Solving Equations using Logarithms - Edexcel Past Exam Questions 2 MARK SCHEME

Question 1

Question number	Scheme	Marks
(a)	$\log_3 3x^2 = \log_3 3 + \log_3 x^2$ or $\log y - \log x^2 = \log 3$ or $\log y - \log 3 = \log x^2$	B1
	$\log_3 x^2 = 2\log_3 x$	B1
	Using $\log_3 3 = 1$	B1 (3)
(b)	$3x^2 = 28x - 9$	M1
	Solves $3x^2 - 28x + 9 = 0$ to give $x = \frac{1}{3}$ or $x = 9$	M1 A1 (3) **6**
Notes (a)	**B1** for correct use of addition rule (or correct use of subtraction rule) **B1**: replacing $\log x^2$ by $2\log x$ – **not** $\log 3x^2$ by $2\log 3x$ this is **B0** **These first two B marks are often earned in the first line of working** **B1**. for replacing $\log 3$ by 1 (or use of $3^1 = 3$) If candidate has been awarded 3 marks and their proof includes an error or omission of reference to log$y$ withhold the last mark. So just B1 B1 B0 These marks must be awarded for **work in part (a)** only	
(b)	**M1** for removing logs to get an equation in x– statement in scheme is sufficient. This needs to be accurate without any errors seen in part (b). **M1** for attempting to solve three term quadratic to give $x =$ (see notes on marking quadratics) **A1** for the two correct answers – this depends on second M mark only. Candidates often begin again in part (b) and do not use part (a). If such candidates make errors in log work in part (b) they score first **M0**. The second **M** and the **A** are earned as before. It is possible to get M0M1A1 or M0M1A0.	
Alternative to (b) using y	Eliminates x to give $3y^2 - 730y + 243 = 0$ with no errors is M1 Solves quadratic to find y, then uses values to find x M1 A1 as before **See extra sheet with examples illustrating the scheme.**	

Question 2

Question number	Scheme	Marks
	$2\log x = \log x^2$	B1
	$\log_3 x^2 - \log_3(x-2) = \log_3 \dfrac{x^2}{x-2}$	M1
	$\dfrac{x^2}{x-2} = 9$	A1 o.e.
	Solves $x^2 - 9x + 18 = 0$ to give $x = ...$	M1
	$x = 3$, $x = 6$	A1
		Total 5
Notes	**B1** for **this** correct use of power rule (may be implied) **M1:** for **correct** use of subtraction rule (or addition rule) for logs N.B. $2\log_3 x - \log_3(x-2) = 2\log_3 \dfrac{x}{x-2}$ is **M0** **A1**. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.) **M1** for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics) **A1** for these two correct answers	
Alternative Method	$\log_3 x^2 = 2 + \log_3(x-2)$ is B1, *so* $x^2 = 3^{2+\log_3(x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1 Here M1 is for **complete** method i.e.correct use of powers after logs are used correctly	
Common Slips	$2\log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but **the statement is M0** and so leads to no further marks $2\log_3 x - \log_3(x-2) = 2$ *so* $\log_3 x - \log_3(x-2) = 1$ and $\log_3 \dfrac{x}{x-2} = 1$ can earn M1 for *correct* subtraction rule following error, but no other marks	
Special Case	$\dfrac{\log x^2}{\log(x-2)} = 2$ leading to $\dfrac{x^2}{x-2} = 9$ and then to x=3, x=6, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This *recovery* after uncorrected error is very common] Trial and error, Use of a table or just stating answer **with both** x=3 and x=6 should be awarded B0M0A0 then final M1A1 i.e. 2/5	

Question 3

Question Number	Scheme		Marks
(a)	$2\log(x+15)=\log(x+15)^2$		B1
	$\log(x+15)^2-\log x=\log\dfrac{(x+15)^2}{x}$	Correct use of $\log a-\log b=\log\dfrac{a}{b}$	M1
	$2^6=64$ or $\log_2 64=6$	64 used in the correct context	B1
	$\log_2\dfrac{(x+15)^2}{x}=6\Rightarrow\dfrac{(x+15)^2}{x}=64$	Removes logs correctly	M1
	$\Rightarrow x^2+30x+225=64x$ $or\ x+30+225x^{-1}=64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2-34x+225=0$ *		A1
			(5)
(b)	$(x-25)(x-9)=0\Rightarrow x=25$ or $x=9$	M1: Correct attempt to solve the **given** quadratic as far as $x=$... A1: Both 25 and 9	M1 A1
			(2)
			[7]

Question 4

Question Number	Scheme	Marks
(a)	Either (Way 1) : Attempt $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ $\mathrm{f}(3)=54-45+3a+18=0 \Rightarrow 3a=-27 \Rightarrow a=-9$ * Or (Way 2): Assume a = -9 and attempt $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ f(3) = 0 so (x -3) is factor	M1 A1 * cso (2)
	Or (Way 3): $\left(2x^3-5x^2+ax+18\right)\div(x-3)=2x^2+px+q$ where p is a number and q is an expression in terms of a Sets the remainder 18+3a+9 = 0 and solves to give $a=-9$	M1 A1* cso (2)
(b)	Either (Way 1): $\mathrm{f}(x)=(x-3)(2x^2+x-6)$ $=(x-3)(2x-3)(x+2)$	M1A1 M1A1 (4)
	Or (Way 2) Uses trial or factor theorem to obtain x = -2 **or** x = 3/2 Uses trial or factor theorem to obtain both x = -2 **and** x = 3/2 Puts three factors together (see notes below) Correct factorisation : $(x-3)(2x-3)(x+2)$ or $(3-x)(3-2x)(x+2)$ or $2(x-3)(x-\frac{3}{2})(x+2)$ oe	M1 A1 M1 A1 (4)
	Or (Way 3) No working three factors $(x-3)(2x-3)(x+2)$ otherwise need working	M1A1M1A1
(c)	$\{3^y=3\Rightarrow\}$ $\underline{y=1}$ or $\mathrm{g}(1)=0$ $\{3^y=1.5\Rightarrow\}\log(3^y)=\log 1.5$ or $y=\log_3 1.5$ $\{y=0.3690702...\}\Rightarrow y=$ awrt 0.37	B1 M1 A1 (3) [9]
	Notes for Question	
(a)	M1 for attempting either $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ – with **numbers substituted into expression** A1 for applying $\mathrm{f}(3)$ **correctly**, setting the result **equal to 0**, and manipulating this correctly to give the result given on the paper i.e. $a=-9$. (Do not accept x = -9) Note that the answer is given in part (a). If they **assume** a = -9 and **verify** by factor theorem or division they must state $(x-3)$ **is a factor** for A1 (or equivalent such as QED or a tick).	
(b)	1st M1: attempting to divide by $(x-3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$. (Could divide by $(3-x)$, in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. 1st A1: usually for $2x^2+x-6$ ... Credit when seen and use isw if miscopied 2nd M1: for a *valid** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) NB: $(x-3)(x-\frac{3}{2})(x+2)$ is M1A1M0A0, $(x-3)(x-\frac{3}{2})(2x+4)$ is M1A1M1A0, but $2(x-3)(x-\frac{3}{2})(x+2)$ is M1A1M1A1.	
(c)	B1: $\underline{y=1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for g(1) = 0. M1: Attempt to take logs to solve $3^y=\alpha$ or even $3^{2y}=\alpha$, but not $6^y=\alpha$ where $\alpha>0$ and $\alpha\neq 3$ & was a root of f(x) = 0 (ft their factorization) A1: for an answer that rounds to 0.37. If a third answer is included (and not "rejected") such as ln(-2) lose final A mark	

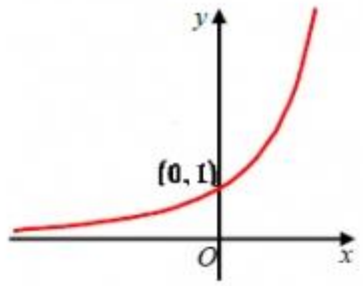
Question 5

Question Number	Scheme	Marks
(i) Method 1	$\log_2\left(\frac{2x}{5x+4}\right) = -3$ or $\log_2\left(\frac{5x+4}{2x}\right) = 3$, or $\log_2\left(\frac{5x+4}{x}\right) = 4$ (see special case 2)	M1
	$\left(\frac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\frac{5x+4}{2x}\right) = 2^3$ or $\left(\frac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\frac{2x}{5x+4}\right)\right) = \log_2\left(\frac{1}{8}\right)$	M1
	$16x = 5x+4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent)	dM1
	$x = \frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	A1 cso **(4)**
(i) Method 2	$\log_2(2x) + 3 = \log_2(5x+4)$	
	So $\log_2(2x) + \log_2(8) = \log_2(5x+4)$ (3 replaced by $\log_2 8$)	2nd M1
	Then $\log_2(16x) = \log_2(5x+4)$ (addition law of logs)	1st M1
	Then final M1 A1 as before	dM1A1
(ii)	$\log_a y + \log_a 2^3 = 5$	M1
	$\log_a 8y = 5$ — Applies product law of logarithms.	dM1
	$y = \frac{1}{8}a^5$ — $y = \frac{1}{8}a^5$	A1cao
		(3) **[7]**
	Notes for Question	
(i)	1st M1: Applying the subtraction or addition law of logarithms correctly to make **two log terms in** x **into one** log term in x 2nd M1: For RHS of either 2^{-3}, 2^3, 2^4 or $\log_2\left(\frac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an earlier error. **Use of 3^2 is M0** 3rd dM1: Obtains **correct** linear equation in x, usually the one in the scheme and attempts $x =$ A1: cso Answer of 4/11 with **no** suspect log work preceding this.	
(ii)	M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ *or* $\log_a 8$ dM1: (should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$	
(i)	**Special case 1:** $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ or $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \log_2\frac{2x}{5x+4} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ each attempt scores M0M1M1A0 – special case	
	Special case 2: $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \log_2 2 + \log_2 x = \log_2(5x+4) - 3$, is M0 until the two log terms are combined to give $\log_2\left(\frac{5x+4}{x}\right) = 3 + \log_2 2$. This earns M1 Then $\left(\frac{5x+4}{x}\right) = 2^4$ or $\log_2\left(\frac{5x+4}{x}\right) = \log_2 2^4$ gets second M1. Then scheme as before.	

Question 6

Question Number	Scheme	Marks
(a)	Way 1: $\log_3(9x) = \log_3 9 + \log_3 x$ or Way 2: $\log_3(9x) = \log_3 3^{a+2}$	M1
	$= 2 + a$	A1 (2)
(b)	Way 1: $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$ or Way 2 $= \log_3 \frac{3^{5a}}{3^4}$	M1
	$\log x^5 = 5\log x$ or log81=4log3 or log81=4 ; $= \log_3 3^{5a-4}$	M1
	$= 5a - 4$	A1 cso (3)
(c)	$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$	
	Method 1 / Method 2	
	$\Rightarrow 2 + a + 5a - 4 = 3$; $\log_3\left(9x.\frac{x^5}{81}\right) = (3 \text{ or } \log 27)$	M1
	$\Rightarrow a = \frac{5}{6}$; $\log_3\left(\frac{x^6}{9}\right) = 3 \text{ or } \log 27$	A1
	$\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a\log_{10} 3$ so $x =$; $\Rightarrow \frac{x^6}{9} = 3^3 \Rightarrow x^6 = 3^5 \Rightarrow x =$	M1
	$x = 2.498$ **or awrt** ; $x = 2.498$ **or awrt**	A1 (4)
	If $x = -2.498$ appears as well or instead this is A0	**Total 9**
	Notes for Question	
(a)	Way 1: M1: Use of log(ab) = log(a) + log(b) A1: must be $a + 2$ or $2 + a$ **Way 2: Uses** $x = 3^a$ to give $\log_3(9x) = \log_3 3^{a+2}$, A1 for $a + 2$ or $2 + a$	
(b)	Way 1: M1: Use of log(a/b) = log(a) – log(b) M1: Use of nlog(a) = log(a)^n Way 2: M1 Use of correct powers of 3 in numerator and denominator M1: Subtracts powers A1: No errors seen	
(c)	**Method 1**: M1: Uses (a) and (b) results to form an equation in a (may not be linear) A1: a = awrt 0.833 M1: Finds x by use of 3 to a power, or change of base performed correctly A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…) **Method 2:** M1: Use of log(ab) = log(a) + log(b) in an equation (RHS may be wrong) A1: Equation correct and simplified M1: Tries to undo log by 3 to power correctly, and uses root to obtain x A1: $x = 2.498$ (accept answer which round to this value from 2.498049533…) Lose this mark if negative answer is given as well as or instead of positive answer.	

Question 7

Question Number	Scheme		Marks
	Graph of $y=3^x$ and solving $3^{2x}-9(3^x)+18=0$		
(a)		At least two of the three criteria correct. **(See notes below.)**	B1
		All three criteria correct. **(See notes below.)**	B1
		Criteria number 1: Correct shape of curve for $x \geq 0$ and at least touches the positive y-axis. **Criteria number 2**: Correct shape of curve for $x<0$. Must not touch the x-axis or have any turning points. **Criteria number 3**: $(0, 1)$ stated or in a table or 1 marked on the y-axis. Allow $(1, 0)$ rather than $(0, 1)$ if marked in the "correct" place on the y-axis.	
			[2]
(b)	$(3^x)^2-9(3^x)+18=0$ or $y=3^x \Rightarrow y^2-9y+18=0$	Forms a quadratic of the correct form in 3^x or in "y" where $"y"=3^x$ or even in x where $"x"=3^x$	M1
	$\{(y-6)(y-3)=0$ or $(3^x-6)(3^x-3)=0\}$		
	$y=6,\ y=3$ or $3^x=6,\ 3^x=3$	**Both** $y=6$ and $y=3$.	A1
	$\{3^x=6 \Rightarrow\}\ x\log 3=\log 6$ or $x=\dfrac{\log 6}{\log 3}$ or $x=\log_3 6$	A valid method for solving $3^x=k$ where $k>0, k\neq 1, k\neq 3$ **to give either** $x\log 3=\log k$ or $x=\dfrac{\log k}{\log 3}$ or $x=\log_3 k$	dM1
	$x=1.63092...$	awrt 1.63	A1cso
	Provided the first M1A1 is scored, the second M1A1 can be implied by awrt 1.63		
	$x=1$	$x=1$ stated as a solution from *any* working.	B1
			[5]
			Total 7

Question 8

Question Number	Scheme		Marks
	$5^y = 8$		
(i)	$y\log 5 = \log 8$	$y\log 5 = \log 8$ or $y = \log_5 8$	M1
	$\left\{y = \dfrac{\log 8}{\log 5}\right\} = 1.2920...$	awrt 1.29	A1
	Allow correct answer only		
			[2]
	$\log_2(x+15) - 4 = \frac{1}{2}\log_2 x$		
(ii)	$\log_2(x+15) - 4 = \log_2 x^{\frac{1}{2}}$	Applies the power law of logarithms seen **at any point in their working**	M1
	$\log_2\left(\dfrac{x+15}{x^{\frac{1}{2}}}\right) = 4$	Applies the subtraction or addition law of logarithms **at any point in their working**	M1
	$\left(\dfrac{x+15}{x^{\frac{1}{2}}}\right) = 2^4$	Obtains a correct expression with logs removed and no errors	M1
	$x - 16x^{\frac{1}{2}} + 15 = 0$ or e.g. $x^2 + 225 = 226x$	Correct **three** term quadratic in any form	A1
	$(\sqrt{x}-1)(\sqrt{x}-15) = 0 \Rightarrow \sqrt{x} = ...$	A **valid** attempt to factorise or solve their **three term quadratic** to obtain $\sqrt{x} = ...$ or $x = ...$ Dependent on all previous method marks.	dddM1
	$\{\sqrt{x} = 1, 15\}$		
	$x = 1,\ 225$	**Both $x = 1$ and $x = 225$ (If both are seen, ignore any other values of $x \le 0$ from an otherwise correct solution)**	A1
			[6]
			Total 8
	Alternative:		
	$2\log_2(x+15) - 8 = \log_2 x$		
	$\log_2(x+15)^2 - 8 = \log_2 x$	Applies the power law of logarithms	M1
	$\log_2\left(\dfrac{(x+15)^2}{x}\right) = 8$	Applies the subtraction law of logarithms	M1
	$\dfrac{(x+15)^2}{x} = 2^8$	Obtains a correct expression with logs removed	M1
	$x^2 + 30x + 225 = 256x$		
	$x^2 - 226x + 225 = 0$	Correct three term quadratic in any form	A1
	$(x-1)(x-225) = 0 \Rightarrow x = ...$	A valid attempt to factorise or solve their 3TQ to obtain $x = ...$ Dependent on all previous method marks.	dddM1
	$x = 1,\ 225$	**Both $x = 1$ and $x = 225$ (If both are seen, ignore any other values of $x \le 0$ from an otherwise correct solution)**	A1

Question 9

Question Number	Scheme	Marks
(i)	$8^{2x+1}=24$ $(2x+1)\log 8 = \log 24$ or $(2x+1)=\log_8 24$ or $8^{2x}=3$ and so $(2x)\log 8 = \log 3$ or $(2x)=\log_8 3$ $x=\frac{1}{2}\left(\frac{\log 24}{\log 8}-1\right)$ or $x=\frac{1}{2}(\log_8 24-1)$ $x=\frac{1}{2}\left(\frac{\log 3}{\log 8}\right)$ or $x=\frac{1}{2}(\log_8 3)$ o.e. $=0.264$	M1 dM1 A1 (3)
(ii)	$\log_2(11y-3)-\log_2 3-2\log_2 y=1$ $\log_2(11y-3)-\log_2 3-\log_2 y^2=1$ $\log_2\frac{(11y-3)}{3y^2}=1$ or $\log_2\frac{(11y-3)}{y^2}=1+\log_2 3=2.58496501$ $\log_2\frac{(11y-3)}{3y^2}=\log_2 2$ or $\log_2\frac{(11y-3)}{y^2}=\log_2 6$ (allow awrt 6 if replaced by 6 later) Obtains $6y^2-11y+3=0$ o.e. i.e. $6y^2=11y-3$ for example Solves quadratic to give $y=$ $y=\frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	M1 dM1 B1 A1 ddM1 A1 (6) [9]
Notes (i) (ii)	**M1**: Takes logs and uses law of powers correctly. (Any log base may be used) Allow lack of brackets. **dM1**: Make x subject of their formula correctly (may evaluate the log before subtracting 1 and calculate e.g. (1.528 -1)/2) **A1**: Allow answers which round to 0.264 **M1**: Applies power law of logarithms replacing $2\log_2 y$ by $\log_2 y^2$ **dM1**: Applies quotient or product law of logarithms correctly to the three log terms including term in y^2. (dependent on first M mark) or applies quotient rule to two terms and collects constants (allow "triple" fractions) $1+\log_2 3$ on RHS is not sufficient – need $\log_2 6$ or 2.58… e.g. $\log_2(11y-3)=\log_2 3+\log_2 y^2+\log_2 2$ becoming $\log_2(11y-3)=\log_2 6y^2$ **B1**: States or uses $\log_2 2=1$ or $2^1=2$ at any point in the answer so may be given for $\log_2(11y-3)-\log_2 3-2\log_2 y=\log_2 2$ or for $\frac{(11y-3)}{3y^2}=2$, for example (Sometimes this mark will be awarded before the second M mark, and it is possible to score M1M0B1in some cases) Or may be given for $\log_2 6=2.584962501..$ or $2^{2.584962501..}=6$ **A1**: This or equivalent quadratic equation (does not need to be in this form but should be equation) **ddM1:** (dependent on the two previous M marks) Solves their quadratic equation following reasonable log work using factorising, completion of square, formula or implied by both answers correct. **A1:** Any equivalent correct form – need both answers- allow awrt 0.333 for the answer 1/3 *NB: If "=0" is missing from the equation but candidate continues correctly and obtains correct answers then allow the penultimate A1 to be implied (Allow use of x or other varable instead of y throughout)	

Question 10

Question Number	Scheme		Marks
(i)	Two Ways of answering the question are given in part (i)		
Way 1	$\log_3\left(\dfrac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\dfrac{a-2}{3b+1}\right) = 1$	Applying the subtraction law of logarithms	M1
	$\dfrac{3b+1}{a-2} = 3^{-1}\left\{= \dfrac{1}{3}\right\}$ or $\left(\dfrac{a-2}{3b+1}\right) = 3$	Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3 = a-2 \Rightarrow\}\ b = \dfrac{1}{9}a - \dfrac{5}{9}$	$b = \dfrac{1}{9}a - \dfrac{5}{9}$ or $b = \dfrac{a-5}{9}$	A1 oe
			[3]
	In Way 2 a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs		
(i) Way 2	Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$		2nd M1
	$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\dfrac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$		1st M1
	$\{3b+1 = \dfrac{a-2}{3}\}\ b = \dfrac{1}{9}a - \dfrac{5}{9}$		A1
			[3]
	Five Ways of answering the question are given in part (ii)		
(ii) Way 1 See also common approach below in notes	$32(2^{2x}) - 7(2^x) = 0$	Deals with power 5 correctly giving $\times 32$	M1
	So, $2^x = \dfrac{7}{32}$	$2^x = \dfrac{7}{32}$ or $y = \dfrac{7}{32}$ or awrt 0.219	A1 oe
	$x\log 2 = \log\left(\dfrac{7}{32}\right)$ or $x = \dfrac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x = \log_2\left(\dfrac{7}{32}\right)$	A valid method for solving $2^x = \frac{7}{32}$ Or $2^x = k$ to achieve $x = \ldots$	dM1
	$x = -2.192645\ldots$	awrt -2.19	A1
			[4]
	Begins with $2^{2x+5} = 7(2^x)$ (for Way 2 and Way 3) (see notes below)		
(ii) Way 2	$(2x+5)\log 2 = \log 7 + x\log 2$	Correct application of either the power law or addition law of logarithms	M1
		Correct result after applying the power **and** addition laws of logarithms.	A1
	$2x\log 2 + 5\log 2 = \log 7 + x\log 2$ $\Rightarrow x = \dfrac{\log 7 - 5\log 2}{\log 2}$	Multiplies out, collects x terms to achieve $x = \ldots$	dM1
	$x = -2.192645\ldots$	awrt -2.19	A1
			[4]
(ii) Way 3	$2x + 5 = \log_2 7 + x$	Evidence of $\log_2$ and either $2^{2x+5} \to 2x+5$ or $7(2^x) \to \log_2 7 + \log_2(2^x)$	M1
		$2x + 5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$	Collects x terms to achieve $x = \ldots$	dM1
	$x = -2.192645\ldots$	awrt -2.19	A1
			[4]

(ii) Way 4	$2^{2x+5} = 7(2^x) \Rightarrow 2^{x+5} = 7$	
	$x+5 = \log_2 7$ or $\dfrac{\log 7}{\log 2}$ — Evidence of $\log_2$ and either $2^{x+5} \to x+5$ or $7 \to \log_2 7$	M1
	$x+5 = \log_2 7$ oe.	A1
	$x = \log_2 7 - 5$ — Rearranges to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
Way 5 (similar to Way 3)	$2^{2x+5} = 2^{\log_2 7}(2^x)$ — 7 is replaced by $2^{\log_2 7}$	M1
	$2x+5 = \log_2 7 + x$ — $2x+5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow x = \log_2 7 - 5$ — Collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$ — awrt -2.19	A1
		[4]
		7

	Question Notes	
(i)	1st M1	Applying either the addition or subtraction law of logarithms correctly to combine any **two** log terms into **one** log term.
	2nd M1	For making a correct connection between log base 3 and 3 to a power.
	A1	$b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$ o.e. e.g. Accept $b = \frac{1}{3}\left(\frac{a}{3} - \frac{5}{3}\right)$ but not $b = \frac{a-2}{9} - \frac{3}{9}$ nor $b = \dfrac{\left(\frac{a}{3} - \frac{5}{3}\right)}{3}$
(ii)	1st M1	First step towards solution – an equation with one side or other correct or one term dealt with correctly (see five* possible methods above)
	1st A1	Completely correct first step – giving a correct equation as shown above
	dM1	Correct complete method (all log work correct) and working to reach $x =$ in terms of logs reaching a correct expression or one where the only errors are slips solving linear equations
	2nd A1	Accept answers which round to -2.19 If a second answer is also given this becomes A0
	Special Case in (i)	Writes $\dfrac{\log_3(3b+1)}{\log_3(a-2)} = -1$ and proceeds to $\dfrac{3b+1}{a-2} = 3^{-1}\left\{= \dfrac{1}{3}\right\}$ and to correct answer- Give M0M1A1 (special case)
	Common approach to part (ii)	Let $2^x = y$ Treat this as **Way 1** They get $32y^2 - 7y = 0$ for M1 and need to reach $y = \dfrac{7}{32}$ for A1. Then back to **Way 1** as before. Any letter may be used for the new variable which I have called y. If they use x and obtain $x = \dfrac{7}{32}$, this may be awarded M1A0M0A0. Those who get $y^2 - 7y + 32 = 0$ or $y^7 - 7y = 0$ will be awarded M0,A0,M0,A0
	Common Present-ation of Work in ii	**Many begin with** $\log\left(2^{2x+5}\right) - \log\left(7(2^x)\right) = 0$. It is possible to reach this in two stages correctly so do not penalise this and award the full marks if they continue correctly as in **Way 2**. If however the solution continues with $(2x+5)\log 2 - x\log 14 = 0$ or with $(2x+5)\log 2 - 7x\log 2 = 0$ (both incorrect) then they are awarded M1A0M0A0 just getting credit for the $(2x+5)\log 2$ term.
	Note	N.B. The answer (+)2.19 results from "algebraic errors solving linear equations" leading to $2^x = \frac{32}{7}$ and gets M1A0M1A0

Question 11

Question Number	Scheme	Marks
(i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x =)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \frac{(9y+b)}{(2y-b)} = 2$ — Applies quotient law of logarithms	M1
	$\frac{(9y+b)}{(2y-b)} = 3^2$ — Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ — Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or : $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ — 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ — 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ — Multiplies across and makes y the subject	M1 A1cso **(4)** [7]

	Notes
(i)	1st M1: Applies power law of logarithms correctly to one side of the equation M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x+a=\sqrt{16a^6}$ isw so allow $x+a=\pm\ 4a^3$ for Method mark. Also allow $x+a=4a^4$ or $x+a=\pm\ 4a^{5.5}$ or even $x+a=16a^3$ as there is evidence of attempted square root. May see the correct $x+a=10^{(\log 4+3\log a)}$ *so* $x=-a+10^{(\log 4+3\log a)}$ which gains M1A0 unless followed by the answer in the scheme. A1: Do not allow $x=\pm\ 4a^3-a$ for accuracy mark. You may see the factorised $a(2a+1)(2a-1)$ o.e.
(ii)	M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in y M1: Uses $\log_3 3^2 = 2$ 3rd M1: Obtains **correct** linear equation in y usually the one in the scheme and attempts $y =$ A1cso: $y=\frac{10}{9}b$ or correct equivalent after **completely correct** work. **Special case:** $\frac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3\frac{(9y+b)}{(2y-b)} = 2$ Candidates may then write $\frac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.