A level Best Guess Paper 2 June 2023 Mark Scheme

Question 1

Question Number	Scheme	Marks
(a)	(i) $a_2 = 1$	B1
	(ii) $a_{107} = 3$	B1
		(2)
(b)	$\sum_{n=1}^{200}(2a_n - 1) = 5+1+5+1+.......5+1 = 100\times(5+1)$	M1
	$= 600$	A1
		(2)
		(4 marks)

Notes

(a) (i)

B1 $a_2 = 1$ Accept the sight of 1. Ignore incorrect working

(a)(ii)

B1 $a_{107} = 3$ Accept sight of just 3. Ignore incorrect working
If there are lots of 1's and 3's without reference to any suffices they need to choose 3.

(b)

M1 Establishes an attempt to find the sum of a series with two distinct terms.
Look for $100\times a + 100\times b$ or $200\times a + 200\times b$ where a and b are allowable terms.
Examples of allowable terms are

$a, b = 1, 5$ (which are correct)

$a, b = 1, 3$ (which are the values for (a))

$a, b = 3, 7$ (which is using $2a_n + 1$)

$a, b = 0, 5$ (which is a slip on the first value)

Methods using AP (and GP) formulae are common and score 0 marks.

A1 600.
600 should be awarded both marks as long as no incorrect working is seen

Question 2

Question Number	Scheme	Marks
	$kx^2 + 6kx + 5 = 0$	
	$b^2 - 4ac = (6k)^2 - 4 \times k \times 5$	M1
	$b^2 - 4ac = (6k)^2 - 4 \times k \times 5 ... 0 \Rightarrow k...$	dM1
	$k < \frac{5}{9}$	A1
	$0 < k < \frac{5}{9}$	A1
		(4 marks)

M1: Attempts to use $b^2 - 4ac$ for the given quadratic with $b = 6k$, a = k and $c = 5$.
May be seen as part of the quadratic formula or may be implied by an attempt to solve e.g. $b^2 = 4ac$
Condone attempts where the "6" isn't squared e.g. $6k^2 - 4 \times k \times 5$ but the k must be squared.

dM1: Dependent upon the previous M mark, it is for setting $b^2 - 4ac...0$ leading to a non-zero value for k from an "equation" of the form $\alpha k^2 - \beta k...0$
Condone any of e.g. "=", "<", ">" etc. for "..." for this mark.

A1: For obtaining an upper limit for k of $\frac{5}{9}$ (not just the value) but condone $k \leqslant \frac{5}{9}$ which may be implied by e.g. $0 < k < \frac{5}{9}$ or $0 < k \leqslant \frac{5}{9}$. Allow exact equivalents for $\frac{5}{9}$ e.g. $\frac{10}{18}$ etc. Condone the use of x for this mark so allow e.g. $x \leqslant \frac{5}{9}$, $x < \frac{5}{9}$ Allow $0.\dot{5}$ for $\frac{5}{9}$

Allow the inequalities to be on separate lines e.g. $k < \frac{5}{9}$
$k > 0$

Question 3

Question Number	Scheme	Notes	Marks
(a)	$h = 0.5$	Correct strip width	B1
	$A \approx \frac{1}{2} \times \frac{1}{2}\{6.792 + 5.113 + 2(6.298 + 5.858 + 5.466)\}$ Correct application of the trapezium rule with their h		M1
	= 11.79	Cao	A1
			(3)
(b)(i)	$A \approx 2 \times "11.79"$	Multiplies their answer to (a) by 2	M1
	= 23.58	Correct answer or correct ft	A1ft
(b)(ii)	$A \approx "11.79" + 6$	Adds 6 to their answer to (a)	M1
	= 17.79	Correct answer or correct ft	A1ft
			(4)
			Total 7

Question 4

Question Number	Scheme	Notes	Marks
	$f'(x) = 6x^2 + 14x^{-3}$	At least one of either $2x^3 \to \pm Ax^2$ or $-\frac{7}{x^2} \to \pm Bx^{-3}$; $A, B \neq 0$	M1
		Correct differentiation which can be simplified or un-simplified	A1
	$\left\{\beta \simeq 0.65 - \frac{f(0.65)}{f'(0.65)}\right\} \Rightarrow \beta \simeq 0.65 - \frac{-0.01879733728...}{53.51360719...}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $f(0.65)$ and $f'(0.65)$	dM1
	$\{\beta = 0.6503512623...\} \Rightarrow \beta = 0.6504$ (4 dp)	**dependent on all 3 previous marks** 0.6504 on their first iteration (Ignore any subsequent iterations)	A1 **cso** **cao**
	Correct differentiation followed by a correct answer of 0.6504 scores full marks in part (b) **Correct answer with <u>no</u> working scores no marks in part (b)**		**(4)**
			7

	dM1	This mark can be implied by applying at least one correct *value* of either $\mathrm{f}(0.65)$ or their $\mathrm{f}'(0.65)$ (where $\mathrm{f}'(0.65)$ is found using their $\mathrm{f}'(x)$) to 1 significant figure in $0.65-\dfrac{\mathrm{f}(0.65)}{\mathrm{f}'(0.65)}$. So just $0.65-\dfrac{\mathrm{f}(0.65)}{\mathrm{f}'(0.65)}$ with an incorrect answer and no other evidence scores final dM0A0.
	Note	If you see $0.65-\dfrac{\mathrm{f}(0.65)}{\mathrm{f}'(0.65)}=0.6504$ with no algebraic differentiation, then send the response to review.
	Note	You can imply the M1 A1 marks for algebraic differentiation for either • $\mathrm{f}'(0.65)=6(0.65)^2+14(0.65)^{-3}$ • $\mathrm{f}'(0.65)$ applied correctly in $\beta \simeq 0.65-\dfrac{2(0.65)^3-\dfrac{7}{(0.65)^2}+16}{6(0.65)^2+14(0.65)^{-3}}$
	Note	**Differentiating INCORRECTLY to give** $\mathrm{f}'(x)=6x^2-14x^{-3}$ leads to $\beta \simeq 0.65-\dfrac{-0.01879733728...}{-48.44360719...}=0.6496119749...=0.6496$ (4 dp) **This response should be awarded M1 A0 dM1 A0**
	Note	**Differentiating INCORRECTLY to give** $6x^2-14x^{-3}$ and $\beta \simeq 0.65-\dfrac{\mathrm{f}(0.65)}{\mathrm{f}'(0.65)}=0.6496$ is **M1 A0 dM1 A0**

Question 5

Question Number	Scheme	Marks
(a)	$4^{-\frac{1}{2}}$ or $\frac{1}{4^{\frac{1}{2}}}$ or $\frac{1}{2}$	B1
	$(4-5x)^{-\frac{1}{2}}=\frac{1}{2}\left(1-\frac{5x}{4}\right)^{-\frac{1}{2}}$	
	$=...\left(1+\left(-\frac{1}{2}\right)\times\left(-\frac{5x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2!}\times\left(-\frac{5x}{4}\right)^2+...\right)$	M1A1
	$=\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+...$	A1
		(4)
	$\frac{2+kx}{(2-3x)^3}=(2+kx)\left(\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+...\right)$	
(b)	Compares x terms leading to $k=$ .... E.g. $\frac{10}{16}+\frac{k}{2}=\frac{3}{10}\Rightarrow k=-\frac{13}{20}$	M1 A1
		(2)
(c)	Compares x^2 terms leading to $m=$ ... E.g. $m=\frac{75}{128}+\frac{5}{16}\times'-\frac{13}{20}'\Rightarrow m=\frac{49}{128}$	M1 A1
		(2)
		(8 marks)
Alt (a)	$4^{-\frac{1}{2}}$ or $\frac{1}{4^{\frac{1}{2}}}$	B1
	$(4-5x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)+\frac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$	M1A1
	$=\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+...$	A1
		(4)

(a)

B1: For taking out a factor of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

For a direct expansion look for $4^{-\frac{1}{2}}+\dots$ or equivalent.

M1: For the form of the binomial expansion $(1+ax)^{-\frac{1}{2}}$ where $a \neq 1$ or -5

To score M1 it is sufficient to see either term two or term three. Allow a slip on the sign.

So allow for either $\left(-\frac{1}{2}\right)(\pm ax)$ or $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}(\pm ax)^2$

In the alternative version look for $\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)$ or $\frac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$ condoning sign slips

A1: Any (unsimplified) but correct form of the binomial expansion for . $\left(1-\frac{5x}{4}\right)^{-\frac{1}{2}}$

Ignore the factor preceding the bracket for this mark

Score for $1+\left(-\frac{1}{2}\right)\times\left(-\frac{5x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2!}\times\left(-\frac{5x}{4}\right)^2$ o.e.

In the alternative version look for $(4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-5x)+\frac{-\frac{1}{2}\times-\frac{3}{2}}{2}4^{-\frac{5}{2}}(-5x)^2$

A1: cao $\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+\dots$ This must be simplified

$$(2+kx)\left(P+Qx+Rx^2+\dots\right)=1+\frac{3}{10}x+mx^2$$

(b)

M1: For a correct equation in k formed by comparing the x terms. It must lead to a value for k

Follow through on their expansion. So look for $Pk+2Q=\frac{3}{10} \Rightarrow k=\dots$ Condone slips, i.e copying errors.

Condone $Pkx+2Qx=\frac{3}{10}x$ as long as it leads to a value for k

A1: $k=-\frac{13}{20}$

(c)

M1: Correctly compares the x^2 terms, following through on their expansion and their value for k leading to a value for m . Condone slips, i.e copying errors.

Look for $Qk+2R=m$ Condone $Qkx^2+2Rx^2=mx^2$ as long as it leads to a value for m

A1: $m=\frac{49}{128}$ oe Condone sight of $\frac{49}{128}x^2$ as evidence for a correct value for m.

Question 6

Question	Scheme	Marks
(a)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1
	Attempts to solve by multiplying out bracket, collect terms etc. $\frac{3}{2}x=31$	M1
	$x=\frac{62}{3}$ only	A1
		(3)
(b)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k\leqslant 11$	M1
	$5<k\leqslant 11$	A1
		(2)
		(5 marks)

Notes:
(a)
M1: Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving $-2(3-x)+5=\frac{1}{2}x+30$
M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms.
A1: $x=\frac{62}{3}$ only. Do not allow 20.6
(b)
M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k\leqslant 11$
A1: Correct solution only $\{k:k\in\mathbb{R}, 5<k\leqslant 11\}$

Question 7

Question Number	Scheme	Marks
(a)	States or implies that $\log_{10} p = 0.32$ **or** $\log_{10} q = \dfrac{0.56-0.32}{8}$	M1
	$p =$ awrt 2.089 **or** $q =$ awrt 1.072	A1
	States or implies that $\log_{10} p = 0.32$ **and** $\log_{10} q = \dfrac{0.56-0.32}{8}$	M1
	$p =$ awrt 2.089 **and** $q =$ awrt 1.072	A1
		(4)
(b)	States or implies that $\dfrac{dA}{dt} = p \ln q \times q^t$ with their values for p and q	M1 A1
	Rate of increase in pond weed after 6 days is 0.22 (m^2/day)	A1
		(3)
		(7 marks)

(a)

M1: States or implies that $\log_{10} p = 0.32$ **or** $\log_{10} q = \dfrac{0.56-0.32}{8}$ or equivalent equations

A1: $p =$ awrt 2.089 **or** $q =$ awrt 1.072

M1: States or implies that $\log_{10} p = 0.32$ **and** $\log_{10} q = \dfrac{0.56-0.32}{8}$ or equivalent equations

A1: $p =$ awrt 2.089 **and** $q =$ awrt 1.072

(b)

M1: Uses $\dfrac{d}{dt}q^t \to kq^t \quad k \neq 1$

A1: States or implies that $\dfrac{dA}{dt} = p \ln q \times q^t$ with their values for p and q

A1: awrt 0.22 Units are not required

Alt (b) using $\log_{10} A = 0.03t + 0.32$ as a starting point

M1: Attempts to differentiate and reaches $\dfrac{1}{A}\dfrac{dA}{dt} = k$ or equivalent

A1: $\dfrac{1}{A \ln 10}\dfrac{dA}{dt} = 0.03$

A1: awrt 0.22 Units are not required

Question 8

Question Number	Scheme	Marks
(a)	Attempts to find y at -1.25 and -1.2 with one correct to 1sf	M1
	Achieves $y(-1.25) = -0.9$ and $y(-1.2) = 0.2$	
	With reason (change of sign and continuous) and Conclusion	A1
		(2)
(b)	(i) Attempts $\sqrt{12\ln(15)+8}$	M1
	$x_2 =$ awrt 6.3637	A1
	(ii) $x = 6.4142$	B1
		(3)
(c)	$\dfrac{dy}{dx} = \dfrac{12}{2x+3} - x$	M1 A1
	Stationary point when $\dfrac{12}{2x+3} = x \Rightarrow 2x^2 + 3x - 12 = 0 \Rightarrow x =$	dM1
	$\Rightarrow x = \dfrac{-3+\sqrt{105}}{4}$ or awrt 1.81 ONLY	A1
		(4)
		(9 marks)

Question 9

Question Number	Scheme	Notes	Marks
(a)	$x^3-6x+9=-2x^2+7x-1$ $\Rightarrow ...$	Sets $C_1 = C_2$, <u>and collects terms</u>	M1
	$\Rightarrow \pm\left(x^3+2x^2-13x+10\right)=0$	Correct cubic equation. The "= 0" may be implied by their attempt to solve.	A1
	<u>Examples:</u> $x^3+2x^2-13x+10=(x-1)\left(x^2+...x+...\right)=(x-1)(x+...)(x+..)\Rightarrow x=...$ Attempts to factorise using $(x-1)$ as a factor or uses long division by $(x-1)$ to obtain a quadratic factor and proceeds to solve quadratic or factorise and solve NB $x^3+2x^2-13x+10=(x-1)\left(x^2+3x-10\right)$ or $x^3+2x^2-13x+10=(x-1)(x+...)(x+..)\Rightarrow x=...$ Attempts 3 factors directly (by considering roots) or $x^3+2x^2-13x+10=0\Rightarrow x=...$ Solves (using calculator) to obtain 3 roots (may need to check if cubic incorrect)		M1
	$x=2,\ y=5$ or $(2,\ 5)$ Correct values **<u>from a correct cubic.</u>** Allow as a coordinate pair or written separately. If there are any errors in the algebra e.g. wrong factors, wrong working etc. this mark should be withheld even if they have (2, 5) and score as M1A1B1(**Second M on EPEN**)A0		A1
	<u>Special Case</u> If you see: $x^3-6x+9=-2x^2+7x-1\Rightarrow x^3+2x^2-13x+10=0$ $\Rightarrow x=2,\ y=5$ or $(2,\ 5)$ Score M1A1B1(**Second M on EPEN**)A0		
			(4)

(b)	$x^n \to x^{n+1}$	For increasing any power of x by 1 for C_1 or C_2 or for $\pm(C_1 - C_2)$	M1
	$\pm\int\left\{-2x^2+7x-1-\left(x^3-6x+9\right)\right\}\mathrm{d}x = \pm\int\left(-x^3-2x^2+13x-10\right)\mathrm{d}x$ $= \pm\left(-\frac{x^4}{4}-\frac{2x^3}{3}+\frac{13x^2}{2}-10x\right)$ or $\pm\left\{\int\left(-2x^2+7x-1\right)\mathrm{d}x - \int\left(x^3-6x+9\right)\mathrm{d}x\right\}$ $= \pm\left(-\frac{2x^3}{3}+\frac{7x^2}{2}-x-\left(\frac{x^4}{4}-\frac{6x^2}{2}+9x\right)\right)$ or $\int\left(-2x^2+7x-1\right)\mathrm{d}x = -\frac{2x^3}{3}+\frac{7x^2}{2}-x,\ \int\left(x^3-6x+9\right)\mathrm{d}x = \frac{x^4}{4}-\frac{6x^2}{2}+9x$ dM1: For correct integration of 1 term for C_1 **and** one term for C_2 or for correct integration for 2 terms of their $\pm(C_1 - C_2)$ A1: Fully correct integration of both C_1 and C_2 or for $\pm(C_1 - C_2)$. **Award this mark as soon as fully correct integration is seen and ignore subsequent work.**		dM1A1
	$= -\frac{2^4}{4}-\frac{2(2)^3}{3}+\frac{13(2)^2}{2}-10(2)-\left(-\frac{1^4}{4}-\frac{2(1)^3}{3}+\frac{13(1)^2}{2}-10(1)\right)$ Fully correct strategy for the area. Depends on both previous M marks. Uses the limits "2" and 1 in their "changed" expression(s) and subtracts either way round.		ddM1
	$= \frac{13}{12}$ If the attempt is correct apart from subtracting the wrong way round (for limits or functions) and $-\frac{13}{12}$ is obtained, allow recovery if they then make their answer positive.		A1
			(5)
			Total 9

Some values for reference:

$$\left[\frac{-2x^3}{3}+\frac{7x^2}{2}-x\right]_1^2 = \frac{20}{3}-\frac{11}{6} = \frac{29}{6} \qquad \left[\frac{x^4}{4}-\frac{6x^2}{2}+9x\right]_1^2 = 10-\frac{25}{4} = \frac{15}{4}$$

$$= -\frac{2^4}{4}-\frac{2(2)^3}{3}+\frac{13(2)^2}{2}-10(2)-\left(-\frac{1^4}{4}-\frac{2(1)^3}{3}+\frac{13(1)^2}{2}-10(1)\right) = -\frac{10}{3}-\left(-\frac{53}{12}\right)$$

Question 10

Question Number	Scheme	Marks
	$$A = \frac{80pe^{0.15t}}{pe^{0.15t}+4}$$	
(a)	$30 = \frac{80p}{p+4}$	M1
	$30p + 120 = 80p \Rightarrow p = \frac{120}{50} = 2.4$ *	A1*
		(2)
(b)	$50 = \frac{80 \times 2.4e^{0.15T}}{2.4e^{0.15T}+4} \Rightarrow 72e^{0.15T} = 200$	M1 A1
	$\Rightarrow 0.15T = \ln\left(\frac{200}{72}\right) \Rightarrow T = 6.8$	dM1 A1
		(4)
(c)	80 m^2	B1
		(1)
		(7 marks)

(a)

M1: Sets $A = 30$ and $e^{0.15\times 0} = 1$ to set up an equation in p.

A1*: Achieves $p = 2.4$ with no significant (*) errors and with one correct **linear (non fractional)** equation in p. * Condone minor slips as long as they are recovered before reaching the given answer.

An example of this would be

$$p + 4(30) = 80p \Rightarrow 30p + 120 = 80p \Rightarrow 50p = 120 \Rightarrow p = 2.4$$

Alt method

M1: Sets $p = 2.4$, $e^{0.15\times 0} = 1$ and attempts the value of $(A) = \dfrac{80\times 2.4\times 1}{2.4\times 1+4}$

A1*: Achieves $A = 30$ with no errors and concludes that $p = 2.4$. Condone $30 = \dfrac{192}{6.4}$ ✓

(b) Allow $t \leftrightarrow T$ here

M1: Sets $A = 50$, $p = 2.4$ and proceeds to an equation of the form $ce^{0.15t} = d \quad c\times d > 0$

Condone slips, e.g. on the 0.15. You may see $de^{-0.15t} = c \quad c\times d > 0$

A1: Achieves $72e^{0.15t} = 200$ o.e.

dM1: Correct order of operations to find T/t

For example $0.15T = \ln\left(\dfrac{200}{72}\right) \Rightarrow T = ...$ or

$\ln 72 + 0.15T = \ln 200 \Rightarrow 0.15T = ... \Rightarrow T = ...$

A1: AWRT 6.8

(c)

B1: Requires units as well. 80 m^2

Students lacking work in part (b)

Example 1: $50 = \dfrac{80\times 2.4e^{0.15T}}{2.4e^{0.15T}+4} \Rightarrow T = awrt\ 6.8$ can be awarded SC 1000

Example 2: $50 = \dfrac{80\times 2.4e^{0.15T}}{2.4e^{0.15T}+4} \Rightarrow 72e^{0.15T} = 200 \Rightarrow T = awrt\ 6.8$ score M1 A1 via scheme and then SC 10

Question 11

Question Number	Scheme	Marks
(a)	$R = 13$	B1
	$\tan\alpha = \frac{5}{12} \Rightarrow \alpha = \text{awrt } 0.395$	M1A1
		(3)
(b)	$g(\theta) = 10 + 13\sin\left(2\theta - \frac{\pi}{6} - 0.395\right)$	
(i)	(i) Minimum value is -3	B1 ft
(ii)	$2\theta - \frac{\pi}{6} - 0.395 = \frac{3\pi}{2} \Rightarrow \theta = \text{awrt } 2.82$	M1 A1
		(3)
(c)	$h(\beta) = 10 - 169\sin^2(\beta - 0.395)$	
	$-159 \leqslant h \leqslant 10$	M1 A1
		(2)
		(8 marks)

(a)

B1: $R = 13$ ($R = \pm 13$ is B0)

M1: $\tan\alpha = \pm\frac{5}{12}$, $\tan\alpha = \pm\frac{12}{5} \Rightarrow \alpha = ...$

If R is used to find α accept $\sin\alpha = \pm\frac{5}{R}$ or $\cos\alpha = \pm\frac{12}{R} \Rightarrow \alpha = ...$

A1: $\alpha = \text{awrt } 0.395$ Note that the degree equivalent $\alpha = \text{awrt } 22.6°$ is A0

(b)(i)

B1ft: States the value of $10 - R$ following through their R.

(b)(ii)

M1: Attempts to solve $2\theta - \frac{\pi}{6} \pm "0.395" = \frac{3\pi}{2} \Rightarrow \theta = ...$

A1: $\theta = \text{awrt } 2.82$. No other values should be given

(c)

M1: Achieves one of the end values, either -159 (or $10 - (\text{their } R)^2$ evaluated) or 10

A1: Fully correct range $-159 \leqslant h \leqslant 10$, $-159 \leqslant h(\beta) \leqslant 10$, $-159 \leqslant \text{range} \leqslant 10$, $-159 \leqslant h(x) \leqslant 10$, $[-159, 10]$ or equivalent correct ranges.

Question 12

Question	Scheme	Marks
	$\frac{\mathrm{d}}{\mathrm{d}x}(\cos x+\sin x)=-\sin x+\cos x$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(\cos x+\sin x)(3\cos x)-(-\sin x+\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$	M1 A1
	$=\frac{3\cos^2 x+\cancel{3\sin x\cos x}+2\sin x+3\sin^2 x-2\cos x-\cancel{3\sin x\cos x}}{\cos^2 x+2\sin x\cos x+\sin^2 x}$ $=\frac{3+2\sin x-2\cos x}{1+2\sin x\cos x}$	M1
	$=\frac{3+2\sin x-2\cos x}{1+2\sin x\cos x}\times\frac{\sec x}{\sec x}=\frac{3\sec x+2\sec x\sin x-2}{\sec x+2\sin x}$	M1
	$=\frac{2\tan x+3\sec x-2}{\sec x+2\sin x}$	A1
		(6)
		(6 marks)

Notes:

B1: Correct differentiation of $\cos x+\sin x\to-\sin x+\cos x$ seen somewhere in the proof. This can be scored if seen in workings for quotient rule, or even in the denominator of an incorrect attempt at u'/v'.

M1: Differentiates using the quotient rule or product rule. For quotient rule look for

$$\frac{(\cos x+\sin x)(\pm a\cos x)-(\pm\sin x\pm\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$$

For product rule look for $(\cos x+\sin x)^{-1}(\pm a\cos x)\pm(\cos x+\sin x)^{-2}(\pm\cos x\pm\sin x)(2+3\sin x)$

A1: Fully correct derivative.

M1: Expands numerator or denominator and applies $\sin^2 x+\cos^2 x=1$ or other appropriate correct Pythagorean identity at least once in the proof.

M1: Attempts to multiply through by $\sec x$ in numerator and denominator (to achieve the $2\sin x$ term). Not dependent and may be scored before the previous M.

A1: Correct answer. Terms may be in different order. Allow minor slips in notation (e.g. a single missing x) and recovery from missing brackets if the intent is clear, but A0 for persistent incorrect notation throughout (e.g no x's in the trig terms).

Question 13

Question	Scheme		Marks
(a)	$\frac{d\theta}{dt} = \lambda(120-\theta),\ \theta \leqslant 100$		
	$\int \frac{1}{120-\theta}\,d\theta = \int \lambda\,dt$		B1
	$-\ln(120-\theta); = \lambda t + c$	For integrating lhs M1 A1 For integrating rhs M1 A1	M1A1; M1A1
	$\{t=0,\ \theta=20\ \Rightarrow\}\ -\ln(100) = \lambda(0) + c$ $\Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$ $\Rightarrow\ -\lambda t = \ln(120-\theta) - \ln 100$ $\Rightarrow -\lambda t = \ln\left(\frac{120-\theta}{100}\right)$		M1
	$e^{-\lambda t} = \frac{120-\theta}{100}$		dddM1
	$100\,e^{-\lambda t} = 120 - \theta$ leading to $\theta = 120 - 100e^{-\lambda t}$		A1*
			(8)
(b)	$\{\lambda = 0.01,\ \theta = 100 \Rightarrow\}\quad 100 = 120 - 100\,e^{-0.01t}$		M1
	$\Rightarrow 100\,e^{-0.01t} = 120 - 100 \Rightarrow$ $-0.01t = \ln\left(\frac{120-100}{100}\right)$ $t = \frac{1}{-0.01}\ln\left(\frac{120-100}{100}\right)$ $\left\{t = \frac{1}{-0.01}\ln\left(\frac{1}{5}\right) = 100\ln 5\right\}$	Uses correct order of operations by moving from $100 = 120 - 100e^{-0.01t}$ to give $t = \ldots$ and $t = A\ln B$, where $B > 0$	dM1
	$t = 160.94379\ldots$ 161 (s) (nearest second) awrt 161		A1
			(3)
			(11 marks)

Notes:
(a) **B1M1A1M1A1**: Mark as in the scheme. **M1**: Substitutes $t=0$ AND $\theta=20$ in an integrated equation leading to $\pm\lambda t=\ln(\mathrm{f}(\theta))$ **dddM1**: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **A1***: Correct answer with no errors. This is a given answer
(b) **M1**: Substitutes $\lambda=0.01$, $\theta=100$ into given equation **M1**: See scheme **A1**: Awrt 161 seconds.

Question 14

Question Number	Scheme	Marks
(a)	At $t=\frac{\pi}{4}$ $P=\left(\frac{1}{2},2\right)$	**B1**
	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2\sec^2 t}{2\sin t\cos t}=4$ when $t=\frac{\pi}{4}$	**M1 A1**
	Equation of l: $y-2=-\frac{1}{4}\left(x-\frac{1}{2}\right)\Rightarrow 8y-16=-2x+1\Rightarrow 8y+2x=17*$	**dM1 A1 *** cso
		(5)
(b)	$\int y\frac{dx}{dt}\,dt=\int 2\tan t\times 2\sin t\cos t\,dt$	**M1**
	$=\int 4\sin^2 t\,dt$	**A1**
	$=\int 2-2\cos 2t\,dt = 2t-\sin 2t$	**dM1 A1**
	Total area of $S=\left[2t-\sin 2t\right]_0^{\frac{\pi}{4}}+\frac{1}{2}\times 8\times 2=\frac{\pi}{2}-1+8=\frac{\pi}{2}+7$	**M1 A1**
		(6)
		(11 marks)

Notes:

(a)

B1: Correct coordinates for P stated or implied by working.

M1: Attempts to find $\frac{dy}{dx}$ using $\frac{dy/dt}{dx/dt}$ at $t=\frac{\pi}{4}$. Condone poor differentiation. Substitution of the $\frac{\pi}{4}$ is sufficient for the method. Alternatively, may attempt $\frac{dx}{dy}$ or $-\frac{dx}{dy}$. Accept a value following finding $\frac{dy}{dx}$ (or its reciprocal etc) as an attempt to evaluate at $t=\frac{\pi}{4}$ if no contrary working is shown **but check carefully** as the correct answer may arise from incorrect working.

A1: Correct $\frac{dy}{dx}=4$ (oe equation) following correct differentiation. May be implied.

dM1: Attempts to find the equation of the normal at $t=\frac{\pi}{4}$. It is dependent upon the previous M and use of their P. The value of the gradient used must be correct for their differential.

A1*: cso Correct proof leading to $8y+2x=17$

(b)

M1: Attempts $\int y\frac{dx}{dt}\,dt=\int 2\tan t\times\text{"}2\sin t\cos t\text{"}\,dt$ with their $\frac{dx}{dt}$ condoning slips on coefficients.

A1: $\int 4\sin^2 t\,dt$

dM1: Uses $\cos 2t=\pm1\pm2\sin^2 t$ and integrates $\int \pm p\pm q\cos 2t\,dt$ to a form $\pm at\pm b\sin 2t$

See note below.

A1: $\int y\frac{dx}{dt}\,dt=2t-\sin 2t$ See note below.

M1: Full method to find area of region S. Finds the sum of their values for $\int_0^{\frac{\pi}{4}} y\frac{dx}{dt}\,dt$ and $\frac{1}{2}\left(\frac{17}{2}-P_x\right)\times P_y$. Condone poor integration for this mark as long as they are attempting to apply the correct limits to their result. They may attempt the area under the line by integration: $\int_{P_x}^{\frac{17}{2}} -\frac{1}{4}x+\frac{17}{8}\,dx$ In such a method condone minor slips, but must be attempting correct limits.

A1: $\frac{\pi}{2}+7$

Note: If the t's becomes x's during the integration, then allow the M's and the A's if recovered but if $2x-\sin 2x$ or with mixed variables is found and x values substituted then it is M1A0 for the integral and M0 for the method for area.

(a) Alt	At $t=\frac{\pi}{4}$ $P=\left(\frac{1}{2},2\right)$	**B1**
	$y=\frac{2\sin t}{\cos t}=\frac{2\sqrt{x}}{\sqrt{1-x}}$ $y^2=\frac{4x}{1-x}$ $\frac{dy}{dx}=\frac{x^{-\frac{1}{2}}(1-x)^{\frac{1}{2}}-2\sqrt{x}\times-\frac{1}{2}(1-x)^{-\frac{1}{2}}}{1-x}\Rightarrow\left.\frac{dy}{dx}\right\|_{x=\frac{1}{2}}=4$ or $2y\frac{dy}{dx}=\frac{4(1-x)-4x\times-1}{(1-x)^2}\Rightarrow\left.\frac{dy}{dx}\right\|_{x=\frac{1}{2},y=2}=4$ oe	**M1 A1**
	Equation of l: $y-2=-\frac{1}{4}\left(x-\frac{1}{2}\right)\Rightarrow 8y-16=-2x+1\Rightarrow 8y+2x=17*$	**dM1 A1 * cso**
		(5)

(a)

B1: Correct coordinate for $P\left(\frac{1}{2},2\right)$ stated or implied by working.

M1: Attempts to find Cartesian equation for C, any form, and attempts $\frac{dy}{dx}$ (or an equivalent as main scheme) with appropriate differentiation methods for their Cartesian form, allowing for slips and finds x and/or y using $t=\frac{\pi}{4}$ and evaluate the derivative with these values.

A1: Correct $\frac{dy}{dx}=4$ (oe equation) following correct differentiation and from correct work.

dM1: Attempts to find the equation of the normal at their x and y values.
It is dependent upon the previous M and use of their P.

A1*: cso Correct proof leading to $8y+2x=17$