Modelling with Differentiation - Edexcel Past Exam Questions 2 MARK SCHEME

Question 1

Question number	Scheme	Marks
(a)	$kr^2 + cxy = 4$ or $kr^2 + c[(x+y)^2 - x^2 - y^2] = 4$	M1
	$\frac{1}{4}\pi x^2 + 2xy = 4$	A1
	$y = \dfrac{4 - \frac{1}{4}\pi x^2}{2x} = \dfrac{16 - \pi x^2}{8x}$ *	B1 cso (3)
(b)	$P = 2x + cy + k\pi r$ where $c = 2$ or 4 and $k = ¼$ or $½$	M1
	$P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ or $P = \dfrac{\pi x}{2} + 2x + 4\left(\dfrac{16 - \pi x^2}{8x}\right)$ o.e.	A1
	$P = \dfrac{\pi x}{2} + 2x + \dfrac{8}{x} - \dfrac{\pi x}{2}$ so $P = \dfrac{8}{x} + 2x$ *	A1 (3)
(c)	$\left(\dfrac{dP}{dx} = \right) - \dfrac{8}{x^2} + 2$	M1 A1
	$-\dfrac{8}{x^2} + 2 = 0 \Rightarrow x^2 = ..$	M1
	and so $x = 2$ o.e. (ignore extra answer x = -2)	A1
	$P = 4 + 4 = 8$ (m)	B1 (5)
(d)	$y = \dfrac{4 - \pi}{4}$, (and so width) = 21 (cm)	M1, A1 (2) **13**
Notes	(a) **M1**: Putting sum of one or two xy terms and one kr^2 term equal to 4 (k and c may be wrong) **A1**: For any correct form of this equation with x for radius (may be unsimplified) **B1** : Making y the subject of their formula to give this printed answer with no errors (b) **M1** : Uses Perimeter formula of the form $2x + cy + k\pi r$ where $c = 2$ or 4 and $k = ¼$ or $½$ **A1**: Correct unsimplified formula with y substituted as shown, i.e. $c = 4$, $k = ½$, $r = x$ and $y = \dfrac{16 - \pi x^2}{8x}$ or $y = \left(\dfrac{4 - \frac{1}{4}\pi x^2}{2x}\right)$ **A1**: obtains printed answer with at least one line of correct simplification or expansion before giving printed answer or stating result has been shown or equivalent (c) **M1**: At least one power of x decreased by 1 (Allow $2x$ becomes 2) **A1**: accept any equivalent correct answer **M1**: Setting $\dfrac{dP}{dx}$ =0 and finding a value for correct power of x for candidate **A1** : For $x = 2$. (This mark may be given for equivalent and may be implied by correct P) **B1**: 8 (cao) N.B. This may be awarded if seen in part (d) (d) **M1** : Substitute x value found in (c) into equation for y from (a) (or substitute x and P into equation for P from (b)) and evaluate (may see 0.2146 and correct answer implies M1 or need to see substitution if x value was wrong.) **A1** is for 21 or 21cm or 0.21m as this is to nearest cm	

Question 2

Question number	Scheme	Marks
(a)	$(h=)\dfrac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60\div\pi x^2$	B1 (1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines	B1
	Either $(A)=2\pi x^2+2\pi x\left(\dfrac{60}{\pi x^2}\right)$ or As $\pi xh=\dfrac{60}{x}$ then $(A=)2\pi x^2+2\left(\dfrac{60}{x}\right)$	M1
	$A=2\pi x^2+\left(\dfrac{120}{x}\right)$ *	A1 cso (3)
(c)	$\left(\dfrac{dA}{dx}\right)=4\pi x-\dfrac{120}{x^2}$ or $=4\pi x-120x^{-2}$	M1 A1
	$4\pi x-\dfrac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)	M1
	$x=\sqrt[3]{\dfrac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1 (5)
(d)	$A=2\pi(2.12)^2+\dfrac{120}{2.12}$, $=85$ (only ft $x=2$ or 2.1 – both give 85)	M1, A1 (2)
(e)	**Either** $\dfrac{d^2A}{dx^2}=4\pi+\dfrac{240}{x^3}$ and sign considered (May appear in (c)) **Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** *(method 3)* considers value of A either side	M1
	which is >0 and therefore minimum (must substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A, observing greater than minimum value and draws conclusion	A1 (2)
		13 marks
Notes	(a) **B1**: This expression must be correct and in part (a) $\dfrac{60}{\pi r^2}$ is B0 (b) **B1**: Accept any equivalent correct form – may be on two or more lines. **M1** : substitute their expression for h in terms of x into Area formula **of the form** kx^2+cxh **A1**: There should have been no errors in part (b) in obtaining this printed answer (c) **M1**: At least one power of x decreased by 1 **A1** accept any equivalent correct answer **M1**: Setting $\dfrac{dA}{dx}=0$ and finding a value for x^3 ($x^3=$ may be implied by answer). Allow $\dfrac{dy}{dx}=0$ **dM1**: Using **cube root** to find x **A1** : For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark (d) **M1** : Substitute the (+ve) x value found in (c) into equation for A and evaluate . **A1** is for **85 only** (e) **M1**: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered **A1**: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ **must be correct** . Must **not** see 85 substituted)	

Question 3

Question Number	Scheme		Marks
	$y = 6 - 3x - \dfrac{4}{x^3}$		
(a)	$\dfrac{dy}{dx} = -3 + \dfrac{12}{x^4}$ or $-3 + 12x^{-4}$	M1: $x^n \to x^{n-1}$ ($x^1 \to x^0$ or $x^{-3} \to x^{-4}$ or $6 \to 0$) A1: Correct derivative	M1 A1
	$\dfrac{dy}{dx} = 0 \Rightarrow -3 + \dfrac{12}{x^4} = 0 \Rightarrow x = ...$ **or** $\dfrac{dy}{dx} = -3 + \dfrac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\dfrac{dy}{dx} = -3 + \dfrac{12}{x^4} = 0 \Rightarrow \dfrac{12}{x^4} = 3 \Rightarrow x = ...$ **or** Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ **or** $\dfrac{dy}{dx} = -3 + \dfrac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
			(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\dfrac{d^2y}{dx^2} = \dfrac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum		B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow y'' = awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct **and part (b) must be correct** and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow y'' = awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of y or dy/dx either side of $\sqrt{2}$ or their x at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

Question 4

Question Number	Scheme	Marks
(a)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\right\}\ 2x-16x^{-\frac{1}{2}}$	M1 A1
	$2x-16x^{-\frac{1}{2}}=0 \Rightarrow\ x^{\frac{3}{2}}=, x^{-\frac{3}{2}}=$,or $2x-=16x^{-\frac{1}{2}}$ then squared then obtain $x^3=$ [or $2x-16x^{-\frac{1}{2}}=0 \Rightarrow x=4$ (no wrong work seen)]	M1
	$(x^{\frac{3}{2}}=8 \Rightarrow)x=\ 4$	A1
	$x=4,\ y=\ 4^2-32\sqrt{4}+20=-28$ (ignore $y=100$ as second answer)	M1 A1
		(6)
(b)	$\left\{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\right\}\ 2+8x^{-\frac{3}{2}}$	M1 A1
	$(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0 \Rightarrow)\, y$ is a minimum (there should be no wrong reasoning)	A1
		(3) **[9]**
(b)	**Alternative Method: Gradient Test:** M1 for finding the gradient either side of their x-value from part (a). A1 for both gradients calculated correctly to 1 significant figure, then using <0 and >0 respectively *maybe by use of sketch or table*. (See appendix for gradient values. This is **not ft their x**) A1 states minimum needs M1A1 to have been awarded.	
	Notes for Question	
(a)	1st M1: At least one term differentiated correctly, so $x^2 \to 2x$, or $32\sqrt{x} \to 16x^{-\frac{1}{2}}$, or $20 \to 0$ A1: This answer or equivalent e.g. $2x-\frac{16}{\sqrt{x}}$ 2nd M1: Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0, and solves to give $x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$ *or* $x^3=$ after correct squaring or spots $x=4$ (NB $\left\{\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0\right\}$ *so* $2+8x^{-\frac{3}{2}}=0$ is **M0**) N.B. Common error: Putting derivative = 0 and merely obtaining $x=0$ is M0A0, then M0A0 for next two marks. (The first two marks in (a) and marks for second derivative may be earned in part (b).) A1: $x=4$ cao [$x=-4$ is A0 and $x=\pm 4$ is also A0] 3rd M1: Substitutes **their positive found** x **(NOT zero)** into $y=x^2-32\sqrt{x}+20,\ x>0$.**Should follow attempting to set $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and not setting $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$** A1: -28 cao (Does not need to be written as coordinates)	
(b)	M1: Attempts differentiation of their first derivative with at least one term differentiated correctly. Should be seen or referred to (in part (b)) in determining the nature of the stationary point. A1: Answer in scheme or equivalent A1: States minimum (Second derivative should be correct- can follow incorrect positive x. Needs M1A1 to have been awarded- should not follow incorrect reasoning – (need not say $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$ but should not have said $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ for example)	

Question 5

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 2 - 16x^{-3}$	M1 A1
	$2 - 16x^{-3} = 0$ so $x^{-3} =$ or $x^3 =$, or $2 - 16x^{-3} = 0$ so $x = 2$	M1
	$x = 2$ only (after correct derivative)	A1
	$y = 2 \times "2" + 3 + \frac{8}{"2^2"}$	M1
	$= 9$	A1
		(6)
		Total 6

	Notes for Question
	1st M1: At least one term **differentiated (not integrated)** correctly, so $2x \to 2$, or $\frac{8}{x^2} \to -16x^{-3}$, or $3 \to 0$ A1: This answer or equivalent e.g. $2 - \frac{16}{x^3}$ 2nd M1: Sets $\frac{dy}{dx}$ to 0, and solves to give x^3 = value or x^{-3} = value (or states x =2 with no working following correctly stated $2 - 16x^{-3} = 0$) A1: $x = 2$ cso (if $x = -2$ is included this is A0 here) 3rd M1: Attempts to substitutes **their positive** x (found from attempt to differentiate) into $y = 2x + 3 + \frac{8}{x^2}$, $x > 0$ Or may be implied by $y = 9$ or correct follow through from their positive x A1: 9 cao (Does not need to be written as coordinates) (ignore the extra (-2,1) here)

Question 6

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="3">(a)</td><td>$\frac{1}{2}(9x+6x)4x$
or $2x \times 15x$
or $\left(\frac{1}{2}4x \times (9x-6x) + 6x \times 4x\right)$
or $6x^2 + 24x^2$
or $\left(9x \times 4x - \frac{1}{2}4x \times (9x-6x)\right)$
or $36x^2 - 6x^2$</td><td>M1: Correct attempt at the area of a trapezium.
Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x \times 6x$ is M0.
If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.</td><td rowspan="2">M1A1cso</td></tr>
<tr><td>$\Rightarrow 30x^2y = 9600 \Rightarrow y = \dfrac{9600}{30x^2} \Rightarrow y = \dfrac{320}{x^2}$ *</td><td>A1: Correct proof with at least one intermediate step and no errors seen.
"$y =$" is required.</td></tr>
<tr><td colspan="2"></td><td>[2]</td></tr>
<tr><td rowspan="7">(b)</td><td colspan="2">$(S =)\frac{1}{2}(9x+6x)4x + \frac{1}{2}(9x+6x)4x + 6xy + 9xy + 5xy + 4xy$</td><td>M1A1</td></tr>
<tr><td colspan="2">M1: An attempt to find the area of six faces of the prism. The 2 trapezia may be combined as $(9x+6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct.
A1: Correct expression in any form.
Allow just $(S =)\,60x^2 + 24xy$ for M1A1</td><td></td></tr>
<tr><td colspan="2">$y = \dfrac{320}{x^2} \Rightarrow (S =)\,30x^2 + 30x^2 + 24x\left(\dfrac{320}{x^2}\right)$</td><td>M1</td></tr>
<tr><td colspan="2">Substitutes $y = \dfrac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.</td><td></td></tr>
<tr><td>So, $(S =)\,60x^2 + \dfrac{7680}{x}$ *</td><td>Correct solution only.
"$S =$" is not required here.</td><td>A1* cso</td></tr>
<tr><td colspan="2"></td><td>[4]</td></tr>
</table>

(c)	$\frac{dS}{dx} = 120x - 7680x^{-2}$ $\left\{= 120x - \frac{7680}{x^2}\right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value" Setting their $\frac{dS}{dx} = 0$ and "candidate's ft *correct* power of x = a value". **The power of x must be consistent with their differentiation.** If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1cso
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find S – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880 \text{ (cm}^2\text{)}$	Substitute candidate's value of x ($\neq 0$) into a formula for S. **Dependent on both previous M marks.**	ddM1
		2880 cso (**Must come from correct work**)	A1 cao and cso
			[6]
(d)	$\frac{d^2S}{dx^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt S'' $(x^n \to x^{n-1})$ and considers sign. This mark requires an attempt at the second derivative and **some consideration of its sign**. There does not necessarily need to be any substitution. An attempt to solve $S'' = 0$ is M0	M1A1ft
		A1: $120 + \frac{15360}{x^3}$ and > 0 and conclusion. Requires a <u>correct</u> second derivative of $120 + \frac{15360}{x^3}$ (need not be simplified) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been <u>evaluated</u> incorrectly.	
	A correct S'' followed by $S''("4") = "360"$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''("4") = "360"$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14

Question 7

Question Number	Scheme		Marks
(a)	$\{A=\}\ xy+\frac{\pi}{2}\left(\frac{x}{2}\right)^2+\frac{1}{2}x^2\sin 60^\circ$	M1: An attempt to find 3 areas of the form: $xy,\ \ p\pi x^2$ and qx^2 A1: Correct expression for A (terms must be added)	M1A1
	$50=\ xy+\frac{\pi x^2}{8}+\frac{\sqrt{3}x^2}{4} \Rightarrow y=\frac{50}{x}-\frac{\pi x}{8}-\frac{\sqrt{3}x}{4} \Rightarrow y=\frac{50}{x}-\frac{x}{8}\left(\pi+2\sqrt{3}\right)$ * Correct proof with no errors seen		A1 *
			[3]
(b)	$\{P=\}\ \frac{\pi x}{2}+2x+2y$	Correct expression for P in terms of x and y	B1
	$P=\frac{\pi x}{2}+2x+2\left(\frac{50}{x}-\frac{x}{8}\left(\pi+2\sqrt{3}\right)\right)$	Substitutes the given expression for y into an expression for P where P is at least of the form $\alpha x+\beta y$	M1
	$P=\frac{\pi x}{2}+2x+\frac{100}{x}-\frac{\pi x}{4}-\frac{\sqrt{3}}{2}x \Rightarrow P=\frac{100}{x}+\frac{\pi x}{4}+2x-\frac{\sqrt{3}}{2}x$		
	$\Rightarrow P=\frac{100}{x}+\frac{x}{4}\left(\pi+8-2\sqrt{3}\right)$	Correct proof with no errors seen	A1 *
			[3]
	(Note $\frac{\pi+8-2\sqrt{3}}{4}=1.919\ldots\ldots$)		
(c) and (d) can be marked together	$\frac{\mathrm{d}P}{\mathrm{d}x}=-100x^{-2}+\frac{\pi+8-2\sqrt{3}}{4}$	M1: Either $\mu x\to\mu$ or $\frac{100}{x}\to\frac{\pm\lambda}{x^2}$ A1: Correct differentiation (need not be simplified). Allow $-100x^{-2}+(\text{awrt}1{\cdot}92)$	M1A1
	$-100x^{-2}+\frac{\pi+8-2\sqrt{3}}{4}=0\Rightarrow x=\ldots$	Their $P'=0$ and attempt to solve as far as $x=\ldots$. (ignore poor manipulation)	M1
	$\Rightarrow\ x=\sqrt{\frac{400}{\pi+8-2\sqrt{3}}}\ =7.2180574\ldots$	$\sqrt{\frac{400}{\pi+8-2\sqrt{3}}}$ or awrt 7.2 **and no other values**	A1
	$\{x=7.218\ldots\}\ \Rightarrow P=27.708\ldots$ (m)	awrt 27.7	A1
			[5]
	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=\frac{200}{x^3}>0\ \Rightarrow$ Minimum	M1: Finds P'' ($x^n\to x^{n-1}$ allow for constant $\to 0$) **and** considers sign A1ft: $\frac{200}{x^3}$ (need not be simplified) and >0 and conclusion. Only follow through on a correct P'' and a single positive value of x found earlier.	M1A1ft
			[2]
			Total 13

Question 8

Question Number	Scheme	Marks
(a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ or (Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{dC}{dr}=\right\}$ $12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ so $r^k=$ value where $k=\pm2,\pm3,\pm4$	dM1
	Use **cube** root to obtain $r=\left(their\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow $r=3$, and thus $C=$	ddM1
	Then $C=$ awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{d^2C}{dr^2}=\right\}$ $12\pi+\dfrac{600\pi}{r^3}>0$ so **minimum**	B1ft (1) [10]

Notes

(a) **B1:** States $3\times 2\pi r^2$ or states $2\times 2\pi rh$

B1ft: Obtains a **correct** expression for h in terms of r (ft only follows misread of V)

M1: Substitutes their expression for h into **area or cost** expression of form $Ar^2 + Brh$

A1*: Had correct expression for C and achieves **given** answer in part (a) including "$C =$" or "Cost=" and **no errors seen** such as $C =$ area expression without multiples of (£)3 and (£)2 at any point. Cost and area must be perfectly distinguished at all stages for this A mark.

N.B. Candidates using Curved Surface Area = $\frac{2V}{r}$ - please send to review

(b) **M1:** Attempts to differentiate as evidenced by at least one term differentiated correctly

A1ft: Correct derivative – allow $12\pi r - 300\pi r^{-2}$ then isw if the power is misinterpreted (ft only for misread)

dM1: Sets their $\frac{dC}{dr}$ to 0, and obtains r^k = value where $k = 2$, 3 or 4 (needs correct collection of powers of r from their original derivative expression – allow errors dividing by 12π)

ddM1: Uses **cube** root to find r **or** see $r =$ awrt 3 as evidence of cube root and substitutes into correct expression for C to obtain value for C

A1: Accept awrt 483 or 484

(c) **B1ft: Finds** correct expression for $\frac{d^2C}{dr^2}$ and deduces value of $\frac{d^2C}{dr^2} > 0$ so minimum (r may have been wrong)

OR checks gradient to left and right of 2.92 and shows gradient goes from negative to zero to positive so minimum

OR checks value of C to left and right of 2.92 and shows that $C > 483$ so deduces minimum (i.e. uses shape of graph) Only ft on misread of V for each ft mark (see below)

N..B. Some candidates have **misread** the volume as 75 instead of 75π. PTO for marking instruction.

Following this misread candidates cannot legitimately obtain the printed answer in part (a). Either they obtain $C = 6\pi r^2 + \frac{300}{r}$ or they "fudge" their working to appear to give the printed answer.

The policy for a misread is **to subtract 2 marks from A or B marks**. In this case the A mark is to be subtracted from part (a) and the final A mark is to be subtracted from part (b)
The maximum mark for part (a) following this misread is 3 marks. The award is B1 B1 M1 A0 as a maximum.

(a) B1: as before

B1: Uses volume to give $(h =)\frac{75}{\pi r^2}$

M1: $(C) = 6\pi r^2 + 4\pi r\left(\frac{75}{\pi r^2}\right)$

A0: Printed answer is not obtained without error

Most Candidates may then adopt the printed answer and gain up to full marks for the rest of the question so 9 of the 10 marks maximum in all.

Any candidate who proceeds with **their** answer $C = 6\pi r^2 + \frac{300}{r}$ may be awarded up to 4 marks in part (b). These are M1A1dM1ddM1A0 and then the candidate may also be awarded the B1 mark in part (c). So 8 of the 10 marks maximum in all.

(b) M1 A1: $\left\{\frac{dC}{dr} =\right\} 12\pi r - \frac{300}{r^2}$ or $12\pi r - 300r^{-2}$ (then isw)

dM1: $12\pi r - \frac{300}{r^2} = 0$ so $r^k =$ value where $k = 2$, 3 or 4 or $12\pi r - \frac{300}{r^2} = 0$ so $r^k =$ value

ddM1: Use **cube** root to obtain $r = \left(\textit{their}\ \frac{300}{12\pi}\right)^{\frac{1}{3}}$ $(= 1.996)$ - allow $r = 2$, and thus $C = \ldots$ must use $C = 6\pi r^2 + \frac{300}{r}$

A0: Cannot obtain $C = 483$ or 484

(c) B1: $\left\{\frac{d^2C}{dr^2} =\right\} 12\pi + \frac{600}{r^3} > 0$ so **minimum** OR checks gradient to left and right of 1.966 and shows gradient goes from negative to zero to positive so minimum
OR checks value of C to left and right of 1.966 and shows that $C > 225.4$ so deduces minimum (i.e. uses shape of graph)
There is an example in Practice of this misread.

Question 9

Question Number	Scheme		Marks
(a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$	$\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360} \times \pi x^2$ simplified or un-simplified	M1
		$\frac{\pi x^2}{3}$	A1
			[2]
	Parts (b) and (c) may be marked together		
(b)	$\{A =\}\ \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	Attempt to sum 3 areas (at least one correct)	M1
		Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow y = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})$ *	Correct proof.	A1 *
			[3]
(c)	$\{P =\}\ x + x\theta + y + 2x + y\ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	Correct expression in x and y for their θ measured in rads	B1ft
	$...2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})\right)$	Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$		
	$\Rightarrow P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3})$ *	Correct proof.	A1 *
			[3]
	Parts (d) and (e) should be marked together		
(d)	$\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$	$\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$	M1
		Correct differentiation (need not be simplified).	A1;
		Their $P' = 0$	M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ $(= 16.63392808...)$	$\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}(4\pi + 36 - 3\sqrt{3})\right\} \Rightarrow P = 120.236..\ (\text{m})$	awrt 120	A1
			[5]
(e)	$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow \text{Minimum}$	Finds P'' and considers sign.	M1
		$\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
			[2]
			15

	Question Notes	
(a)	M1	Attempts to use Area$(FEA) = \frac{1}{2}x^2 \times \frac{2\pi}{3}$ (using radian angle) or $\frac{120}{360} \times \pi x^2$ (using angle in degrees)
	A1	$\frac{\pi x^2}{3}$ cao (Must be simplified and be their answer in part (a)) Answer only implies M1A1.
		N.B. Area$(FEA) = \frac{1}{2}x^2 \times 120$ is awarded M0A0
(b)	M1	An attempt to sum 3 " areas" consisting of rectangle, triangle and sector (allow slips even in dimensions) but **one** area should be correct
	1st A1	Correct expression for **two** of the three areas listed above. Accept any correct equivalents e.g. two correct from $\frac{1}{2}x^2 \sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{4}x^2\sqrt{3}$, $\frac{1}{2} \times \frac{2}{3}\pi x^2$, $2xy$
	2nd A1*	This is a given answer which should be stated and should be achieved without error so all three areas must have been correct and their sum put equal to 1000 and an intermediate step of rearrangement should be present.

(c)	B1ft	Correct expression for P from arc length, length AB and three sides of rectangle in terms of both x and y with $2y$ (or $y + y$), $3x$ (or $x + 2x$) (or $x + x + x$), and $x\theta$ clearly listed . Allow addition after substitution of y. NB $\theta = \frac{2\pi}{3}$ but allow use of their consistent θ in radians (usually $\theta = \frac{\pi}{3}$) from parts (a) and (b) for this mark. $120x$ or $60x$ do not get this mark.
	M1	Substitutes $y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ or their unsimplified attempt at y from earlier (allow slips e.g. sign slips) into $2y$ term.
	A1*	This is a given answer which should be stated and should be achieved without error
(d)	1st M1	Need to see at least $\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$
	1st A1	Correct differentiation of both terms (need not be simplified) Not follow through. Allow any correct equivalent. e.g. $\frac{dP}{dx} = -1000x^{-2} + \frac{\pi}{3} + 3 - \frac{\sqrt{3}}{4}$ Also allow $\frac{dP}{dx} = -1000x^{-2} + awrt\ 3.61$ Check carefully as there are many correct equivalents and some have two terms in $x\pi$ to differentiate obtaining for example $\frac{2\pi}{3} - \frac{8\pi}{24}$ instead of $\frac{\pi}{3}$
	2nd M1	Setting their $\frac{dP}{dx} = 0$. Do not need to find x, but if inequalities are used this mark cannot be gained until candidate states or uses a value of x without inequalities. May not be explicit but may be implied by correct working and value or expression for x . May result in $x^2 < 0$ so M1A0
	2nd A1	There is no requirement to write down a value for x, so this mark may be implied by a correct value for P. It may be given for a correct expression or value for x of 16.6, 16.7 or 17
	3rd A1	Allow answers wrt 120 but not 121
(e)	M1	Finds P'' **and** considers sign. Follow through **correct** differentiation of their P' (not just reduction of power)
	A1ft	Need $\frac{2000}{x^3}$ and > 0 (or positive value) and conclusion. Only follow through on a correct P'' and a value for x in the range $10 < x < 25$ (need not see x substituted but an x should have been found) If P is substituted then this is awarded M1 A0

	Special case	(d) Some candidates multiply P by 12 to "simplify" If they write $\frac{dP}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$; $= 0$ then solve they will get the correct x and P They should be awarded M1A0M1A1A1 in part (d). If they then do part (e) writing $\frac{d^2P}{dx^2} = \frac{24000}{x^3} > 0 \Rightarrow$ Minimum They should be awarded M1A0 (so lose 2 marks in all) If they wrote $\frac{d(12P)}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$; $= 0$ etc they could get full marks.