Trigonometric Equations and Identities - Edexcel Past Exam Questions 2 MARK SCHEME

Question 1

Question number	Scheme	Marks
(i)	$\sin(3x-15) = \frac{1}{2}$ so $3x-15 = 30$ (α) and $x = 15$	M1 A1
	Need $3x-15 = 180 - \alpha$ or $3x-15 = 540 - \alpha$	M1
	Need $3x-15 = 180 - \alpha$ and $3x-15 = 360 + \alpha$ and $3x-15 = 540 - \alpha$	M1
	$x = 55$ or 175	A1
	$x = 55, 135, 175$	A1 (6)
Notes	M1 Correct order of operation: inverse sine then linear algebra - not just $3x-15 = 30$ (slips in linear algebra lose Accuracy mark) A1 Obtains first solution 15 M1 Uses either $180 - \alpha$ or $540 - \alpha$, M1 uses all three $180 - \alpha$ and $360 + \alpha$ and $540 - \alpha$ A1, for one further correct solution 55 or 175, (depends only on second M1) A1 – all 3 further correct solutions If more than 4 solutions in range, lose last A1 Common slips: Just obtains 15 and 55, or 15 and 175 – usually M1A1M1M0A1A0 Just obtains 15 and 135 is usually M1A1M0M0A0A0 (It is easy to get this erroneously) Obtains 5, 45, 125 and 165 – usually M1A0M1M1A0A0 Obtains 25, 65, 145, (185) usually M1A0M1M1A0A0 Working in radians – lose last A1 earned for $\frac{\pi}{12}, \frac{11\pi}{36}, \frac{3\pi}{4}$ and $\frac{35\pi}{36}$ or numerical equivalents Mixed radians and degrees is usually Method marks only Methods involving no working should be sent to Review	
(ii)	At least one of $\left(\frac{a\pi}{10} - b\right) = 0$ (or $n\pi$) $\left(\frac{a3\pi}{5} - b\right) = \pi$ {or $(n+1)\pi$} or in degrees or $\left(\frac{a11\pi}{10} - b\right) = 2\pi$ {or $(n+2)\pi$}	M1
	If two of above equations used eliminates a or b to find one or both of these or uses period property of curve to find a or uses other valid method to find either a or b (May see $\frac{5\pi}{10} a = \pi$ so $a = $)	M1
	Obtains $a = 2$	A1
	Obtains $b = \frac{\pi}{5}$ (must be in radians)	A1 (4)
Notes	M1: Award for $\left(\frac{a\pi}{10} - b\right) = 0$ or $\frac{a\pi}{10} = b$ BUT $\sin\left(\frac{a\pi}{10} - b\right) = 0$ is M0 M1: As described above but solving $\left(\frac{a\pi}{10} - b\right) = 0$ with $\left(\frac{a3\pi}{5} - b\right) = 0$ is M0 (It gives $a = b = 0$) Special cases: Can obtain full marks here for both correct answers with no working M1M1A1A1 For $a = 2$ only, with no working, award M0M1A1A0 For $b = \frac{\pi}{5}$ only with no working M1M0A0A1	

Question 2

Question number	Scheme		Marks
(a)	States or uses $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$		M1
	$\dfrac{\sin 2x}{\cos 2x} = 5\sin 2x \Rightarrow \sin 2x - 5\sin 2x\cos 2x = 0 \Rightarrow \sin 2x(1 - 5\cos 2x) = 0$ *		A1
			(2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
	$\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) **or** $2x = 281.54$ (or 281.6)		M1
	$x = 39.2$ (or 39.3), 140.8 (or 141)		A1, A1
			(5)
			7 marks

Notes	
	(a) **M1**: Statement that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or Replacement of tan (wherever it appears). Must be a correct statement but may involve θ instead of $2x$.
	A1: the answer is given so all steps should be given.
	N.B. $\sin 2x - 5\sin 2x\cos 2x = 0$ or $-5\sin 2x\cos 2x + \sin 2x = 0$ or $\sin 2x\left(\dfrac{1}{\cos 2x} - 5\right) = 0$ o.e.
	must be seen and be followed by printed answer for A1 mark
	$\sin 2x = 5\sin 2x\cos 2x$ is not sufficient.
	(b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions
	M1: This mark for one of the two statements given (must relate to $2x$ not just to x)
	A1, A1: first A1 for 39.2, second for 140.8
	Special case solving $\cos 2x = -1/5$ giving $2x = 101.5$ or 258.5 is awarded M1A0A0
	140.8 omitted would give M1A1A0
	Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141
	Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14)
	Excess answers **in range** lose last A1 Ignore excess answers outside range.
	All 5 correct answers with no extras and no working gets **full marks** in part (b). The answers imply the method here

Question 3

Question Number	Scheme		Marks
	$\cos^{-1}(-0.4) = 113.58 \ (\alpha)$	Awrt 114	B1
	$3x - 10 = \alpha \Rightarrow x = \dfrac{\alpha + 10}{3}$	Uses their α to find x. Allow $x = \dfrac{\alpha \pm 10}{3}$ **not** $\dfrac{\alpha}{3} \pm 10$	M1
	$x = 41.2$	Awrt	A1
	$(3x - 10 =) 360 - \alpha \ (246.4....)$	$360 - \alpha$ (can be implied by 246.4...)	M1
	$x = 85.5$	Awrt	A1
	$(3x - 10 =) 360 + \alpha \ (= 473.57....)$	$360 + \alpha$ (Can be implied by 473.57...)	M1
	$x = 161.2$	Awrt	A1

Question 4

Question Number	Scheme	Marks
(i)	$(\lvert\alpha\rvert = 56.3099...)$	
	$x = \{\alpha + 40 = 96.309993...\} =$ **awrt 96.3**	B1
	$x - 40^{\circ} = -180 + "56.3099"...$ or $x - 40^{\circ} = -\pi + "0.983"...$	M1
	$x = \{-180 + 56.3099... + 40 = -83.6901...\} =$ **awrt -83.7**	A1
		(3)
(ii)(a)	$\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$	M1
	$\left(\dfrac{1 - \cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$	dM1
	$1 - \cos^2\theta = 3\cos^2\theta + 2\cos\theta \;\Rightarrow\; 0 = 4\cos^2\theta + 2\cos\theta - 1$ *	A1 cso *
		(3)
(b)	$\cos\theta = \dfrac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	M1
	or $4(\cos\theta \pm \tfrac{1}{4})^2 \pm q \pm 1 = 0$, or $(2\cos\theta \pm \tfrac{1}{2})^2 \pm q \pm 1 = 0$, $q \neq 0$ so $\cos\theta = ...$	
	One solution is 72° or 144°, Two solutions are 72° and 144°	A1, A1
	$\theta = \{72, 144, 216, 288\}$	M1 A1
		(5)
		[11]

	Notes for Question	
(i)	B1: 96.3 by any or no method	
	M1: Takes 180 degrees from arctan (1.5) or from their "96.3" May be implied by A1. (Could be obtained by adding 180, then subtracting 360).	
	A1: awrt −83.7	
	Extra answers: ignore extra answers outside range. Any extra answers in range lose final A mark (if earned)	
	Working in radians – could earn M1 for $x - 40^{\circ} = -\pi + "0.983"...$ so B0M1A0	
(ii) (a)	M1: uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or equivalent in equation (not just $\tan = \dfrac{\sin}{\cos}$, with no argument)	
	dM1: uses $\sin^2\theta = 1 - \cos^2\theta$ (quoted correctly) in equation	
	A1: completes proof correctly, with no errors to give printed answer*. Need at least three steps in proof and need to achieve the correct quadratic with all terms on one side and "=0"	
(b)	M1: Attempts to solve quadratic by correct quadratic formula, or completion of the square . Factorisation attempts score M0.	
	1st A1: Either 72 or 144, 2nd A1: both 72 **and** 144 (allow 72.0 etc.)	
	M1: 360 − "a previous solution" (provided that cos was being used) (not dependent on previous M)	
	A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore)	
	(Premature approximation: e.g. 71.9, 144.1, 288.1 and 215.9 – lose first A1 then ft other angles)	
	Do not require degrees symbol for the marks	
	Special case: Working in radians	
	M1: as before, A1 for either $\theta = \tfrac{2}{5}\pi$ or $\theta = \tfrac{4}{5}\pi$ or decimal equivalents, and 2nd A1: both	
	M1: $2\pi - \alpha_1$ or $2\pi - \alpha_2$ then A0 so 4/5	

Question Number	Scheme	Marks
(a)	$\sin(2\theta - 30) = -0.6$ or $2\theta - 30 = -36.9$ or implied by 216.9	B1
	$2\theta - 30 = 216.87 = (180 + 36.9)$	M1
	$\theta = \dfrac{216.87 + 30}{2} = 123.4$ or 123.5	A1
	$2\theta - 30 = 360 - 36.9$ or 323.1	M1
	$\theta = \dfrac{323.1 + 30}{2} = 176.6$	A1 (5)
(b)	$9\cos^2 x - 11\cos x + 3\left(1 - \cos^2 x\right) = 0$ or $6\cos^2 x - 11\cos x + 3\left(\sin^2 x + \cos^2 x\right) = 0$	M1
	$6\cos^2 x - 11\cos x + 3 = 0$ {as $\left(\sin^2 x + \cos^2 x\right) = 1$}	A1
	$(3\cos x - 1)(2\cos x - 3) = 0$ implies $\cos x =$	M1
	$\cos x = \dfrac{1}{3}, \; \left(\dfrac{3}{2}\right)$	A1
	$x = 70.5$	B1
	$x = 360 - "70.5"$ $x = 289.5$	M1 A1cao (7)
		Total 12

	Notes for Question
(a)	B1: This statement seen and must contain no errors or may implied by -36.9
	M1: Uses $180 - \arcsin(-0.6)$ i.e. $180 + 36.9$ (or $\pi + \arcsin(0.6)$ in radians) (in 3rd quadrant)
	A1: allow answers which round to 123.4 or 123.5 must be in degrees
	M1: Uses $360 + \arcsin(-0.6)$ i.e. $360 - 36.9$ (or $2\pi + \arcsin(-0.6)$ in radians) (in 4th quadrant)
	A1: allow answers which round to 176.6 must be in degrees (A1 implies M1)
	Ignore extra answers outside range but lose final A1 for extra answers in the range if both B and A marks have been earned)
	Working in radians may earn B1M1A0M1A0
(b)	M1: Use of $\sin^2 x = \left(1 - \cos^2 x\right)$ or $\left(\sin^2 x + \cos^2 x\right) = 1$ in the given equation
	A1: Correct three term quadratic in any equivalent form
	M1: Uses standard method to solve quadratic and obtains $\cos x =$
	A1: A1 for $\dfrac{1}{3}$ with $\dfrac{3}{2}$ ignored but A0 if $\dfrac{3}{2}$ is incorrect
	B1: 70.5 or answers which round to this value
	M1: $360 - \arccos(their 1/3)$ (or $2\pi - \arccos(their 1/3)$ in radians)
	A1: Second answer
	Working in radians in (b) may earn M1A1M1A1B0M1A0
	Extra values in the range coming from arcos (1/3) – deduct final A mark - so A0

Question 6

Question Number	Scheme		Marks
	(i) $9\sin(\theta + 60°) = 4$; $0 \le \theta < 360°$		
	(ii) $2\tan x - 3\sin x = 0$; $-\pi \le x < \pi$		
(i)	$\sin(\theta + 60°) = \dfrac{4}{9}$, so $(\theta + 60°) = 26.3877...$ $(\alpha = 26.3877...)$	Sight of $\sin^{-1}\left(\dfrac{4}{9}\right)$ or awrt $26.4°$ or 0.461^c Can also be implied for $\theta =$ awrt -33.6 (i.e. $26.4 - 60$)	M1
	So, $\theta + 60° = \{153.6122..., 386.3877...\}$	$\theta + 60° =$ either "$180 -$ their α" or "$360° +$ their α" and not for $\theta =$ either "$180 -$ their α" or "$360° +$ their α". This can be implied by later working. The candidate's α could also be in radians but do not allow mixing of degrees and radians.	M1
	and $\theta = \{93.6122..., 326.3877...\}$	A1: At least one of awrt $93.6°$ or awrt $326.4°$	A1 A1
		A1: Both awrt $93.6°$ and awrt $326.4°$	
	Both answers are cso and must come from correct work		
	Ignore extra solutions outside the range.		
	In an otherwise fully correct solution deduct the final A1 for any extra solutions in range		
			[4]
(ii)	$2\left(\dfrac{\sin x}{\cos x}\right) - 3\sin x = 0$	Applies $\tan x = \dfrac{\sin x}{\cos x}$	M1
	Note: Applies $\tan x = \dfrac{\sin x}{\cos x}$ can be implied by $2\tan x - 3\sin x = 0 \Rightarrow \tan x (2 - 3\cos x)$		
	$2\sin x - 3\sin x \cos x = 0$		
	$\sin x (2 - 3\cos x) = 0$		
	$\cos x = \dfrac{2}{3}$	$\cos x = \dfrac{2}{3}$	A1
	$x =$ awrt $\{0.84, -0.84\}$	A1: One of either awrt 0.84 or awrt -0.84 A1ft: You can apply ft for $x = \pm \alpha$, where $\alpha = \cos^{-1} k$ and $-1 \le k \le 1$	A1A1ft
	In this part of the solution, if there are any extra answers in range in an otherwise correct solution withhold the A1ft.		
	$\{\sin x = 0 \Rightarrow\}\; x = 0$ and $-\pi$	Both $x = 0$ and $-\pi$ or awrt -3.14 from $\sin x = 0$ In this part of the solution, ignore extra solutions in range.	B1
	Note solutions are: $x = \{-3.1415..., -0.8410..., 0, 0.8410...\}$		
	Ignore extra solutions outside the range		
	For all answers in degrees in (ii) M1A1A0A1ftB0 is possible		
	Allow the use of θ in place of x in (ii)		
			[5]
			Total 9

Question 7

Question Number	Scheme		Marks
	$$\dfrac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1; \quad 0 \leqslant \theta < 180^{\circ}$$		
(i)	$\sin 2\theta = \dfrac{1}{3}$	$\sin 2\theta = k$ where $-1 < k < 1$ **Must be 2θ and not θ.**	M1
	$\{2\theta = \{19.4712..., 160.5288...\}\}$		
	$\theta = \{9.7356..., 80.2644...\}$	A1: Either awrt 9.7 or awrt 80.3	A1 A1
		A1: Both awrt 9.7 and awrt 80.3	
	Do not penalise poor accuracy more than once e.g. 9.8 and 80.2 from correct work could score M1A1A0		
	If both answers are correct in radians award A1A0 otherwise A0A0 Correct answers are 0.2 and 1.4		
	Extra solutions in range in an otherwise fully correct solution deduct the last A1		
			[3]
	$5\sin^2 x - 2\cos x - 5 = 0, \quad 0 \leq x < 2\pi.$		
	$5(1 - \cos^2 x) - 2\cos x - 5 = 0$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	$5\cos^2 x + 2\cos x = 0$ $\cos x(5\cos x + 2) = 0$ $\Rightarrow \cos x = ...$	Cancelling out $\cos x$ or a valid attempt at solving the quadratic in $\cos x$ **and** giving $\cos x = ...$ **Dependent on the previous method mark.**	dM1
(ii)	awrt 1.98 or awrt 4.3(0)	Degrees: 113.58, 246.42	A1
	Both 1.98 and 4.3(0)	or their α and their $2\pi - \alpha$, where $\alpha \neq \dfrac{\pi}{2}$. If working in degrees allow $360 - $ their α	A1ft
	awrt 1.57 or $\dfrac{\pi}{2}$ and 4.71 or $\dfrac{3\pi}{2}$ or 90° and 270°	These answers only but ignore other answers **outside** the range	B1
			[5]
	NB: $x = $ awrt $\left\{1.98, 4.3(0), 1.57 \text{ or } \dfrac{\pi}{2}, 4.71 \text{ or } \dfrac{3\pi}{2}\right\}$		8
	Answers in degrees: 113.58, 246.42, 90, 270 Could score M1M1A0A1ftB1 (4/5)		

Question 8

Question Number	Scheme		Marks
(i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = \dfrac{\pi}{3}$	Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\dfrac{1}{2}$ or $\sin 3\theta = \pm\dfrac{\sqrt{3}}{2}$ so $(3\theta) = \dfrac{\pi}{3}$	M1
	Adds π or 2π to previous value of angle (to give $\dfrac{4\pi}{3}$ or $\dfrac{7\pi}{3}$)		M1
	So $\theta = \dfrac{\pi}{9}, \dfrac{4\pi}{9}, \dfrac{7\pi}{9}$ (all three, no extra in range)		A1 (3)
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$ Applies $\sin^2 x = 1 - \cos^2 x$		M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$		dM1
	$\cos x = \dfrac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \dfrac{1}{8} \pm \sqrt{\dfrac{1}{64} + \dfrac{k}{4}}$ or other correct equivalent		A1 (3)
(b)	$\cos x = \dfrac{1 \pm \sqrt{49}}{8} = 1$ and $-\dfrac{3}{4}$ (see the note below if errors are made)		M1
	Obtains two solutions from $0, 139, 221$ (0 or 2.42 or 3.86 in radians)		dM1
	$x = 0$ and 139 and 221 (allow awrt 139 and 221) must be in degrees		A1 (3)
			[9]

(i) **M1**: Obtains $\dfrac{\pi}{3}$. Allow $x = \dfrac{\pi}{3}$ or even $\theta = \dfrac{\pi}{3}$. Need not see working here. May be implied by $\theta = \dfrac{\pi}{9}$ in final answer (allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark)

Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm\dfrac{1}{\sqrt{3}}$

M1: Adding π or 2π to a previous value however obtained. It is not dependent on the previous mark. (May be implied by final answer of $\theta = \dfrac{4\pi}{9}$ or $\dfrac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420).

A1: Need all three correct answers in terms of π and **no extras in range**.

Three correct answers implies M1M1A1

NB : $\theta = 20°$,$80°$, $140°$ earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0

(ii) (a) **M1**: Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$).

This must be awarded in (ii) (a) for an expression with k not after $k = 3$ is substituted.

dM1: Uses formula or completion of square to obtain $\cos x = $ expression in k

(Factorisation attempt is M0) **A1**: cao - award for their final simplified expression

(b) **M1**: **Either** attempts to substitute $k = 3$ into their answer to obtain two values for $\cos x$

Or restarts with $k = 3$ to find two values for $\cos x$ (They cannot earn marks in ii(a) for this)

In both cases they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) **and** correct method for solving their quadratic (usual rules – see notes) The values for $\cos x$ may be >1 or < -1

dM1: Obtains two correct values for x

A1: Obtains **all three correct values** in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.

Question 9

Question Number	Scheme		Marks
	$1 - 2\cos\left(\theta - \dfrac{\pi}{5}\right) = 0$; $-\pi < \theta \leqslant \pi$		
(i)	$\cos\left(\theta - \dfrac{\pi}{5}\right) = \dfrac{1}{2}$	Rearranges to give $\cos\left(\theta - \dfrac{\pi}{5}\right) = \dfrac{1}{2}$ or $-\dfrac{1}{2}$	M1
	$\theta = \left\{-\dfrac{2\pi}{15},\ \dfrac{8\pi}{15}\right\}$	At least one of $-\dfrac{2\pi}{15}$ or $\dfrac{8\pi}{15}$ or $-24°$ or $96°$ or awrt 1.68 or awrt -0.419	A1
		Both $-\dfrac{2\pi}{15}$ and $\dfrac{8\pi}{15}$	A1
			[3]
NB Misread	Misreading $\dfrac{\pi}{5}$ as $\dfrac{\pi}{6}$ or $\dfrac{\pi}{3}$ (or anything else)– treat as misread so M1 A0 A0 is maximum mark		
	$4\cos^2 x + 7\sin x - 2 = 0$, $0 \leqslant x < 360°$		
(ii)	$4(1 - \sin^2 x) + 7\sin x - 2 = 0$	Applies $\cos^2 x = 1 - \sin^2 x$	M1
	$4 - 4\sin^2 x + 7\sin x - 2 = 0$		
	$4\sin^2 x - 7\sin x - 2 = 0$	Correct 3 term, $4\sin^2 x - 7\sin x - 2 \ \{= 0\}$	A1 oe
	$(4\sin x + 1)(\sin x - 2)\ \{=0\}$, $\sin x = ...$	Valid attempt at solving and $\sin x = ...$	M1
	$\sin x = -\dfrac{1}{4},\ \{\sin x = 2\}$	$\sin x = -\dfrac{1}{4}$ (See notes.)	A1 cso
	$x = \text{awrt}\{194.5,\ 345.5\}$	At least one of awrt 194.5 or awrt 345.5 or awrt 3.4 or awrt 6.0	A1ft
		awrt 194.5 and awrt 345.5	A1
			[6]
			9
NB Misread	Writing equation as $4\cos^2 x - 7\sin x - 2 = 0$ with a sign error should be marked by applying the scheme as **it simplifies** the solution (do not treat as misread) Max mark is 3/6		
	$4(1 - \sin^2 x) - 7\sin x - 2 = 0$		M1
	$4\sin^2 x + 7\sin x - 2 = 0$		A0
	$(4\sin x - 1)(\sin x + 2)\ \{=0\}$, $\sin x = ...$	Valid attempt at solving and $\sin x = ...$	M1
	$\sin x = +\dfrac{1}{4},\ \{\sin x = -2\}$	$\sin x = \dfrac{1}{4}$ (See notes.)	A0
	$x = \text{awrt} 165.5$		A1ft
	Incorrect answers		A0

		Question	Notes

<table>
<tr><td>(i)</td><td>M1</td><td colspan="2">Rearranges to give $\cos\left(\theta - \dfrac{\pi}{5}\right) = \pm\dfrac{1}{2}$</td></tr>
<tr><td></td><td>Note</td><td colspan="2">M1 can be implied by seeing either $\dfrac{\pi}{3}$ or $60°$ as a result of taking $\cos^{-1}(...)$.</td></tr>
<tr><td></td><td>A1</td><td colspan="2">Answers may be in degrees or radians for this mark and may have just one correct answer
Ignore mixed units in working if correct answers follow (recovery)</td></tr>
<tr><td></td><td>A1</td><td colspan="2">Both answers correct and in radians as multiples of π $-\dfrac{2\pi}{15}$ and $\dfrac{8\pi}{15}$
Ignore EXTRA solutions outside the range $-\pi < \theta \le \pi$ but lose this mark for extra solutions in this range.</td></tr>
<tr><td>(ii)</td><td>1st M1</td><td colspan="2">Using $\cos^2 x = 1 - \sin^2 x$ on the given equation. [Applying $\cos^2 x = \sin^2 x - 1$, scores M0.]</td></tr>
<tr><td></td><td>1st A1</td><td colspan="2">Obtaining a correct three term equation eg. either $4\sin^2 x - 7\sin x - 2 \{= 0\}$
or $-4\sin^2 x + 7\sin x + 2 \{= 0\}$ or $4\sin^2 x - 7\sin x = 2$ or $4\sin^2 x = 7\sin x + 2$, etc.</td></tr>
<tr><td></td><td>2nd M1</td><td colspan="2">For a valid attempt at solving a 3TQ quadratic in sine. Methods include factorization, quadratic formula, completion of the square (unlikely here) and calculator. (See notes on page 6 for general principles on awarding this mark) Can use any variable here, s, y, x or $\sin x$, and an attempt to find at least one of the solutions for $\sin x$. This solution may be outside the range for $\sin x$</td></tr>
<tr><td></td><td>2nd A1</td><td colspan="2">$\sin x = -\dfrac{1}{4}$ BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer
of $\sin x = 2$, but penalise if candidate states an incorrect result. e.g. $\sin x = -2$.</td></tr>
<tr><td></td><td>Note</td><td colspan="2">$\sin x = -\dfrac{1}{4}$ can be implied by later correct working if no errors are seen.</td></tr>
<tr><td></td><td>3rd A1ft</td><td colspan="2">At least one of awrt 194.5 or awrt 345.5 or awrt 3.4 or awrt 6.0. This is a limited follow through.
Only follow through on the error $\sin x = \dfrac{1}{4}$ and allow for 165.5 special case (as this is equivalent work)   This error is likely to earn M1A1M1A0A1A0 so 4/6 or M1A0M1A0A1A0 if the quadratic had a sign slip.</td></tr>
<tr><td></td><td>4th A1
Note</td><td colspan="2">awrt 194.5 and awrt 345.5
If there are any EXTRA solutions inside the range $0 \le x < 360°$ and the candidate would otherwise score FULL MARKS then withhold the final A1 mark.
Ignore EXTRA solutions outside the range $0 \le x < 360°$.</td></tr>
<tr><td></td><td>Special Cases</td><td colspan="2">Rounding error Allow M1A1M1A1A1A0 for those who give two correct answers but wrong accuracy e.g. awrt 194, 346 (Remove final A1 for this error)
Answers in radians:– lose final mark so either or both of 3.4, 6.0 gets A1ftA0
It is possible to earn M1A0A1A1 on the final 4 marks if an error results fortuitously in $\sin x = -1/4$ then correct work follows.</td></tr>
</table>

Question Number	Scheme	Marks
	$\dfrac{\sin x}{16} = \dfrac{\sin 50^0}{13}$	M1
	$(\sin x) = \dfrac{16 \times \sin 50}{13}$ (= 0.943 but accept 0.94)	A1
		dM1 A1
	x = awrt 70.5(3) **and** 109.5 or 70.6 and 109.4	(4) [4]

<div align="center">Notes</div>

M1: use sine formula correctly **in any form**. Allow awrt 0.77 for $\sin 50°$

A1: give the correct value or correct expression for $\sin x$ (this implies the M1 mark).

If it is given as expression they do not need degrees symbol. $\dfrac{\sin 50 \times 16}{13}$ is fine,

If this is given as a decimal allow answers which round to 0.94.

Allow awrt −0.323 (radians) here but no further marks are available.

If they give this as x (not sin$x$) and do not recover this is A0

dM1: Correct work leading to x=… (via inverse sin) expression or value for sin$x$

If the previous A mark has been awarded for a correct expression then this is for getting to awrt 70.5 or 109.5 (allow for 70.6 or 109.4).

If the previous A mark was not gained, e.g. rounding errors were made in rearranging the correct sine formula then award dM1 for evidence of use of **inverse sin in degrees** on their value for sin$x$ (may need to check on calculator).

NB 70.5 following a correct sine formula will gain M1A1M1.

A1: deduce and state **both of the answers** x = 70.5 and 109.5 (do not need degrees) Accept awrt these. Also accept 70.6 and 109.4.

(Second answer is sometimes obtained by a long indirect route but still scores A1)

If working in radians throughout, answers are 1.23 and 1.91 and this can be awarded M1 A1 M1 A0 (Working with 50 radians gives probable answers of -0.3288 and 3.47 giving M1A1M0A0)

Special case: Wrong labelling of triangle. This simplifies the problem as there is only one solution for angle x. So it is not treated as a misread. If they find the missing side as awrt 12.6 then proceed to find an angle or its sine or cosine then give M1A0M0A0

Alternative Method using cosine rule

Let $BC = a$.

M1: uses the cosine rule to form to form a three term quadratic equation in a (e.g.

$a^2 - 32a\cos 50° + 87 = 0$ or $a^2 - \text{awrt } 20.6a + 87 = 0$ though allow slips in signs rearranging)

A1: Solves and obtains a correct value for a of awrt 14.6 or awrt 5.95.

dM1: A **correct full** method to find (at least) one of the two angles. May use cosine rule again, or find angle BAC and then use sine rule. As in the main scheme, if the previous A mark has been awarded then they should obtain one of the correct angles for this mark.

A1: deduces both correct answer as in main scheme.

NB obtaining only one correct angle will usually score M1A1M1A0 in any method.

Question 11

Question Number	Scheme		Marks

(a)	**Way 1**
	
	$1-\sin^2 x = 8\sin^2 x - 6\sin x$
	
	E.g. $9\sin^2 x - 6\sin x = 1$ or
	   $9\sin^2 x - 6\sin x - 1 = 0$ or
	   $9\sin^2 x - 6\sin x + 1 = 2$
	
	So $9\sin^2 x - 6\sin x + 1 = 2$ or
	   $(3\sin x - 1)^2 - 2 = 0$
	so $(3\sin x - 1)^2 = 2$ or
	$2 = (3\sin x - 1)^2$ *
		Way 2
		$2 = (3\sin x - 1)^2$ gives $9\sin^2 x - 6\sin x + 1 = 2$
		so $\sin^2 x + 8\sin^2 x - 6\sin x + 1 = 2$
		
		so $8\sin^2 x - 6\sin x = 1 - \sin^2 x$
		
		$8\sin^2 x - 6\sin x = \cos^2 x$ *
			B1
			
			M1
			
			A1cso*
			
			(3)

(b)	**Way 1:** $(3\sin x - 1) = (\pm)\sqrt{2}$ $\sin x = \dfrac{1 \pm \sqrt{2}}{3}$ or awrt 0.8047 **and** awrt -0.1381 $x = 53.58$, 126.42 (or 126.41), 352.06, 187.94	**Way 2:** Expands $(3\sin x - 1)^2 = 2$ and uses quadratic formula on 3TQ	M1 A1 dM1A1 A1 (5) [8]

Notes

(a) **Way 1**

B1: Uses $\cos^2 x = 1 - \sin^2 x$

M1: Collects $\sin^2 x$ terms to form a three term quadratic or into a suitable completed square format. May be sign slips in the collection of terms.

A1*: cso This needs an intermediate step from 3 term quadratic and no errors in answer and printed answer stated but allow $2 = (3\sin x - 1)^2$. If sin is used throughout instead of sinx it is A0.

Way 2

B1: **Needs correct expansion** and split

M1: Collects $1 - \sin^2 x$ together

A1*: Conclusion and no errors seen

(b) M1: Square roots both sides(Way 1), or expands and uses quadratic formula (Way 2) Attempts at factorization after expanding are M0.

A1: **Both** correct answers for sinx (need plus and minus). Need not be simplified.

dM1: Uses inverse sin to give one of the given correct answers

1st A1: Need two correct angles (allow awrt) Note that the scheme allows 126.41 in place of 126.42 though 126.42 is preferred

A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore)

(Premature approximation:– in the final three marks lose first A1 then ft other angles for second A mark)

Do **not** require degrees symbol for the marks

Special case: Working in radians

M1A1A0 for the *correct* 0.94, 2.21, 6.14, 3.28