A level Best Guess Paper 1 June 2023 Mark Scheme

Question 1

Question Number	Scheme	Marks
(a)	Sets $f(3)=0 \rightarrow$ equation in k Eg. $27k-135-96-12=0$	M1
	$\Rightarrow 27k=243 \Rightarrow k=9$ * (= 0 must be seen)	A1*
		(2)
(b)	$9x^3-15x^2-32x-12=(x-3)(9x^2+12x+4)$	M1 A1
	$=(x-3)(3x+2)^2$	dM1 A1
		(4)
(c)	Attempts $\cos\theta=-\frac{2}{3}$	M1
	$\theta=131.8°, 228.2°$ (awrt)	A1
		(2)
		(8 marks)

Notes

(a)

M1 Attempts to set $f(3)=0 \rightarrow$ equation in k Eg. $27k-135-96-12=0$. Condone slips.

Score when you see embedded values within the equation or two correct terms on the lhs of the equation. It is implied by sight of $27k-243=0$ or $27k=135+96+12$.

A1* Completes proof with at least one intermediate "solvable" line namely $27k=243 \Rightarrow k=9$ or $27k-243=0 \Rightarrow k=9$. This is a given answer so there should be no errors.

It is a "show that" question so expect to see

(i) Either $f(3)=0$ explicitly stated or implied by sight of $27k-135-96-12=0$ or $27k-243=0$

(ii) One solvable intermediate line followed by $k=9$

A candidate could use $k=9$ and start with $f(x)=9x^3-15x^2-32x-12$

M1 For attempting $f(3)=9\times 3^3-15\times 3^2-32\times 3-12$.

Alt attempts to divide $f(x)$ by $(x-3)$. See below on how to score such an attempt

A1* Shows that $f(3)=0$ and makes a minimal statement to the effect that "so $k=9$"

If division is attempted it must be correct and a statement is required to the effect that there is no remainder, " so $k=9$"

If candidates have divided (correctly) in part (a) they can be awarded the first two marks in (b) when they start factorising the $9x^2+12x+4$ term.

(b)

M1 Attempt to divide or factorise out $(x-3)$. Condone students who use a different value of k.

For factorisation look for first and last terms $9x^3-15x^2-32x-12=(x-3)\left(\pm 9x^2 .... \pm 4\right)$

For division look for the following line

$$\begin{array}{r} 9x^2 \; + ... \\ x-3\overline{)9x^3-15x^2-32x-12} \\ \underline{9x^3-27x^2} \end{array}$$

A1 Correct quadratic factor $9x^2+12x+4$.

You may condone division attempts that don't quite work as long as the correct factor is seen.

dM1 Attempt at factorising their $9x^2+12x+4$ Apply the usual rules for factorising

A1 $(x-3)(3x+2)^2$ or $(x-3)(3x+2)(3x+2)$ on one line.

Accept $9(x-3)\left(x+\frac{2}{3}\right)^2$ oe. It must be seen as a product

Remember to isw for candidates who go on to give roots $f(x)=(x-3)(3x+2)^2 \Rightarrow x=....$

Note: Part (b) is "Hence" so take care when students **write down the answer** to (b) without method

If candidates state $x=-\frac{2}{3},3 \Rightarrow f(x)=\left(x+\frac{2}{3}\right)\left(x+\frac{2}{3}\right)(x-3)$ score 0 0 0 0

If candidates state $x=-\frac{2}{3},3 \Rightarrow f(x)=(3x+2)(3x+2)(x-3)$ they score SC 1010.

If candidates state $x=-\frac{2}{3},3 \Rightarrow f(x)=9\left(x+\frac{2}{3}\right)\left(x+\frac{2}{3}\right)(x-3)$ they score SC 1010.

If candidate writes down $f(x)=(3x+2)(3x+2)(x-3)$ with no working they score SC 1010.

If a candidate writes down $(x-3)(3x+2)$ are factors it is 0000

(c)

M1 A correct attempt to find one value of θ in the given range for their $\cos\theta=-\frac{2}{3}$

(You may have to use a calculator). So if (b) is factorised correctly the mark is for one of the values.

This can be implied by sight of awrt 132 or 228 in degrees or awrt 2.3 which is the radian solution.

A1 CSO awrt $\theta=131.8°, 228.2°$ with no additional solutions within the range $0 \leqslant \theta < 360°$

Watch for correct solutions appearing from $3\cos\theta-2=0 \Rightarrow \cos\theta=\frac{2}{3}$. This is M0 A0

Answers without working are acceptable.

M1 For one correct answer

M1 A1 For two correct answers with no additional solutions within the range.

Question 2

Question Number	Scheme	Marks
(a)	$\mathrm{f}(x)=\dfrac{5x-3}{x-4}\Rightarrow \mathrm{f}'(x)=\dfrac{5(x-4)-(5x-3)}{(x-4)^2}=\dfrac{k}{(x-4)^2}$	M1 dM1
	States that $\mathrm{f}'(x)=\dfrac{-17}{(x-4)^2}\Rightarrow \mathrm{f}'(x)<0$ hence decreasing * cso	A1*
		(3)
(b)	$y=\dfrac{5x-3}{x-4}\Rightarrow xy-4y=5x-3\Rightarrow xy-5x=4y-3$	M1
	$\Rightarrow x=\dfrac{4y-3}{y-5}$ So $\mathrm{f}^{-1}(x)=\dfrac{4x-3}{x-5}$	A1
	Domain $x>5$	B1
		(3)
(c) (i)	$\mathrm{ff}(x)=\dfrac{5\times\dfrac{5x-3}{x-4}-3}{\dfrac{5x-3}{x-4}-4}$	M1
	$\mathrm{ff}(x)=\dfrac{5\times(5x-3)-3(x-4)}{5x-3-4(x-4)}=\dfrac{22x-3}{x+13}$	dM1 A1
(ii)	$5<\mathrm{ff}(x)<22$	B1, B1
		(5)
		(11 marks)

Question 3

Question Number	Scheme	Marks
(a)	Attempts $(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})-(8\mathbf{i}-5\mathbf{j}+3\mathbf{k})$	M1
	$\overrightarrow{RQ}=-6\mathbf{i}+2\mathbf{j}+\mathbf{k}$	A1
		(2)
(b)	Attempts $\overrightarrow{PQ}\cdot\overrightarrow{RQ}=2\times-6+-3\times2+4\times1$	M1
	Full attempt to find $\cos PQR$ E.g. $2\times-6+-3\times2+4\times1=\sqrt{29}\sqrt{41}\cos PQR$	dM1
	Angle PQR = 114°	A1
		(3)
		(5 marks)

(a)

M1: Attempts to subtract vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ either way around. Look for $\overrightarrow{PQ}-\overrightarrow{PR}$ or $\overrightarrow{PR}-\overrightarrow{PQ}$. If a method is not shown it can be implied by two correct components of $\pm6\mathbf{i}\pm2\mathbf{j}\pm\mathbf{k}$ Note that an attempt such as $\overrightarrow{PR}-\overrightarrow{QP}$ is M0

A1: $\overrightarrow{RQ}=-6\mathbf{i}+2\mathbf{j}+\mathbf{k}$ o.e. such as $\begin{pmatrix}-6\\2\\1\end{pmatrix}$ Do not accept coordinates or indeed $\begin{pmatrix}-6\mathbf{i}\\2\mathbf{j}\\\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts scalar product of $\overrightarrow{PQ}$ and their $\overrightarrow{RQ}$. Look for an attempt at multiplying together the components and adding. There will be some confusion over direction so allow for sight of

$$(\pm2\times"\pm6")+(\pm3\times"\pm2")+(\pm4\times"\pm1")$$

This cannot be scored if they attempt a scalar product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ for instance.

dM1: Full attempt to find $\cos PQR$ using $\mathbf{a}.\mathbf{b}=|\mathbf{a}|\,|\mathbf{b}|\cos\theta$ using vectors $\pm\overrightarrow{PQ}$ and their $\pm\overrightarrow{RQ}$.

There must be an attempt at both moduli with at least one correct (which may be unsimplified) but you should ft on their $\overrightarrow{RQ}$. Don't be concerned whether the angle is acute or obtuse.

A1: Angle PQR = awrt 114° ISW after sight of this, e.g followed by 66°

Alt (b)

M1: Attempts all three lengths or all three lengths ^{2} using Pythagoras' Theorem. Look for an attempt to square and add with at least one modulus or modulus^2 correct.

dM1: Attempts to use the cosine rule with the lengths in the correct positions in order to find angle PQR

Look for $\cos PQR = \dfrac{PQ^2 + QR^2 - PR^2}{2 \times PQ \times QR}$

There are more round about methods including finding other angles first and then using the sine rule but this method mark can only be awarded when an attempt is made at angle PQR

A1: Angle PQR = awrt 114°

It must be found using "correct" vectors, e.g . $\pm(-6\mathbf{i}+2\mathbf{j}+\mathbf{k})$ and $\pm(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})$

If, for example $\overrightarrow{RQ}$ is incorrect, e.g. $6\mathbf{i}-2\mathbf{j}+\mathbf{k}$, A0 will be awarded even if 114° is stated

Question 4

Question	Scheme	Marks
(a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$	M1
	(10, 8)	A1
		(2)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25\ (= r^2)$ **or** $(r^2 =)$ "100"+"64" − 139	M1
	$r = 5$*	A1
		(2)
(c)	Substitute $x = 13$ into the equation of circle and solve quadratic to give $y =$	M1
	e.g. $x = 13 \Rightarrow (13-10)^2 + (y-8)^2 = 25 \Rightarrow (y-8)^2 = 16$ so $y = 4$ **or** 12 **N.B. This can be attempted via a 3, 4, 5 triangle so spotting this and achieving one value for y is M1 A1. Both values scores M1 A1 A1**	A1 A1
		(3)
(d)	$OC = \sqrt{10^2+8^2} = \sqrt{164}$	M1
	Length of tangent $= \sqrt{164-5^2} = \sqrt{139}$	M1 A1
		(3)
		(10 marks)

Notes:

(a)

M1: Obtains $(x \pm 10)^2$ **and** $(y \pm 8)^2$ May be implied by one correct coordinate

A1: (10, 8) Answer only scores both marks.

Alternative: *Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$

M1: Obtains $(\pm 10, \pm 8)$

A1: Centre is $(-g, -f)$, **and so centre is** (10, 8).

(b)

M1: For a correct method leading to $r = \ldots$, or $r^2 =$

Allow "100"+"64"−139 or an attempt at using $(x \pm 10)^2 + (y \pm 8)^2 = r^2$ form to identify $r=$

A1*: $r = 5$ This is a printed answer, so a correct method must be seen.

Alternative:

(b)

M1: Attempts to use $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)$"100"+"64"−139

A1*: $r = 5$ following a correct method.

(c)

M1: Substitutes $x = 13$ into either form of the circle equation, forms and solves the quadratic equation in y

A1: Either $y = 4$ or 12

A1: Both $y = 4$ **and** 12

(d)

M1: Uses Pythagoras' Theorem to find length OC using their $(10, 8)$

M1: Uses Pythagoras' Theorem to find OX. Look for $\sqrt{OC^2 - r^2}$

A1: $\sqrt{139}$ only

Question 5

Question	Scheme	Marks
	For question many variations on the proof are possible. Below is a general outline with some examples, which cover many cases. If you see an approach you do not know how to score, consult your team leader. **M1:** Will be scored for setting up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction and is scored at the point you can see each of these elements. A formal statement of the assumption is not required at this stage. **A1:** Scored for a correct statement from which it is possible to draw a contradiction. **dM1;** For making a complete argument that leads to a (full) contradiction of the initial statement, though may be allowed if there are minor gaps or omissions. **A1:** Correct and complete work with contradiction drawn and conclusion made. There must have been a statement of assumption at the start for which to draw the contradiction, though it may not be technicality a correct assumption as long as a relevant assumption has been made. E.g. Accept "Assume $n^2 - 2$ is divisible be 4 for all n"	
	(Assume that there is an n with $n^2 - 2$ is divisible by 4 so) $n^2 - 2 = 4k$	M1
	then $n^2 = 4k + 2 = 2(2k+1)$ (so is even)	A1
	Hence n^2 is even so n (=$2m$) is even hence n^2 is a multiple of 4 As n^2 is a multiple of 4 then $n^2 - 2 = 4m^2 - 2 = 2(2m^2 - 1)$ cannot be a multiple of 4 (as $2m - 1$ is odd) so there is a contradiction.	dM1
	So the original assumption has been shown false. Hence "$n^2 - 2$ is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes
M1: Sets up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2-2=4k$ **A1:** Reaches $n^2=2(2k+1)$ **dM1:** For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Accept explanations such as "as n^2 is even then n is even hence n^2 is a multiple of 4 so n^2-2 cannot be a multiple of 4 (as 4 does not divide 2)" **A1*:** Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all n"

Alt 1	(Assume that n^2-2 is divisible by 4 for some n,) so $\frac{n^2-2}{4}$ is an integer. Then if n is even $n=2m$ (m integer) so $\frac{n^2-2}{4}=\frac{(2m)^2-2}{4}$ (oe with odd)	M1
	$=m^2-\frac{1}{2}$ (which is not an integer)	A1
	Since m^2 is an integer, $m^2-\frac{1}{2}$ is not, hence n cannot be even, but if n is odd then $\frac{n^2-2}{4}=\frac{(2m+1)^2-2}{4}=m^2+m-\frac{1}{4}$, which is again not an integer (since m^2+m is)	dM1
	Hence there is a contradiction (as n cannot be an integer) Hence "n^2-2 is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes
M1: Sets up an algebraic statement in terms of a variable (integer) m or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this Alt, consider case use of $n=2m$ or $n=2m+1$ in $\frac{n^2-2}{4}$ is sufficient **A1:** Reaches $m^2-\frac{1}{2}$ for n even or $m^2+m-\frac{1}{4}$ for n odd. **dM1:** For a complete argument that leads to a contradiction in both cases. See scheme. Allow if minor details are omitted as long as the overall argument is clear. **A1*:** Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all n"

Alt 2	(Assume that $n^2 - 2$ is divisible by 4) $\Rightarrow n^2 - 2 = 4k$	M1
	$\Rightarrow n^2 = 4k + 2 \Rightarrow n = 2\sqrt{k + \frac{1}{2}}$ or $n = \sqrt{2}\sqrt{2k+1}$	A1
	So for some integer m $\sqrt{k + \frac{1}{2}} = \frac{\mathrm{m}}{2} \Rightarrow 2k + 1 = \frac{m^2}{2}$ but m^2 is odd if m is odd so $\frac{m^2}{2}$ not an integer, or m^2 is a multiple of 4 if m even, so odd=even or $2k + 1$ is odd, so does not have a factor 2 to combine with the $\sqrt{2}$ outside, hence n must be irrational	dM1
	Hence we have a contradiction. So "$n^2 - 2$ is never divisible by 4" is true for all n *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) k or any other variable aside n that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing divisibility by 4 by stating $n^2 - 2 = 4k$

A1: Reaches $n = 2\sqrt{k + \frac{1}{2}}$ or $n = \sqrt{2}\sqrt{2k+1}$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Must be a valid attempt to show that $2\sqrt{k + \frac{1}{2}}$ / $\sqrt{2}\sqrt{2k+1}$ is not an integer, and this method is a hard route.

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all n. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume $n^2 - 2$ is divisible be 4 for all n"

Alt 3	(Assume that n^2-2 is divisible by 4) then for *n* even we have (for some integer *m*) $n^2-2=4m^2-2$ **or** for *n* odd $n^2-2=4\left(m^2+m\right)-1$	M1
	$4m^2-2$ or $4\left(m^2+m\right)-1$	A1
	Since 4 divides n^2-2 and $4m^2$ thus for *n* even, 4 must divide 2, a contradiction, so *n* cannot be even, and also 4 divides $4\left(m^2+m\right)$ so for *n* odd, 4 divides 1, also a contradiction.	dM1
	Hence we have a contradiction for both cases (and as *n* must be either even or odd). so "n^2-2 is never divisible by 4" is true for all *n* *	A1*
		(4)
		(4 marks)

Notes

M1: Sets up an algebraic statement in terms of a variable (integer) *m* or any other variable aside *n* that engages with divisibility by 4 in some way and can lead to a contradiction. No need for explicit statement of assumption - accept if just a suitable equation is set up. In this case supposing using *n* odd or *n* even to form an expression for n^2-2 of the form $4\times\text{integer}\pm\text{non-mulitple of }4$

A1: Reaches $4m^2-2$ or $4\left(m^2+m\right)-1$

dM1: For a complete argument that leads to a contradiction. See scheme. Allow if minor details are omitted as long as the overall argument is clear. Both cases must be considered with a reason for the contradiction given (not just stated not divisible by 4).

A1*: Draws the contradiction to their initial assumption and concludes the statement is true for all *n*. There must have been a clear assumption at the start that is contradicted, and all working must have been correct. For the assumption be generous with the technicality as long as a relevant assumption has been made. E.g. Accept "Assume n^2-2 is divisible be 4 for all *n*"

Question 6

Question	Scheme	Marks
(a)	$\frac{dA}{dt} = -0.5$	B1
	$A = \pi x^2 \Rightarrow \frac{dA}{dx} = 2\pi x$	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = \frac{"-0.5"}{"2\pi x"} \quad \left(= \frac{-1}{4\pi x}\right)$	M1
	$\frac{dx}{dt} = -0.011368...$	A1cso
		(4)
(b)	$V = \pi x^2(3x) = 3\pi x^3$	B1
	$\frac{dV}{dx} = 9\pi x^2$	B1ft
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 9\pi x^2 \times "-\frac{1}{4\pi x}" \ (= -2.25x)$	M1
	$\left(\frac{dV}{dt} =\right) -9 \Rightarrow$ (Rate of decrease $=$) $9 \ \left(\text{mm}^3 \text{ s}^{-1}\right)$	A1
		(4)
		(8 marks)

Notes

(a)

B1: $\frac{dA}{dt} = -0.5$ seen or implied from working

B1: $\frac{dA}{dx} = 2\pi x$ seen or implied from working. Must be in terms of x, but allow recovery if in terms of r and later work uses $r = 7$ to achieve a solution.

M1: Attempts to use an appropriate chain rule with their $\frac{dA}{dt}$ and $\frac{dA}{dx}$ e.g. $\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = ...$

A1: awrt -0.0114 or $-\frac{1}{28\pi}$ cso (must have the negative sign)

(b)

B1: $V = \pi x^2(3x)$ or $V = 3\pi x^3$

B1ft: $\frac{dV}{dx} = 9\pi x^2$ or ft from their equation for V in one variable

M1: Their $\frac{dV}{dx} \times$ their $\frac{dx}{dt}$. Note the $\frac{dx}{dt}$ must be in terms of x or with $x = 4$ substituted first, M0 if they use their answer to (a).

A1: (Rate of decrease $=$) $9 \ \left(\text{mm}^3 \text{ s}^{-1}\right)$ (with or without the negative sign). May be scored following $\frac{dA}{dt} = 0.5$ in part (a)

Question 7

(a)

M1 Uses a valid method to differentiate. This could be:

(i) using the product rule on $\mathrm{f}(x)=\left(x^{3}-4x\right)\mathrm{e}^{-\frac{1}{2}x}$ so score for an expression of the form $(\mathrm{f}'(x)=)\ \pm A\left(x^{3}-4x\right)\mathrm{e}^{-\frac{1}{2}x}\pm\left(Bx^{2}\pm C\right)\mathrm{e}^{-\frac{1}{2}x}$. $(A, B, C\neq 0)$ Condone the squared missing on the Bx^{2} term

(ii) using the quotient rule on $\mathrm{f}(x)=\dfrac{x^{3}-4x}{\mathrm{e}^{\frac{1}{2}x}}$ so score for an expression of the form

$$(\mathrm{f}'(x)=)\ \frac{\pm\mathrm{e}^{\frac{1}{2}x}\left(Bx^{2}\pm C\right)-A\mathrm{e}^{\frac{1}{2}x}\left(x^{3}-4x\right)}{\left(\mathrm{e}^{\frac{1}{2}x}\right)^{2}}$$

$(A, B, C\neq 0)$ Condone the squared missing on the Bx^{2} term or an attempt at (iii) $\mathrm{f}'(x)=uvw'+uv'w+u'vw$ which could look like:

$$(x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}+x\frac{\mathrm{d}\left((x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}\right)}{\mathrm{d}x}=\pm\ldots(x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}\pm x(\ldots x)\mathrm{e}^{-\frac{1}{2}x}\pm\ldots x(x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}$$

A1 Correct $\mathrm{f}'(x)$ but may be unsimplified. Isw after a correct unsimplified expression

$$\left(3x^{2}-4\right)\mathrm{e}^{-\frac{1}{2}x}-\frac{1}{2}\left(x^{3}-4x\right)\mathrm{e}^{-\frac{1}{2}x}\ \text{ or }\ (x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}+x\frac{\mathrm{d}\left((x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}\right)}{\mathrm{d}x}=(x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}+x(2x)\mathrm{e}^{-\frac{1}{2}x}-\frac{1}{2}x(x^{2}-4)\mathrm{e}^{-\frac{1}{2}x}$$

(b) Note on EPEN it is M1A1M1A1 but we are marking this M1B1M1A1

M1 Full method to find the equation of the normal through O.

Look for an attempt at $f'(0)$ followed by the equation $y = -\frac{1}{f'(0)}x$

B1 Equation of normal is $y = \frac{1}{4}x$ (seen or implied) (which may follow an incorrect

$f'(x)$ from part (a))

M1 Equates their $y = \frac{1}{4}x$ (which must be a straight line through the origin) with

$f(x) = x\left(x^2 - 4\right)e^{-\frac{1}{2}x}$, divides through or factorises out the x term and attempts to make

x^2 (or allow $4x^2$) the subject

A1* Full proof showing all steps. There is no requirement to justify the − sign. Note that A1* cannot be scored if A0 in part (a), unless they restart in (b).

(c)(i)

M1 Substitutes $x = -2$ into the iteration formula and finds x_2. May also be implied by -2.0228 or -2.0229

A1 awrt -2.0229

(ii)

A1 $(x =) -2.0226$ correct to 4 dp

Question 8

Question Number	Scheme	Notes	Marks
(a)	$u_{20} = 100 + 19(-2) = 62*$	Correct method shown	B1*
			(1)
(b)	$S_{20} = \frac{1}{2}(20)\{2 \times 100 + 19(-2)\} = \ldots$ **or** $S_{20} = \frac{1}{2}(20)\{100 + 62\} = \ldots$	Applies a correct AP sum formula with $n = 20$, $a = 100$ and $d = -2$ **or** $n = 20$, $a = 100$ and $l = 62$	M1
	$= 1620$ (mm)	Correct value	A1
			(2)
(c)	$62 \times r^2 = 60 \Rightarrow r^2 = \ldots$	Correct strategy to find r	M1
	$r^2 = \frac{60}{62} \Rightarrow r = \sqrt{\frac{60}{62}}$ $= 0.983738\ldots$	r = awrt 0.984	A1
			(2)
(d)	Total distance from GS hits = $\frac{62 \times 0.983\ldots(1 - 0.983\ldots^n)}{1 - 0.983\ldots}$		M1
	$1620 + \frac{62 \times 0.983\ldots(1 - 0.983\ldots^n)}{1 - 0.983\ldots} > 3000$	Correct equation set up with their r and suitable a	M1
	$0.983\ldots^n < 0.63207\ldots \Rightarrow n = \frac{\log(0.63207\ldots)}{\log(0.983\ldots)}$	Fully correct processing to find n from an equation of suitable form,	M1
	$n = 27.98\ldots \Rightarrow N = 20 + 28 = 48$	N = 48 only	A1cso
			(4)
	(d) Alternative taking 20th hit as first term of GP		
	Total distance from GS hits = $\frac{62(1 - 0.983\ldots^n)}{1 - 0.983\ldots}$		M1
	$1620 - 62 + \frac{62(1 - 0.983\ldots^n)}{1 - 0.983\ldots} > 3000$	Correct equation set up with their r and suitable a	M1
	$0.983\ldots^n < 0.62179\ldots \Rightarrow n = \frac{\log(0.62179\ldots)}{\log(0.983\ldots)}$	Fully correct processing to find n from an equation of suitable form,	M1
	$n = 28.98\ldots \Rightarrow N = 19 + 29 = 48$	N = 48 only	A1cso
			Total 9

Question 9

Question Number	Scheme	Marks
(a)	$x = y\mathrm{e}^{2y} \Rightarrow \quad \dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$	M1, A1
	$\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{x}{y} + 2x$	
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\dfrac{x}{y} + 2x}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x+2xy} = \dfrac{y}{x(1+2y)}$ *	A1*
		(4)
(b)	Deduces $y = -\dfrac{1}{2}$	B1
	Substitutes $y = -\dfrac{1}{2} \Rightarrow x = -\dfrac{1}{2\mathrm{e}}$ Range for k $\quad -\dfrac{1}{2\mathrm{e}} < k < 0$	M1, A1
		(3)
		(7 marks)

Question 10

Question Number	Scheme	Marks
(a)	Uses $\sin 2x = 2\sin x\cos x$ **AND** $\cos 2x = 1-2\sin^2 x$ o.e. in $\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}$	M1
	$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{2\sin x\cos x}{\cos x}+\frac{1-2\sin^2 x}{\sin x}$ $=\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}+\frac{1}{\sin x}-\frac{2\sin^{\cancel{2}} x}{\cancel{\sin x}}=\frac{1}{\sin x}=\operatorname{cosec} x$ *	dM1 A1*
		(3)
(b)	Uses part (a) $\Rightarrow 7+\operatorname{cosec} 2\theta = 3\cot^2 2\theta$	B1
	Either Uses $\pm1\pm\cot^2 2\theta = \pm\operatorname{cosec}^2 2\theta \ \rightarrow$ 3TQ in $\operatorname{cosec} 2\theta$ **Or alternatively** replaces $\operatorname{cosec}2\theta$ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta/\sin^2 2\theta$, multiplies by $\sin^2 2\theta$ and uses $\pm\cos^2 2\theta = \pm1\pm\sin^2 2\theta \ \rightarrow$ 3TQ in $\sin 2\theta$	M1
	$3\operatorname{cosec}^2 2\theta - \operatorname{cosec} 2\theta - 10 = 0$ **or** $10\sin^2 2\theta + \sin 2\theta - 3 = 0$	A1
	$(3\operatorname{cosec} 2\theta+5)(\operatorname{cosec} 2\theta-2)=0$ **or** $(5\sin 2\theta+3)(2\sin 2\theta-1)=0$ $\Rightarrow \operatorname{cosec} 2\theta = -\frac{5}{3}, 2$ **or** $\Rightarrow \sin 2\theta = -\frac{3}{5}, \frac{1}{2}$ $\Rightarrow \sin 2\theta = -\frac{3}{5}, \frac{1}{2} \Rightarrow \theta = \ldots$	dM1
	$\theta = \frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$ (awrt these values)	A1, A1
		(6)
		Total 9

(a)
M1: Uses
- $\sin 2x = 2\sin x\cos x$
- AND $\cos 2x = 1-2\sin^2 x$ or equivalent. Condone sign slips on the versions of $\cos 2x$

in an attempt to write $\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}$ as an expression in $\sin x$ and $\cos x$

dM1: Adopts a valid approach that can be followed and completes the proof .
All necessary steps may not be shown and condone errors such as writing cos for $\cos x$ or $\sin x^2$ for $\sin^2 x$

A1*: Correct proof showing all necessary intermediate steps with no errors (seen within the body of the solution) or omissions of any of the steps shown. The LHS starting point does not need to be seen
See main mark scheme and below for examples showing all steps and scoring full marks

$$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{\sin x\sin 2x+\cos x\cos 2x}{\sin x\cos x}$$
$$=\frac{2\sin^2 x\cos x+\cos x(1-2\sin^2 x)}{\sin x\cos x}$$
$$=\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}$$
$$=\operatorname{cosec}x$$

$$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}+\frac{\cos 2x}{\sin x}$$
$$=\frac{2\sin^2 x+(\cos^2 x-\sin^2 x)}{\sin x}$$
$$=\frac{\sin^2 x+\cos^2 x}{\sin x}=\frac{1}{\sin x}$$
$$=\operatorname{cosec}x$$

Alt part (a)

M1: For using compound angle formula $\sin x \sin 2x + \cos x \cos 2x = \cos(2x - x)$

dM1: As in the main scheme, it is for adopting a valid approach that can be followed and completing the proof

A1: Correct proof showing all necessary steps (See below) with no errors or omissions

$$\begin{aligned}\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}&=\frac{\sin x\sin 2x+\cos x\cos 2x}{\sin x\cos x}\\&=\frac{\cos(2x-x)}{\sin x\cos x}\\&=\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}\\&=\operatorname{cosec}x\end{aligned}$$

(b)

B1: States $7+\operatorname{cosec} 2\theta = 3\cot^2 2\theta$ or exact equivalent which may be implied by subsequent work
OR $7+\operatorname{cosec} x = 3\cot^2 x$ with $x = 2\theta$
Watch for and do not allow $7+\operatorname{cosec}\,\theta 2\theta 3\cot^2$

M1: Attempts to use part (a) and uses $\pm 1 \pm \cot^2 2\theta = \pm\operatorname{cosec}^2 2\theta$ to form a 3TQ in $\operatorname{cosec} 2\theta$
Condone $3\cot^2 2\theta$ being replaced by $3\times \pm\operatorname{cosec}^2 2\theta \pm 1$ with or without the bracket.
Condone when the "7" is missing but these attempts will score a maximum of 2 marks. This mark and dM1
The terms don't need to be collected for this mark.
Alternatively replaces $\operatorname{cosec}2\theta$ with $1/\sin 2\theta$, $\cot^2 2\theta$ with $\cos^2 2\theta/\sin^2 2\theta$ within an equation of the form $a + b\operatorname{cosec} 2\theta = c\cot^2 2\theta$ multiplies by $\sin^2 2\theta$ and uses $\pm\cos^2 2\theta = \pm 1 \pm \sin^2 2\theta \rightarrow$ 3TQ in $\sin 2\theta$

A1: Correct equation $3\operatorname{cosec}^2 2\theta - \operatorname{cosec} 2\theta - 10 = 0$ **or** $10\sin^2 2\theta + \sin 2\theta - 3 = 0$
The $= 0$ may be implied by further work, e.g solution of the equation
Allow this mark even for the correct equation in a different forms. E.g. $3\operatorname{cosec}^2 2\theta - \operatorname{cosec} 2\theta = 10$

dM1: For a correct attempt to solve their 3TQ $\sin 2\theta$ or $\operatorname{cosec} 2\theta$ leading to a value for θ
If they state that $\sin\theta = -\frac{3}{5}, \frac{1}{2}$ and do not proceed to take arcsin and $\div 2$ it is M0

A1: For two of awrt $\theta = \frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$

A1: For awrt $\theta = \frac{\pi}{12}(0.262),\ \frac{5\pi}{12}(1.31),\ -0.322,\ -1.25$ with no additional values within the range.

If you see other worthwhile solutions and the scheme cannot be applied, e.g t formula, please send to review

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How to mark when other variables are used, e.g. $x = 2\theta$

B1: $7+\operatorname{cosec} x = 3\cot^2 x$

M1: Uses $\pm 1 \pm \cot^2 x = \pm\operatorname{cosec}^2 x$ to form 3TQ in $\operatorname{cosec} x$ ....or the equivalent in $\sin x$

A1: Correct equation $3\operatorname{cosec}^2 x - \operatorname{cosec} x - 10 = 0$ or $10\sin^2 x + \sin x - 3 = 0$

ddM1: For this to be scored there must be an attempt to halve the values, otherwise M0.

Allow full marks to be scored for a candidate who uses a different variable correctly and reaches 4 correct answers

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Question 11

Question	Scheme	Marks
(a)	$1=2\sin u \Rightarrow p=\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$	B1
	$x=2\sin u \Rightarrow \frac{dx}{du}=2\cos u$ oe	M1
	$\int \frac{3x+2}{\left(4-x^2\right)^{\frac{3}{2}}}\,dx=\int \frac{6\sin u+2}{\left(4-4\sin^2 u\right)^{\frac{3}{2}}}\,2\cos u\,du=\int \frac{6\sin u+2}{\left(4\cos^2 u\right)^{\frac{3}{2}}}\,2\cos u\,du$	M1
	$=\int \frac{12\sin u}{8\cos^2 u}+\frac{2}{4\cos^2 u}\,du=\int_0^{\frac{\pi}{6}}\left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)\,du$ *	A1*
		(4)
(b)	$\int\left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)\,du=\frac{3}{2}\sec u+\frac{1}{2}\tan u$	M1A1
	$\left[\frac{3}{2}\sec u+\frac{1}{2}\tan u\right]_0^{"\frac{\pi}{6}"}=\left(\frac{3}{2}\sec\left("\frac{\pi}{6}"\right)+\frac{1}{2}\tan\left("\frac{\pi}{6}"\right)\right)-\left(\frac{3}{2}\sec 0+\frac{1}{2}\tan 0\right)=\ldots$	M1
	$=\sqrt{3}+\frac{\sqrt{3}}{6}-\frac{3}{2}=\frac{7\sqrt{3}}{6}-\frac{3}{2}\quad\left(=\frac{7\sqrt{3}-9}{6}\right)$	A1
		(4)
		(8 marks)

Notes
(a)
B1: $p=\frac{\pi}{6}$ Allow if seen anywhere, even in (b). $p=30$ is B0.
M1: $x=2\sin u \Rightarrow \frac{dx}{du}=\pm\ldots\cos u$ or any rearrangement of this equation.
M1: **Full** substitution from an integral in terms of x to an integral in terms of u and **uses** the identity $\sin^2 u+\cos^2 u=1$ in the denominator. Do not be concerned with the limits for this mark.
A1*: Achieves given answer include du (with their p) with no errors and at least one intermediate step with the fractional power simplified. Condone missing du in intermediate lines.
(b)
M1: $\int\left(\frac{3}{2}\sec u\tan u+\frac{1}{2}\sec^2 u\right)\,du=\ldots\sec u+\ldots\tan u$
A1: $\frac{3}{2}\sec u+\frac{1}{2}\tan u$ ignore any constant c
M1: Depends on having one term of the correct form, attempts to substitute in their p ($\neq 1$) and 0, subtracting either way round. The substitution must be seen or clearly implied, e.g. by correct values for each term in an intermediate step before the answer (allowing missing 0's).
A1: $\frac{7\sqrt{3}}{6}-\frac{3}{2}$ or exact equivalent eg $\frac{7\sqrt{3}-9}{6}$ Allow if $p=30^{\circ}$ was used.

Question 12

Question	Scheme	Marks
(a)	$\frac{dx}{dt}=\cos t+6\cos t\sin t \qquad \frac{dy}{dt}=3\cos t-2\sin t$	B1B1
	$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}=\frac{3\cos t-2\sin t}{\cos t+6\cos t\sin t}=\frac{3\cos\pi-2\sin\pi}{\cos\pi+6\cos\pi\sin\pi}=3$ *	M1A1*
		(4)
(b)	When $t=\pi,\ x=-3,\ y=-2$	B1
	$y-"-2"=3(x-"-3")$	M1
	$y=3x+7$	A1
		(3)
(c)	$y=3x+7\Rightarrow 3\sin t+2\cos t=3\left(\sin t-3\cos^2 t\right)+7$ or $y=3\left(x+3\cos^2 t\right)+2\cos t\Rightarrow 3x+7=3x+9\cos^2 t+2\cos t$	M1
	$\Rightarrow 9\cos^2 t+2\cos t-7=0$ *	A1*
		(2)
(d)	$\cos t=\frac{7}{9}$	B1
	$y=3\times\frac{\sqrt{32}}{9}+2\times\frac{7}{9}=\frac{4\sqrt{2}}{3}+\frac{14}{9}$	M1A1
		(3)
		(12 marks)

Notes
(a) **B1**: $\left(\frac{dx}{dt}=\right)\cos t + 6\cos t\sin t$ or $\cos t + 3\sin 2t$ **B1:** $\left(\frac{dy}{dt}=\right)3\cos t - 2\sin t$ **M1:** Attempts $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ using their $\frac{dx}{dt}$ and their $\frac{dy}{dt}$ and substitutes $t=\pi$. (May substitute π before dividing.) **A1*:** Achieves $\frac{dy}{dx}=3$ with full working shown and no errors. **(b)** **B1:** $x=-3$, $y=-2$ which may be seen within their working **M1:** Attempts to find the equation of the tangent with gradient 3. If they use $y=mx+c$ they must proceed as far as $c=...$ **A1:** $y=3x+7$ **(c)** **M1:** A full attempt to solve simultaneously the given parametric equations with their equation of the tangent **A1*:** Achieves $9\cos^2 t+2\cos t-7=0$ with no errors **(d)** **B1**: $\cos t=\frac{7}{9}$ seen or implied. Allow if seen in (c). **M1:** Attempts to find the y coordinate Must attempt to evaluate trig terms. If no substitution/working shown, then score for awrt 3.44 following a correct value for cos t **A1:** $\frac{4\sqrt{2}}{3}+\frac{14}{9}$ or exact equivalent. Withhold if additional answers are given.