Modelling with Differentiation - Edexcel Past Exam Questions 2

1.

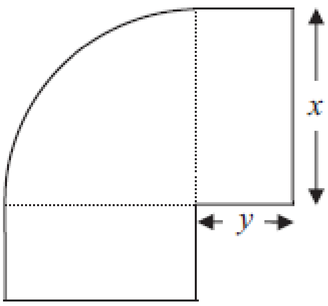

Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4m^2,

(*a*) show that $$y = \frac{16 - \pi x^2}{8x}.$$ **(3)**

(*b*) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x.$$ **(3)**

(*c*) Use calculus to find the minimum value of P. **(5)**

(*d*) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre. **(2)**

Jan 12 Q8

2.

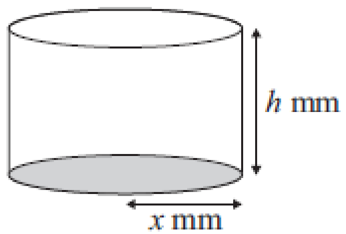

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(*a*) express h in terms of x, **(1)**

(*b*) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \frac{120}{x}$. **(3)**

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(*c*) Use calculus to find the value of x for which A is a minimum. **(5)**

(*d*) Calculate the minimum value of A, giving your answer to the nearest integer. **(2)**

(*e*) Show that this value of A is a minimum. **(2)**

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3. The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$.

(*a*) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$. **(4)**

(*b*) Find the x-coordinate of the other turning point Q on the curve. **(1)**

(*c*) Find $\frac{d^2y}{dx^2}$. **(1)**

(*d*) Hence or otherwise, state with justification, the nature of each of these turning points P and Q. **(3)**

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4. The curve with equation

$$y = x^2 - 32\sqrt{x} + 20, \qquad x > 0,$$

has a stationary point P.

Use calculus

(*a*) to find the coordinates of P, **(6)**

(*b*) to determine the nature of the stationary point P. **(3)**

June 13 Q9

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5. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \qquad x > 0$$ **(6)**

June 13(R) Q1

6.

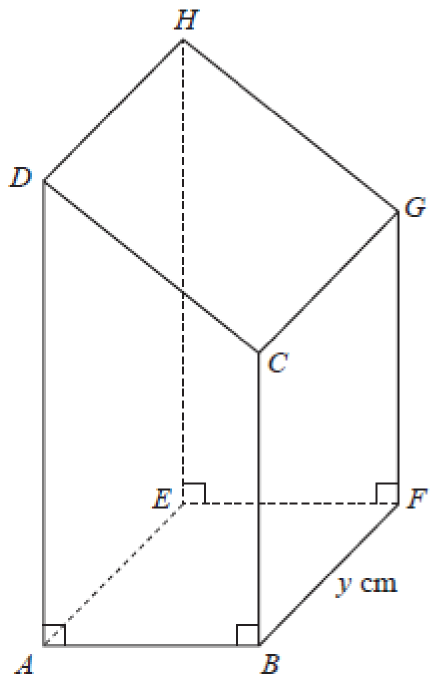

Figure 4

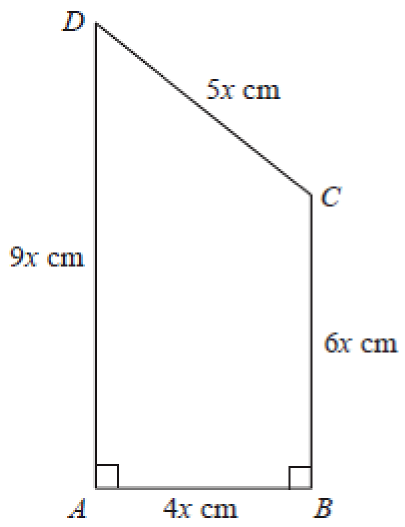

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.

The angle $DAB = 90°$ and the angle $ABC = 90°$. The volume of the letter box is 9600 cm^3.

(*a*) Show that $y = \dfrac{320}{x^2}$. **(2)**

(*b*) Hence show that the surface area of the letter box, S cm^2, is given by $S = 60x^2 + \dfrac{7680}{x}$. **(4)**

(*c*) Use calculus to find the minimum value of S. **(6)**

(*d*) Justify, by further differentiation, that the value of S you have found is a minimum. **(2)**

June 14 Q10

7.

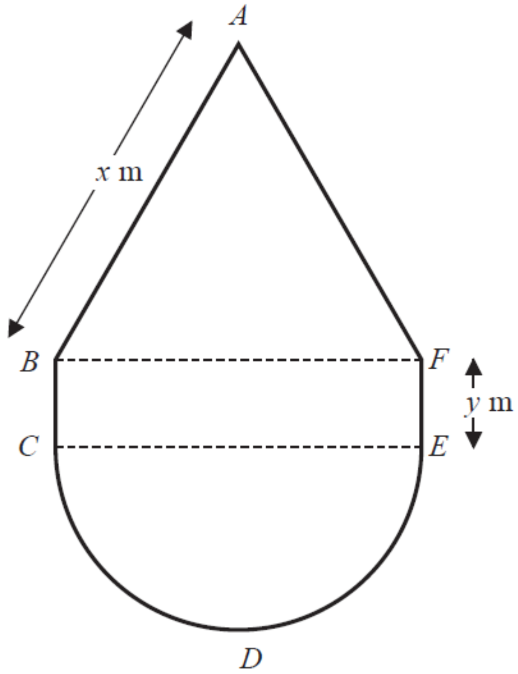

Figure 4

Figure 4 shows the plan of a pool.

The shape of the pool *ABCDEFA* consists of a rectangle *BCEF* joined to an equilateral triangle *BFA* and a semi-circle *CDE*, as shown in Figure 4.

Given that *AB* = *x* metres, *EF* = *y* metres, and the area of the pool is 50 m^2,

(*a*) show that

$$y = \frac{50}{x} - \frac{x}{8}\left(\pi + 2\sqrt{3}\right)$$

(3)

(*b*) Hence show that the perimeter, *P* metres, of the pool is given by

$$P = \frac{100}{x} + \frac{x}{4}\left(\pi + 8 - 2\sqrt{3}\right)$$

(3)

(*c*) Use calculus to find the minimum value of *P*, giving your answer to 3 significant figures.

(5)

(*d*) Justify, by further differentiation, that the value of *P* that you have found is a minimum.

(2)

June 14(R) Q9

8. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.

The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(*a*) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}.$$ **(4)**

(*b*) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound. **(5)**

(*c*) Justify that the answer that you have obtained in part (*b*) is a minimum. **(1)**

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9.

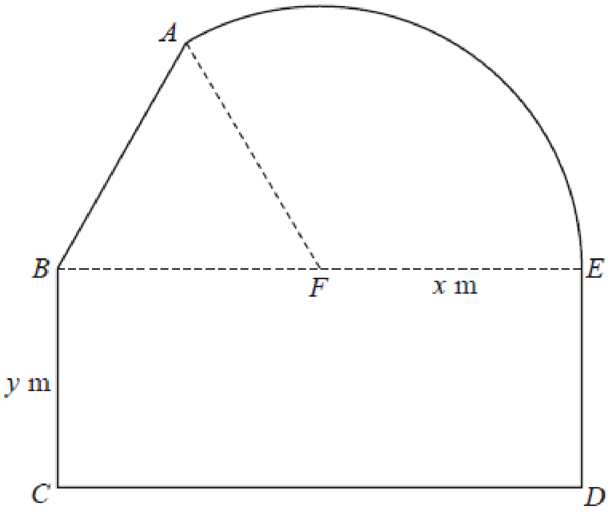

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leq x \leq 25$.

(*a*) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form. **(2)**

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2,

(*b*) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right).$$

(3)

(*c*) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right).$$

(3)

(*d*) Use calculus to find the minimum value of P, giving your answer to the nearest metre. **(5)**

(*e*) Justify, by further differentiation, that the value of P you have found is a minimum. **(2)**

June 16 Q9