“Best Guess” A level Mathematics

Paper 1

June 2023

Time: 2 hours

Information for Candidates

- There are 12 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Disclaimer: There is no guarantee that any specific topic will be examined this way in the summer and you cannot rely on this as your only source of revision.

In 2022 I wrote a predicted paper and some questions reflected the real exam paper. It was easier as we were provided with advance information on all the topics. This year is different, nobody can predict a paper. However, this paper is created based on **high frequency of topics and trend** from previous years. Some topics or similar skills from this paper may appear in Paper 2 and vice versa, or may not.

Visit www.naikermaths.com for more practice papers and plenty of revision resources to help you in your revision.

Question 1

$$f(x) = kx^3 - 15x^2 - 32x - 12 \qquad \text{where } k \text{ is a constant}$$

Given $(x - 3)$ is a factor of $f(x)$,

(a) show that $k = 9$ **(2)**

(b) Using algebra and showing each step of your working, fully factorise $f(x)$. **(4)**

(c) Solve, for $0 \leq \theta < 360°$, the equation

$$9\cos^3\theta - 15\cos^2\theta - 32\cos\theta - 12 = 0$$

giving your answers to one decimal place. **(2)**

(Total for question = 8 marks)

Question 2

The function f is defined by

$$f(x) = \frac{5x - 3}{x - 4} \qquad x > 4$$

(a) Show, by using calculus, that f is a decreasing function. **(3)**

(b) Find f^{-1} **(3)**

(c) (i) Show that $ff(x) = \dfrac{ax + b}{x + c}$ where a, b and c are constants to be found.

(ii) Deduce the range of ff. **(5)**

(Total for question = 11 marks)

Question 3

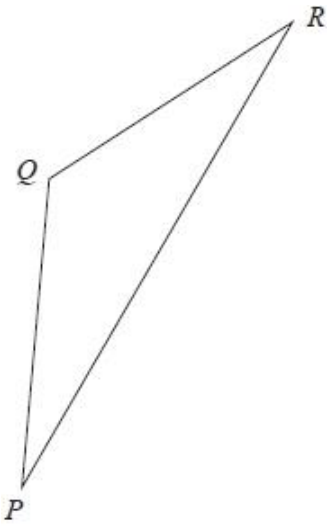

Figure 1

Figure 1 shows a sketch of triangle *PQR*.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\overrightarrow{RQ}$ **(2)**

(b) Find the size of angle *PQR*, in degrees, to three significant figures. **(3)**

(Total for question = 5 marks)

Question 4

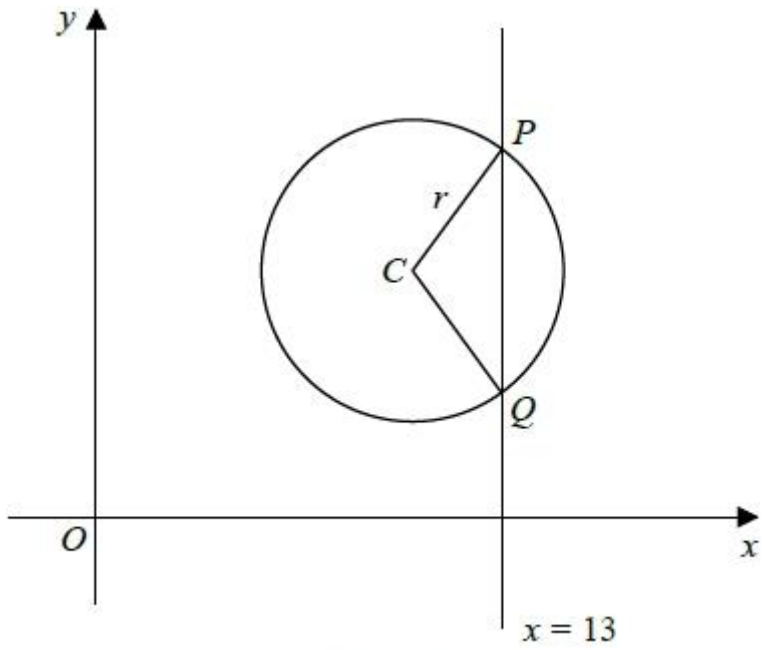

Figure 1

The circle with equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

had centre C and radius r.

(a) Find the coordinates of C. **(2)**

(b) Show that $r = 5$ **(2)**

The line with equation $x = 13$ crosses the circle at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q. **(3)**

A tangent to the circle from O touches the circle at point X.

(d) Find, in surd form, the length OX. **(3)**

(Total for question = 10 marks)

Question 5

Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(Total for question = 4 marks)

Question 6

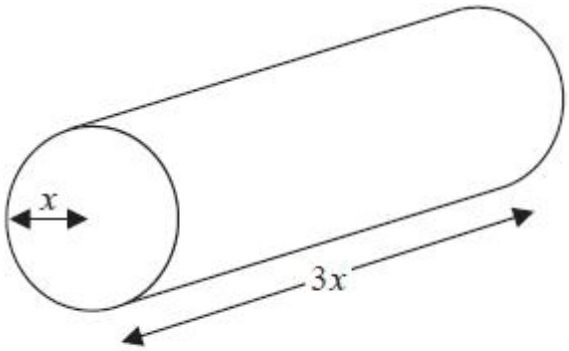

Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of 0.5 mm^2s^{-1}

(a) Find $\frac{dx}{dt}$ when $x = 7$ **(4)**

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$ **(4)**

(Total for question = 8 marks)

Question 7

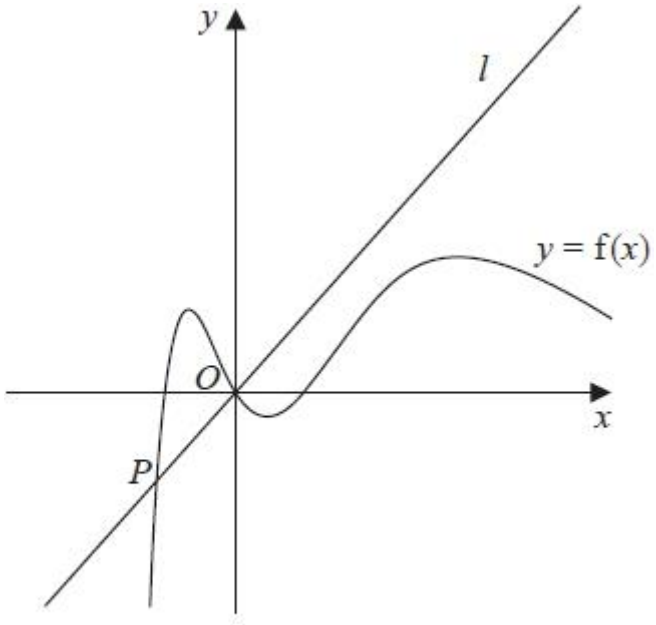

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x(x^2 - 4)e^{-\frac{1}{2}x}$$

(a) Find $f'(x)$. **(2)**

The line l is the normal to the curve at O and meets the curve again at the point P.

The point P lies in the 3rd quadrant, as shown in Figure 3.

(b) Show that the x coordinate of P is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x}}$$

(4)

(c) Using the iterative formula

$$x_{n+1} = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x_n}} \quad \text{with } x_1 = -2$$

find, to 4 decimal places,

(i) the value of x_2

(ii) the x coordinate of P. **(3)**

(Total for question = 9 marks)

Question 8

A metal post is repeatedly hit in order to drive it into the ground.

Given that

- on the 1st hit, the post is driven 100 mm into the ground
- on the 2nd hit, the post is driven an **additional** 98 mm into the ground
- on the 3rd hit, the post is driven an **additional** 96 mm into the ground
- the **additional** distances the post travels on each subsequent hit form an arithmetic sequence

(a) show that the post is driven an **additional** 62 mm into the ground with the 20th hit. **(1)**

(b) Find the **total distance** that the post has been driven into the ground after 20 hits. **(2)**

Given that for each subsequent hit after the 20th hit

- the **additional** distances the post travels form a geometric sequence with common ratio r
- on the 22nd hit, the post is driven an **additional** 60 mm into the ground

(c) find the value of r, giving your answer to 3 decimal places. **(2)**

After a total of N hits, the post will have been driven more than 3 m into the ground.

(d) Find, showing all steps in your working, the smallest possible value of N. **(4)**

(Total for question = 9 marks)

Question 9

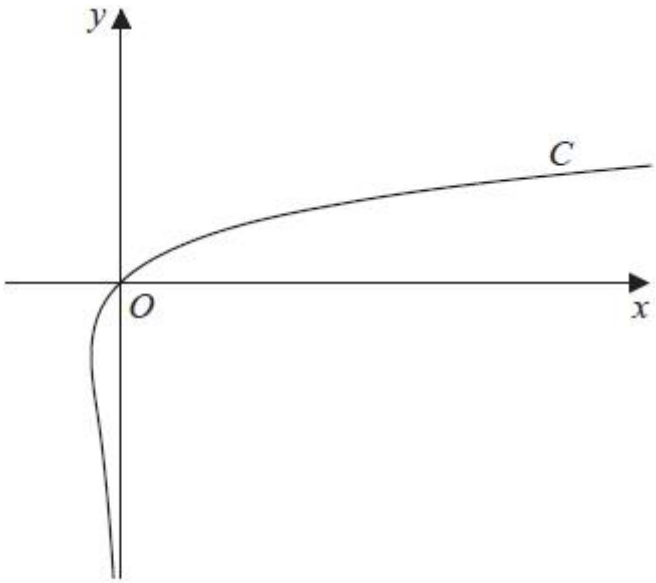

Figure 3

Figure 3 shows a sketch of the curve C with equation

$$x = ye^{2y} \qquad y \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{y}{x(1+2y)}$$

(4)

Given that the straight line with equation $x = k$, where k is a constant, cuts C at exactly two points,

(b) find the range of possible values for k. **(3)**

(Total for question = 7 marks)

Question 10

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x} \equiv \operatorname{cosec} x \qquad x \neq \frac{n\pi}{2} \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$$7 + \frac{\sin 4\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin 2\theta} = 3\cot^2 2\theta$$

giving your answers in radians to 3 significant figures where appropriate. **(6)**

(Total for question = 9 marks)

Question 11

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2 \sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x = \int_0^p \left(\frac{3}{2}\sec u \tan u + \frac{1}{2}\sec^2 u\right)\mathrm{d}u$$

where p is a constant to be found. **(4)**

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

(4)

(Total for question = 8 marks)

Question 12

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3\cos^2 t \qquad y = 3\sin t + 2\cos t \qquad 0 \leq t \leq 5$$

(a) Show that $\frac{dy}{dx} = 3$ where $t = \pi$ **(4)**

The point P lies on C where $t = \pi$

(b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found. **(3)**

Given that the tangent to the curve at P cuts C at the point Q

(c) show that the value of t at point Q satisfies the equation

$$9\cos^2 t + 2\cos t - 7 = 0$$ **(2)**

(d) Hence find the exact value of the y coordinate of Q **(3)**

(Total for question = 12 marks)

TOTAL FOR PAPER IS 100 MARKS