“Best Guess” A level Mathematics

Paper 2

June 2023

Time: 2 hours

Information for Candidates

- There are 14 questions in this question paper
- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
- Full marks may be obtained for answers to ALL questions

Disclaimer: There is no guarantee that any specific topic will be examined this way in the summer and you cannot rely on this as your only source of revision.

In 2022 I wrote a predicted paper and some questions reflected the real exam paper. It was easier as we were provided with advance information on all the topics. This year is different, nobody can predict a paper. However, this paper is created based on **high frequency of topics and trend** from previous years. Some topics or similar skills from this paper may appear in Paper 1 and vice versa, or may not.

Visit www.naikermaths.com for more practice papers and plenty of revision resources to help you in your revision.

Question 1

A sequence a_1, a_2, a_3, ... is defined by

$$a_{n+1} = 4 - a_n$$

$$a_1 = 3$$

Find the value of

(a) (i) a_2

(ii) a_{107} **(2)**

(b) $\sum_{n=1}^{200} (2a_n - 1)$ **(2)**

(Total for question = 4 marks)

Question 2

Given that the equation

$$kx^2 + 6kx + 5 = 0 \text{ where } k \text{ is a non zero constant}$$

has no real roots, find the range of possible values for k.

(Total for question = 4 marks)

Question 3

The table below shows corresponding values of x and y for

$$y = 2^{5-\sqrt{x}}$$

The values of y are given to 3 decimal places.

x	5	5.5	6	6.5	7
y	6.792	6.298	5.858	5.466	5.113

Using the trapezium rule with all the values of y in the given table,

(a) obtain an estimate for

$$\int_5^7 2^{5-\sqrt{x}}\,dx$$

giving your answer to 2 decimal places. **(3)**

(b) Using your answer to part (a) and making your method clear, estimate

(i) $\int_5^7 2^{6-\sqrt{x}}\,dx$

(ii) $\int_5^7 \left(3 + 2^{5-\sqrt{x}}\right)dx$ **(4)**

(Total for question = 7 marks)

Question 4

$$f(x) = 2x^3 - \frac{7}{x^2} + 16, \qquad x \neq 0$$

The equation f(x) = 0 has a single root β in the interval [0.6, 0.7].

Taking 0.65 as a first approximation to β, apply the Newton-Raphson procedure once to f(x) to obtain a second approximation to β, giving your answer to 4 decimal places. **(4)**

(Total for question = 4 marks)

Question 5

(a) Use the binomial expansion to expand

$$(4 - 5x)^{-\frac{1}{2}} \qquad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2 giving each coefficient as a fully simplified fraction. **(4)**

$$f(x) = \frac{2 + kx}{\sqrt{4 - 5x}} \qquad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of f(x), in ascending powers of x, is

$$1 + \frac{3}{10}x + mx^2 + \dots \qquad \text{where } m \text{ is a constant}$$

(b) find the value of k, **(2)**

(c) find the value of m. **(2)**

(Total for question = 8 marks)

Question 6

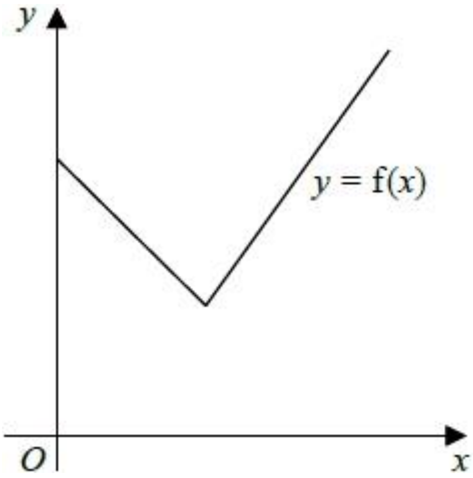

Figure 1

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2|3 - x| + 5 \qquad x \geqslant 0$$

(a) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(b) state the set of possible values for k. **(2)**

(Total for question = 5 marks)

Question 7

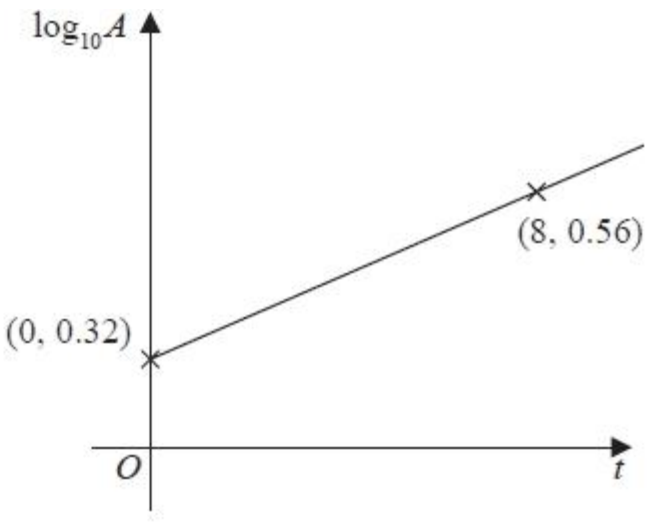

Figure 1

The growth of duckweed on a pond is being studied.

The surface area of the pond covered by duckweed, A m^2, at a time t days after the start of the study is modelled by the equation

$$A = pq^t \qquad \text{where } p \text{ and } q \text{ are positive constants}$$

Figure 1 shows the linear relationship between $\log_{10}A$ and t.
The points (0, 0.32) and (8, 0.56) lie on the line as shown.
(a) Find, to 3 decimal places, the value of p and the value of q. **(4)**

Using the model with the values of p and q found in part (a),

(b) find the rate of increase of the surface area of the pond covered by duckweed, in m^2 / day, exactly 6 days after the start of the study.
Give your answer to 2 decimal places. **(3)**

(Total for question = 7 marks)

Question 8

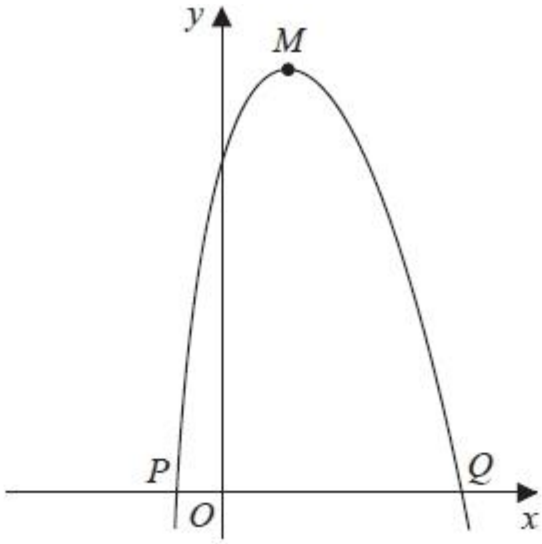

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 6\ln(2x+3) - \frac{1}{2}x^2 + 4 \qquad x > -\frac{3}{2}$$

The curve cuts the negative x-axis at the point P, as shown in Figure 1.

(a) Show that the x coordinate of P lies in the interval [–1.25, –1.2] **(2)**

The curve cuts the positive x-axis at the point Q, also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12\ln(2x_n + 3) + 8} \quad \text{with} \quad x_1 = 6$$

(b) (i) find, to 4 decimal places, the value of x^2

(ii) find, by continued iteration, the x coordinate of Q. Give your answer to 4 decimal places.

(3)

The curve has a maximum turning point at M, as shown in Figure 1.

(c) Using calculus and showing each stage of your working, find the x coordinate of M. **(4)**

(Total for question = 9 marks)

Question 9

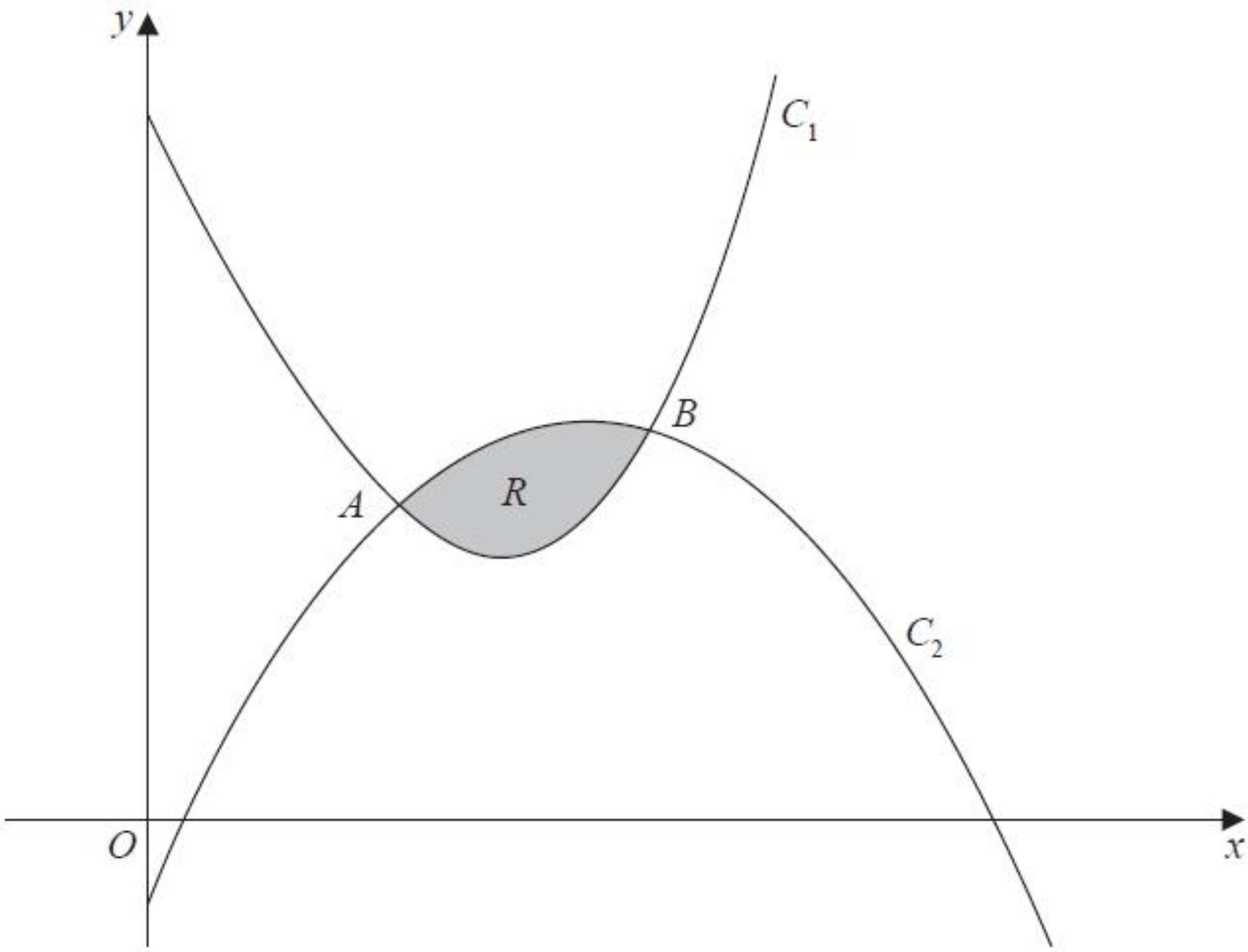

Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2 with equations

$$C_1 : y = x^3 - 6x + 9 \qquad x \geqslant 0$$
$$C_2 : y = -2x^2 + 7x - 1 \qquad x \geqslant 0$$

The curves C_1 and C_2 intersect at the points A and B as shown in Figure 1.

The point A has coordinates (1, 4).

Using algebra and showing all steps of your working,

(a) find the coordinates of the point B. **(4)**

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R. **(5)**

(Total for question = 9 marks)

Question 10

The growth of a weed on the surface of a pond is being studied.

The surface area of the pond covered by the weed, A m^2, is modelled by the equation

$$A = \frac{80pe^{0.15t}}{pe^{0.15t} + 4}$$

where p is a positive constant and t is the number of days after the start of the study.

Given that

- 30 m^2 of the surface of the pond was covered by the weed at the start of the study
- 50 m^2 of the surface of the pond was covered by the weed T days after the start of the study

(a) show that $p = 2.4$ **(2)**

(b) find the value of T, giving your answer to one decimal place.

(*Solutions relying entirely on graphical or numerical methods are not acceptable*.) **(4)**

The weed grows until it covers the surface of the pond.

(c) Find, according to the model, the maximum possible surface area of the pond. **(1)**

(Total for question = 7 marks)

Question 11

(a) Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians, to 3 decimal places. **(3)**

The function g is defined by

$$g(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \qquad \theta > 0$$

Find

(b) (i) the minimum value of $g(\theta)$

(ii) the smallest value of θ at which the minimum value occurs. **(3)**

The function h is defined by

$$h(\beta) = 10 - (12\sin\beta - 5\cos\beta)^2$$

(c) Find the range of h. **(2)**

(Total for question = 8 marks)

Question 12

$$y = \frac{2 + 3\sin x}{\cos x + \sin x}$$

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a\tan x + b\sec x + c}{\sec x + 2\sin x}$$

where a, b and c are integers to be found. **(6)**

(Total for question = 6 marks)

Question 13

Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at time t is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \lambda(120 - \theta) \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off. **(3)**

(Total for question = 11 marks)

Question 14

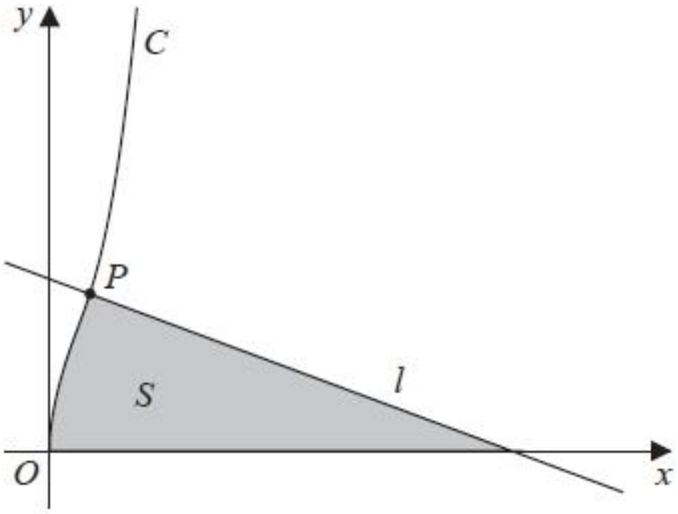

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve *C* has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point *P* with parameter $t = \dfrac{\pi}{4}$ lies on *C*.

The line *l* is the normal to *C* at *P*, as shown in Figure 3.

(a) Show, using calculus, that an equation for *l* is

$$8y + 2x = 17$$ **(5)**

The region *S*, shown shaded in Figure 3, is bounded by *C*, *l* and the *x*-axis.

(b) Find, using calculus, the exact area of *S*. **(6)**

(Total for question = 11 marks)

TOTAL FOR PAPER IS 100 MARKS