Based on the 2022 Advanced Information from Edexcel exam board

Predicted A level Mathematics Paper 2 June 2022

Set A

Time: 2 hours

Information for Candidates

- This predicted paper is based on the 2022 advance information from Edexcel exam board
- There are 12 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Advice to candidates:

- You must ensure that your answers to parts of questions are clearly labelled.
- You must show sufficient working to make your methods clear to the Examiner
- Answers without working may not gain full credit

Disclaimer: There is no guarantee that any specific topic will be examined this way in the summer and you cannot rely on this as your only source of revision. Visit www.naikermaths.com for more practice papers and plenty of revision resources to help you in your revision.

Question 1

A sequence a_1, a_2, a_3, ... is defined by

$$a_{n+1} = 2(a_n + 3)^2 - 7$$
$$a_1 = p - 3$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form. (1)

Given that an $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2

(6)

(Total for question = 7 marks)

Question 2

The adult population of a town at the start of 2019 is 25 000

A model predicts that the adult population will increase by 2% each year, so that the number of adults in the population at the start of each year following 2019 will form a geometric sequence.

(a) Find, according to the model, the adult population of the town at the start of 2032 (3)

It is also modelled that every member of the adult population gives £5 to local charity at the start of each year.

(b) Find, according to these models, the total amount of money that would be given to local charity by the adult population of the town from 2019 to 2032 inclusive. Give your answer to the nearest £1 000 (3)

(Total for question = 6 marks)

Question 3

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(\frac{1}{4} - 5x\right)^{\frac{1}{2}} \qquad |x| < \frac{1}{20}$$

giving each coefficient in its simplest form. **(5)**

By substituting $x = \frac{1}{100}$ into the answer for (a),

(b) find an approximation for $\sqrt{5}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers to be found. **(2)**

(Total for question = 7 marks)

Question 4

The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \qquad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2}\ln x \qquad x > 0, x \in \mathbb{R}$$

(a) Find, in simplest form, the value of $fg(e^2)$ **(2)**

(b) Find f^{-1} **(3)**

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x)$$

(3)

(Total for question = 8 marks)

Question 5

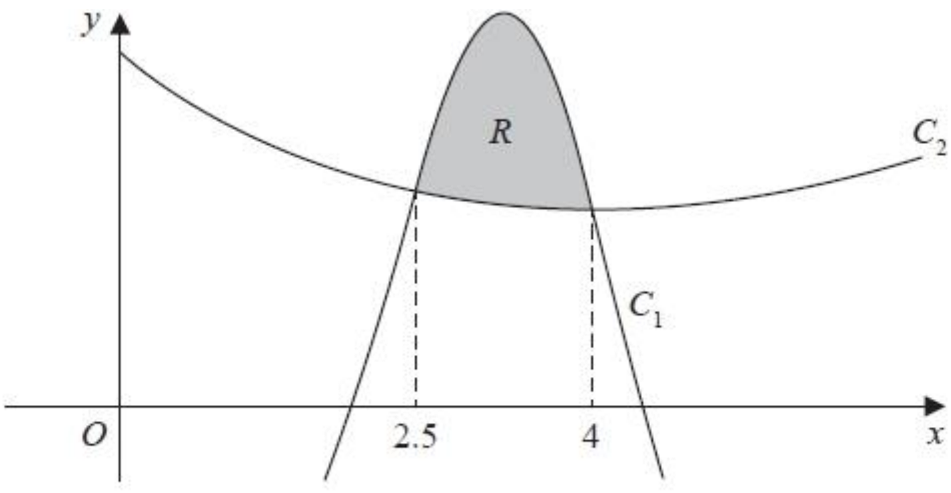

Figure 2

Figure 2 shows a sketch of part of the graph of the curves C_1 and C_2

The curves intersect when $x = 2.5$ and when $x = 4$

A table of values for some points on the curve C_1 is shown below, with y values given to 3 decimal places as appropriate.

x	2.5	2.75	3	3.25	3.5	3.75	4
y	5.453	7.764	9.375	9.964	9.367	7.626	5

Using the trapezium rule with all the values of y in the table,

(a) find, to 2 decimal places, an estimate for the area bounded by the curve C_1, the line with equation $x = 2.5$, the x-axis and the line with equation $x = 4$

(4)

The curve C_2 has equation

$$y = x^{\frac{3}{2}} - 3x + 9 \qquad x > 0$$

(b) Find $\displaystyle\int\left(x^{\frac{3}{2}} - 3x + 9\right)\mathrm{d}x$

(3)

The region R, shown shaded in Figure 2, is bounded by the curves C_1 and C_2

(c) Use the answers to part (a) and part (b) to find, to one decimal place, an estimate for the area of the region R.

(3)

(Total for question = 10 marks)

Question 6

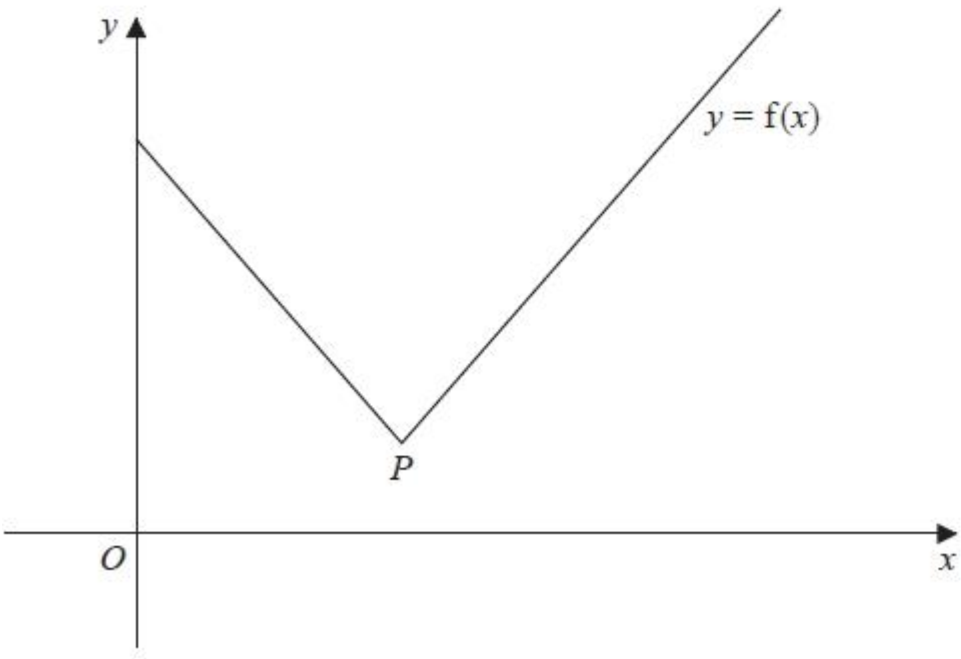

Figure 2

Figure 2 shows part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|2x - 5| + 3 \qquad x \geqslant 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P. **(2)**

(b) Solve the equation $\mathrm{f}(x) = 3x - 2$

(4)

Given that the equation

$$\mathrm{f}(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k. **(3)**

(Total for question = 9 marks)

Question 7

$f(x) = \ln(2x - 5) + 2x^2 - 30, \qquad x > 2.5.$

(*a*) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]. **(2)**

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(*b*) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures. **(2)**

(*c*) Show that α is the only root of $f(x) = 0$. **(2)**

(Total for Question 8 is 6 marks)

Question 8

Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$.

You may assume the formula for $\sin(A \pm B)$ and that, as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$.

(Total for Question 10 is 5 marks)

Question 9

(a) Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians, to 3 decimal places. **(3)**

The function g is defined by

$$g(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \qquad \theta > 0$$

Find

(b) (i) the minimum value of $g(\theta)$

(ii) the smallest value of θ at which the minimum value occurs. **(3)**

The function h is defined by

$$h(\beta) = 10 - (12\sin\beta - 5\cos\beta)^2$$

(c) Find the range of h. **(2)**

(Total for question = 8 marks)

Question 10

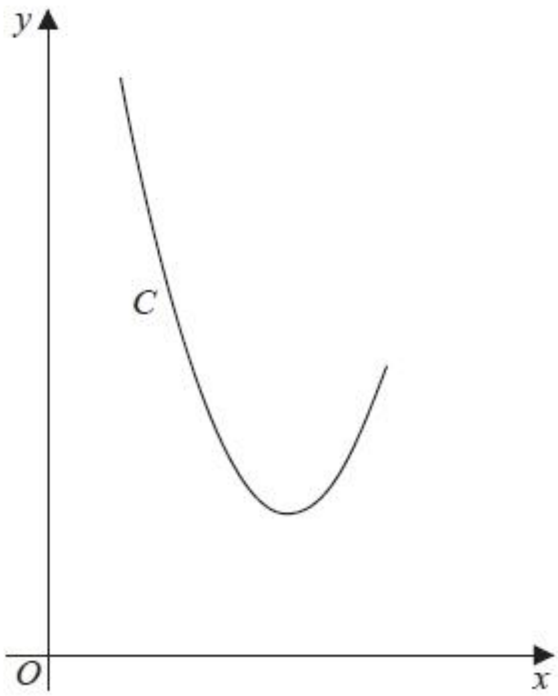

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$ **(4)**

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found. **(3)**

(c) Find the range of f. **(2)**

(Total for question = 9 marks)

Question 11

(i)

$$\mathrm{f}(x) = \frac{(2x+5)^2}{x-3} \qquad x \neq 3$$

(a) Find $\mathrm{f}'(x)$ in the form $\dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(b) Hence find the range of values of x for which $\mathrm{f}(x)$ is increasing. **(6)**

(ii)

$$\mathrm{g}(x) = x\sqrt{\sin 4x} \qquad 0 \leqslant x < \frac{\pi}{4}$$

The curve with equation $y = \mathrm{g}(x)$ has a maximum at the point M.

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

where k is a constant to be found. **(5)**

(Total for question = 11 marks)

Question 12

(a) Write $\dfrac{1}{(H-5)(H+3)}$ in partial fraction form. **(3)**

The depth of water in a storage tank is being monitored.

The depth of water in the tank, *H* metres, is modelled by the differential equation

$$\frac{dH}{dt} = -\frac{(H-5)(H+3)}{40}$$

where *t* is the time, in days, from when monitoring began.

Given that the initial depth of water in the tank was 13 m,

(b) solve the differential equation to show that

$$H = \frac{10 + 3e^{-0.2t}}{2 - e^{-0.2t}}$$

(7)

(c) Hence find the time taken for the depth of water in the tank to fall to 8 m.

(Solutions relying entirely on calculator technology are not acceptable.) **(3)**

According to the model, the depth of water in the tank will eventually fall to *k* metres.

(d) State the value of the constant *k*. **(1)**

(Total for question = 14 marks)

TOTAL FOR PAPER IS 100 MARKS