1 mark for the correct equation of the straight line (usually seen but may be implied by correct line - curve equation) and second B1 if they also use limits 1 and 2 to obtain 4.5 (or find the triangle area 12.5).</p>	<p>B1</p> <p>B1</p> <p>M1A1</p> <p>dM1</p> <p>ddM1</p> <p>A1</p> <p>M1A1</p>
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Notes

- (a) M1: Attempt at differentiation - all powers reduced by 1 with $8 \rightarrow 0$.
 A1: the derivative must be correct and uses derivative = 0 to find x or substitutes $x = 1$ to give 0. Ignore any reference to the other root ($-5/2$) for this mark.
 A1cso: obtains $x = 1$ from correct work, or deduces turning point (if substitution used – may be implied by a preamble e.g. $dy/dx = 0$ at T.P.)
 N.B. If their factorisation or their second root is incorrect then award A0cso.
 If however their factorisation/roots are correct, it is not necessary for them to comment that -2.5 is outside the range given.
- (b) **Way 1:**
 B1: Obtains $y = -25$ when $x = 1$ (may be seen anywhere – even in (a)) or finds correct equation of line is $y = 25x - 50$
 B1: Obtains area of triangle = 12.5 (may be seen anywhere). Allow -12.5 . Accept $\frac{1}{2} \times 1 \times 25$
 M1: Attempt at integration of cubic; two correct terms for their integration. No limits needed
 A1: completely correct integral for the cubic (**may be unsimplified**)
 dM1: We are looking for the start of a correct method here (dependent on previous M). It is for substituting 1 and $-1/4$ and subtracting. May use 2 and $-1/4$ and also 2 and 1 **AND subtract** (which is equivalent)
 ddM1 (depends on both method marks) **Correct method to obtain shaded area so adds two positive numbers (areas) together – one is area of triangle, the other is area of region obtained from integration of correct function with correct limits (may add two negatives then makes positive)**
Way 2: This is a long method and needs to be a correct method
 B1: Finds $y = -25$ at $x = 1$, or correct equation of line is $y = 25x - 50$
 B1: May be implied where WAY 2 is used and final correct answer obtained so award of final A1 results in the award of this B1. It may also be implied by correct integration of line equation or of curve minus line expression between limits 1 and 2. So if only slip is final subtraction (giving final A0, this mark may still be awarded) So may be implied by 4.5 seen for area of “segment shaped” region between line and curve.
 M1: Attempt at integration of given cubic or after attempt at subtracting their line equation (no limits needed). Two correct terms needed
 A1: Completely correct integral for their cubic (**may be unsimplified**) – **may have wrong coefficients of x and wrong constant term through errors in subtraction**
 dM1: Use **limits for original curve between $-1/4$ and 2 and use limits of 1 and 2 for area between line and curve**– needs completely correct limits– see scheme- this is dependent on two integrations
 ddM1: (depends on both method marks) Subtracts “their 37.0195” – “their 4.5” Needs consistency of signs.
 A1: 32.52 or awrt 32.52 e.g. $32\frac{133}{256}$ **NB: This correct answer implies the second B mark**
 (Trapezium rule gets no marks after first two B marks) The first two B marks may be given wherever seen. The integration of a cubic gives the following M1 and correct integration of their cubic

$$\int (4x^3 + 9x^2 + Ax + B) dx = x^4 + \frac{9}{3}x^3 + \frac{Ax^2}{2} + Bx \{+ c\}$$
 gives the A1